Spin Hall Effect and Optical Detection in Semiconductor Quantum Wells

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Outline

- **Introduction**
- **Nonlinear Rashba model and Spin Relaxation**
- **Spin Hall Effect in narrow band-gap QW**
- **Directly optical detection of pure spin current**
- **Conlusions**

Collaborators

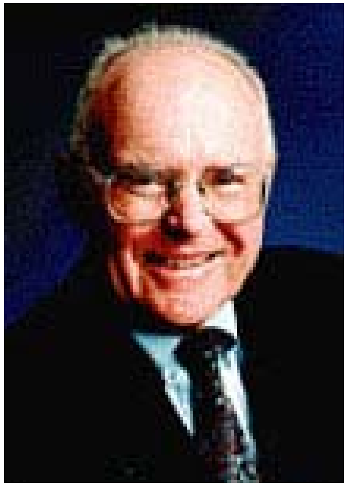
W. Yang, J. T. Liu, Z. Zhang, J. Li (SKLSM, IOS)

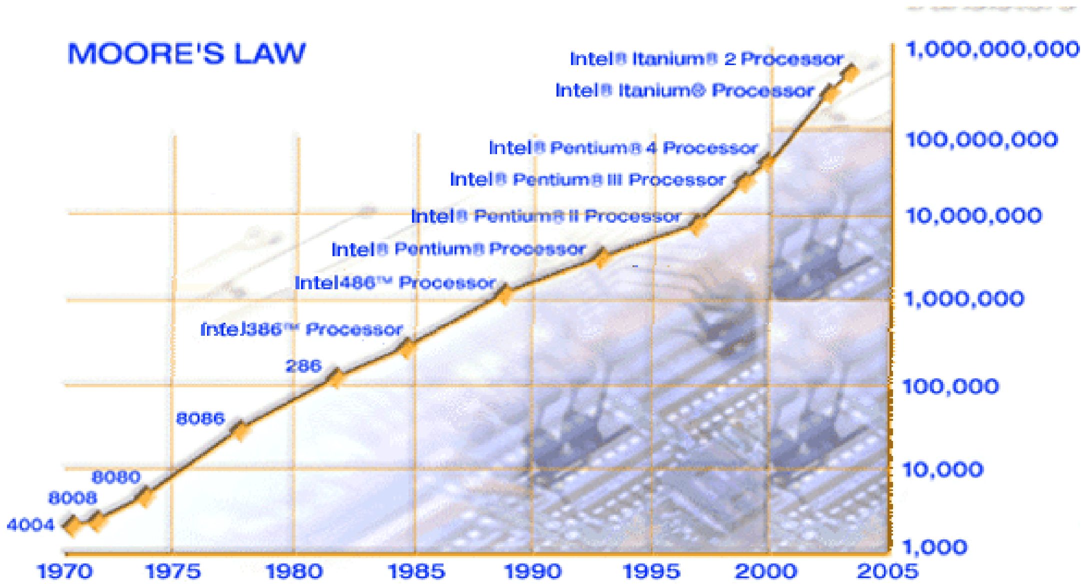
Prof. S. C. Zhang (Stanford University)

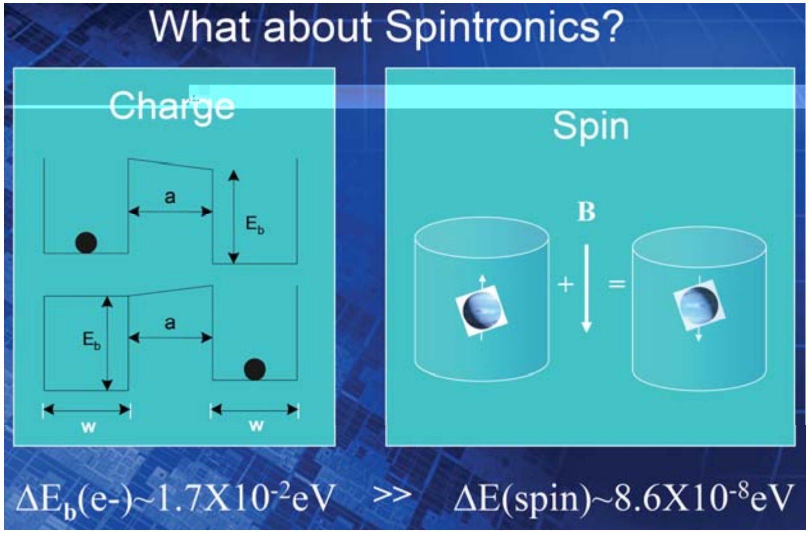
Acknowledgments: NSFC Grant No. 60525405 and

The innovation project of knowledge from CAS

Introduction...







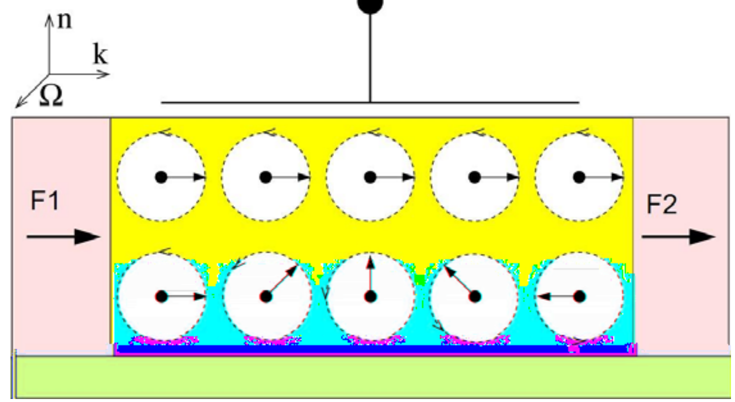
1、Power consumption---heat
2、Quantum effect

Possible solution: Spintronics
Limits: Room temperature
spin injection and relaxation

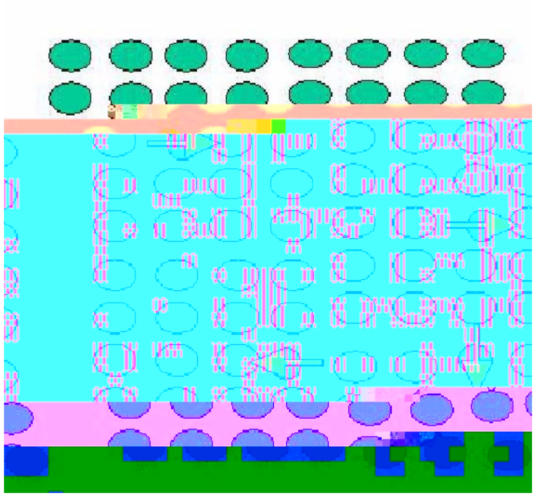
Manipulation of spin in Semiconductors

1, sp-d exchange interaction:
DMS, e.g., GaMnAs
2, Laser pulse:
Optical Stark effect
3, Relativistic effect:
Spin-orbit interaction

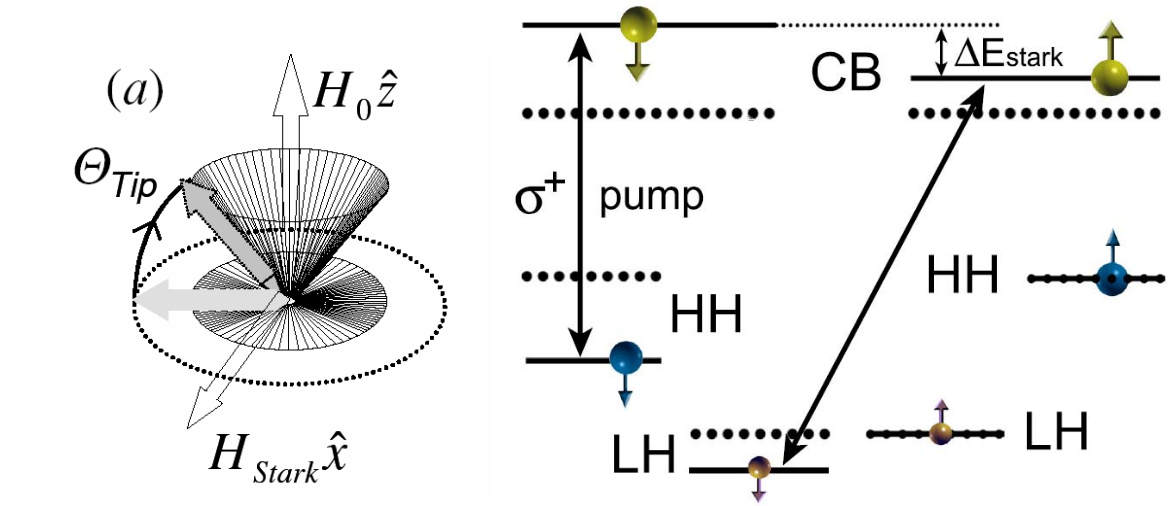
Datta-Das transistor, APL(1990)



DMS

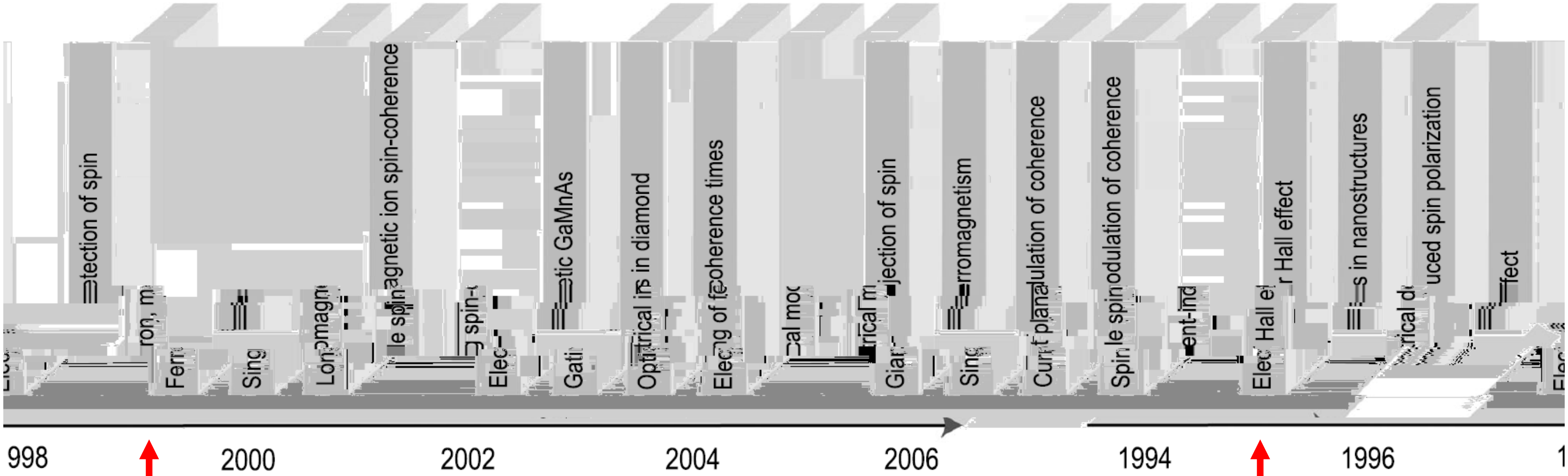


Optical Stark effect



Progress of spintronics

Milestones



From D. D. Awaschlom

Spin Hall Effect

Spin coherence

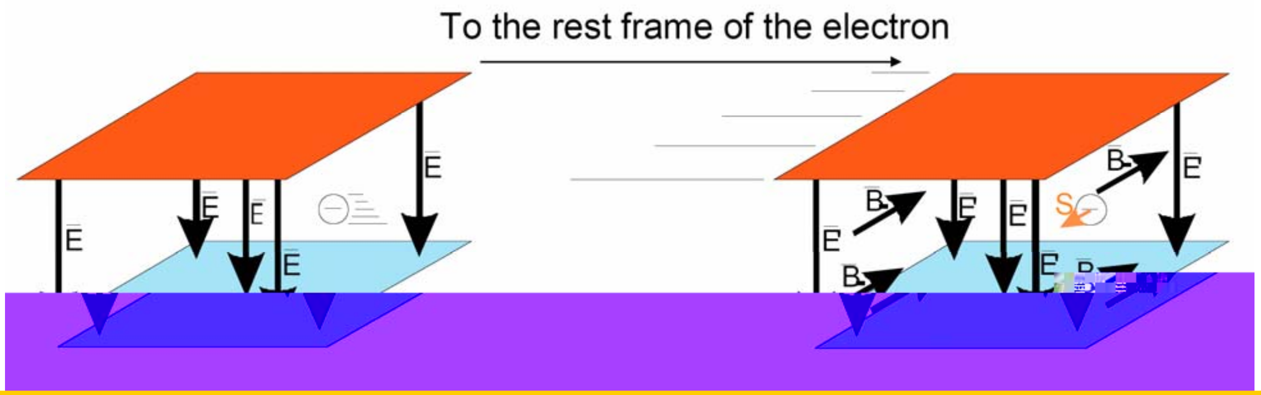
Detection of spin

The spin-orbit interaction

Origin of Rashba spin-orbit coupling

Relativistic effect

$$\vec{B}_{ef} = -\frac{1}{c^2}\vec{v}\times\vec{E}$$



$$H_{\text{Dirac}} \mid \begin{pmatrix} mc^2\, 2\, V(\mathbf{r}) & c\omega\bullet\phi \\ c\omega\bullet\phi & 4mc^2\, 2\, V(\mathbf{r}) \end{pmatrix} \Downarrow H_{\text{eff}} \mid e^S H_{\text{Dirac}} e^{4S} \mid \begin{pmatrix} H & 0 \\ 0 & \bar{H} \end{pmatrix}$$

orbital **spin**

$$H \mid mc^2\, 2\, V(\mathbf{r})\, 2\, \frac{\phi^2}{2m}\, 2\, \sigma_B \omega\bullet\mathbf{B}\, 2\, \frac{\hbar^2}{4m^2c^2}(\subseteq V\, \Delta\, \phi) \notin \omega\, 2\, \frac{\hbar^2}{8m^2c^2}\subseteq^2 V\, 4\, \frac{1}{8m^3c^2}(\omega\bullet\phi)^4$$

Symmetry

$$|\mathbf{x}\rangle \Downarrow |\mathbf{x}\rangle^{\phi} \;|\; |4\mathbf{x}\rangle,$$
$$|\mathbf{p}\rangle \Downarrow |\mathbf{p}\rangle^{\phi} \;|\; |4\mathbf{p}\rangle,$$
$$|j,m\rangle \Downarrow |j,m\rangle^{\phi} \;|\; |j,m\rangle$$

$$|\mathbf{x}\rangle \Downarrow |\mathbf{x}\rangle^{\chi} \;|\; |\mathbf{x}\rangle,$$
$$|\mathbf{p}\rangle \Downarrow |\mathbf{p}\rangle^{\chi} \;|\; |4\mathbf{p}\rangle,$$
$$|j,m\rangle \Downarrow |j,m\rangle^{\chi} \;|\; D(\mathbf{e}_y,4\phi)|j,m\rangle \nabla |j,4m\rangle,$$

Space inversion

$$H|n\mathbf{k}s\rangle \;|\; E_{n\mathbf{k}s}|n\mathbf{k}s\rangle$$

Time reversal

$$H|n\mathbf{k}s\rangle^{\phi} \;|\; E_{n\mathbf{k}s}|n\mathbf{k}s\rangle^{\phi}$$
$$|n\mathbf{k}s\rangle^{\phi} \;|\; |n,4\mathbf{k},s\rangle$$
$$\blacklozenge$$
$$E_{n\mathbf{k}s} \;|\; E_{n,4\mathbf{k},s}$$

With both symmetry

$$E_{n\mathbf{k}s} \;|\; E_{n,\mathbf{k},4s}$$

$$H|n\mathbf{k}s\rangle^{\chi} \;|\; E_{n\mathbf{k}s}|n\mathbf{k}s\rangle^{\chi}$$
$$|n\mathbf{k}s\rangle^{\chi} \nabla |n,4\mathbf{k},4s\rangle$$
$$\blacklozenge$$
$$E_{n\mathbf{k}s} \;|\; E_{n,4\mathbf{k},4s}$$

Spin splitting induced by the symmetry breaking

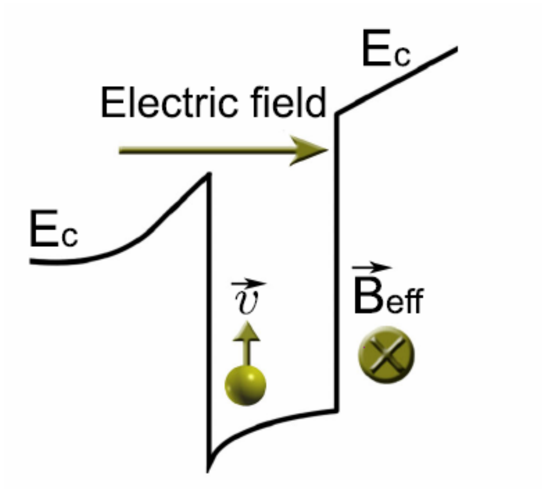
Spin-orbit interactions

E. I. Rashba, JETP 1961

G. Dresselahus, Phys Rev 1955

Rashba SOI

Structural asymmetry (SIA)



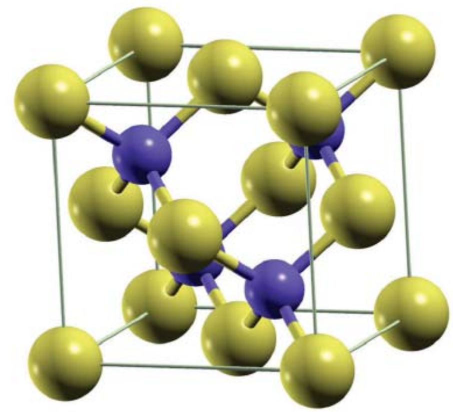
$$H_R = \alpha\left(\vec{\sigma}\times\vec{k}\right)\cdot\hat{z} = \alpha\left(\sigma_x k_y - \sigma_y k_x\right)$$

$$\vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$$

R. Winkler, *Spin-Orbit Coupling Effects in Two-Dimensional Electron and Hole System, Springer ,2003*

Dresselhaus SOI

Crystal inversion asymmetry(BIA)



$$H_D = \gamma\,\vec{\sigma}\cdot\vec{\kappa}$$

$$\kappa_x = k_y k_x k_y - k_z k_x k_z$$

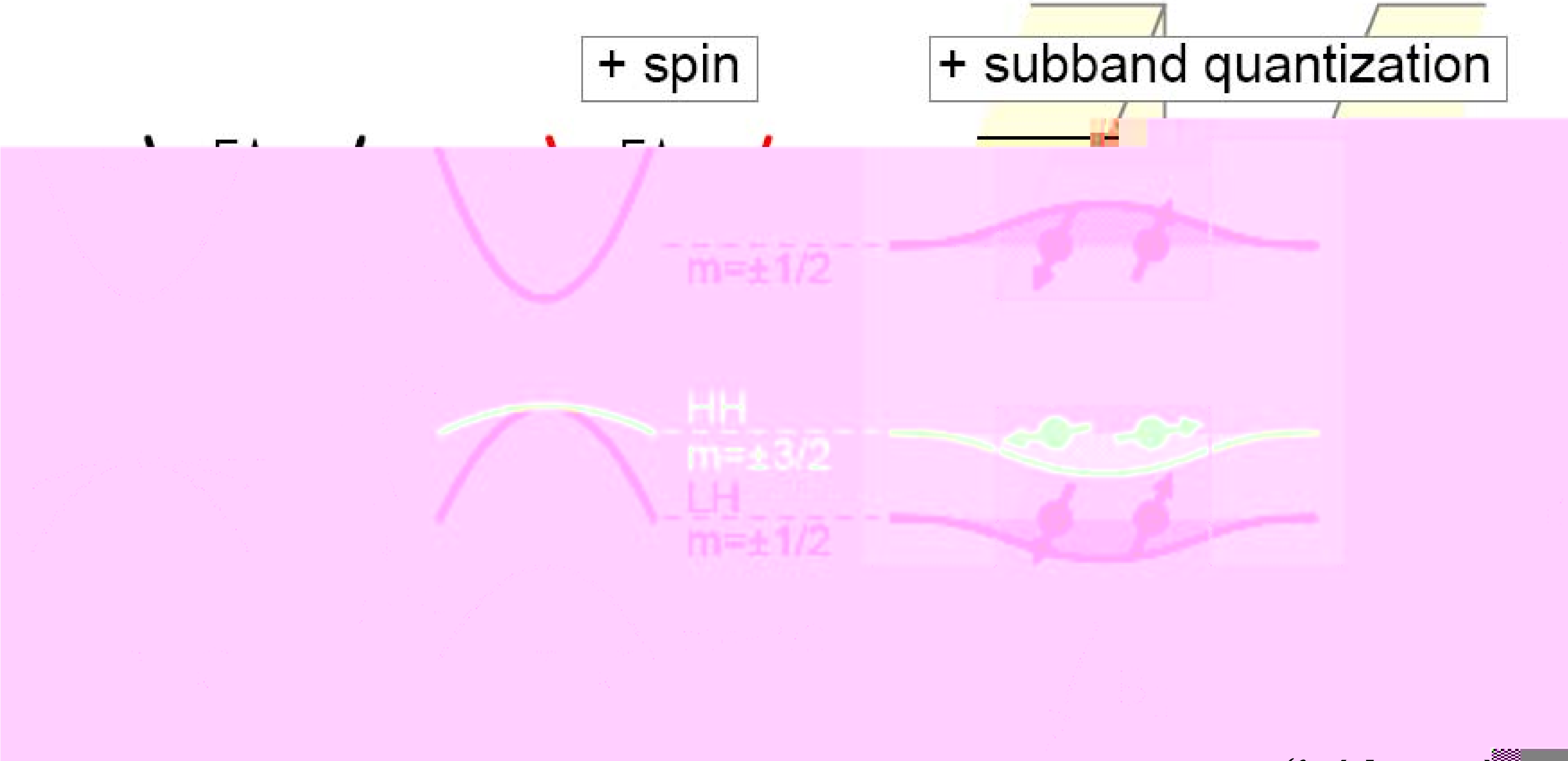
$$\kappa_y = k_z k_y k_z - k_x k_y k_x$$

$$\kappa_z = k_x k_z k_x - k_y k_z k_y$$

Linear Rashba model

Semiconductor quantum well

$$\mathcal{H}_{SO} \sim \sigma \cdot \mathbf{B}(\mathbf{k})$$

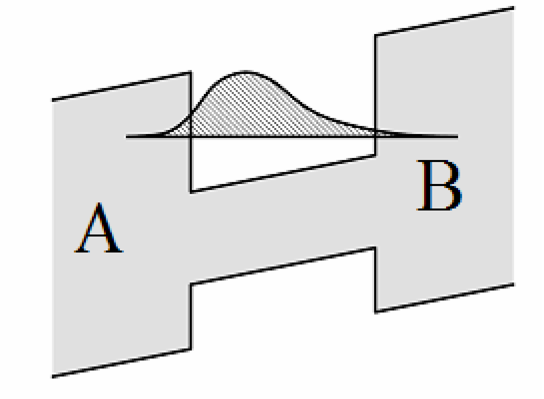


$$\mathbf{B}_R = \zeta\,(\,\mathbf{k} \times \mathbf{z}\,)$$

Linear Rashba model

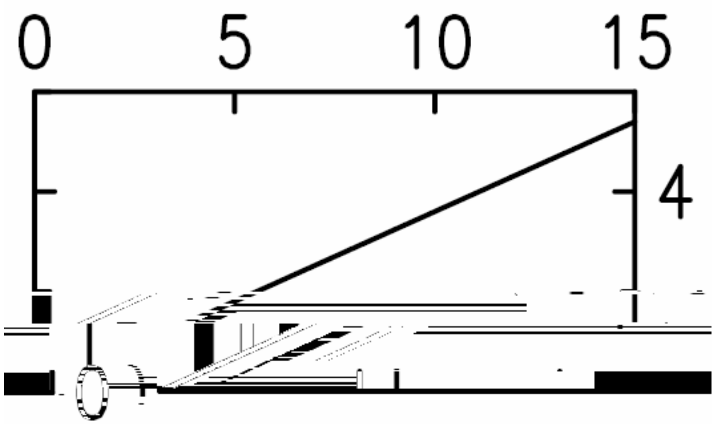
$$H_R = \alpha\left(\vec{\sigma}\times\vec{k}\right)\cdot\hat{z} = \alpha\left(\sigma_x k_y - \sigma_y k_x\right)$$

$$\vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$$



Spin splitting

$$\Delta E = 2\alpha k_{//}$$



R. Winkler and U. Rössler, PRB 48, 8918 (1993)

E. A. de Andrada e Silva et al., PRB 55, 16293 (1997)

R. Winkler and U. Rössler, PRB 62, 4245 (2000)

S. Lamari, PRB 64, 245340 (2001)

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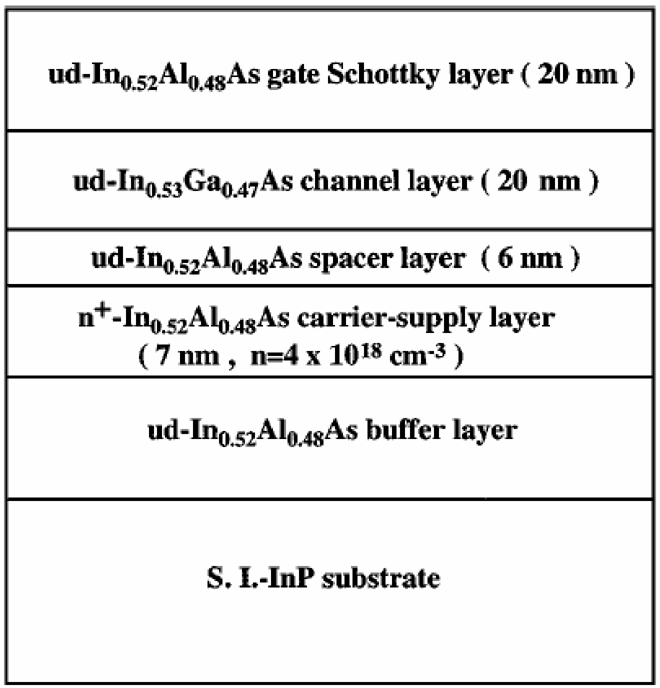
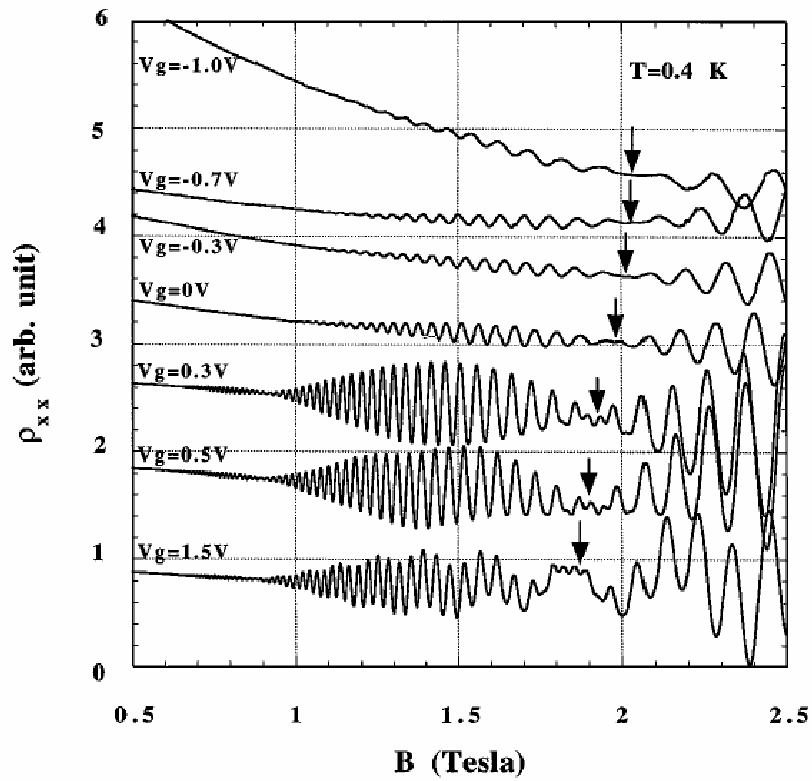
Experiment

Gate Control of Spin-Orbit Interaction in an Inverted $In_{0.53}Ga_{0.47}As/In_{0.52}Al_{0.48}As$ Heterostructure

Junsaku Nitta, Tatsushi Akazaki, and Hideaki Takayanagi
NTT Basic Research Laboratories, 3-1 Wakamiya, Morinosato, Atsugi-shi, Kanagawa 243-01, Japan

Takatomo Enoki
NTT System Electronics Laboratories, 3-1 Wakamiya, Morinosato, Atsugi-shi, Kanagawa 243-01, Japan
(Received 23 July 1996)





Effective-mass theory

Effective mass k·p theory

the Bloch functions $e^{i\boldsymbol{k}\cdot\boldsymbol{r}} u_{\nu\boldsymbol{k}}(\boldsymbol{r}) \equiv e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \langle \boldsymbol{r} | \nu\boldsymbol{k} \rangle$

$$\left[\frac{p^2}{2m_0} + V_0(\boldsymbol{r})\right] e^{i\boldsymbol{k}\cdot\boldsymbol{r}} u_{\nu\boldsymbol{k}}(\boldsymbol{r}) = E_\nu(\boldsymbol{k})\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}} u_{\nu\boldsymbol{k}}(\boldsymbol{r}) \,.$$

$$\left[\frac{p^2}{2m_0} + V_0 + \frac{\hbar^2 k^2}{2m_0} + \frac{\hbar}{m_0}\boldsymbol{k}\cdot\boldsymbol{p}\right] |\nu\boldsymbol{k}\rangle = E_\nu(\boldsymbol{k})\, |\nu\boldsymbol{k}\rangle$$

Considering SOI

$$[\frac{p^2}{2m_0} 2 V_0 2 \frac{k^2}{2m_0} 2 \frac{\hbar}{m_0} k \notin \phi \, 2 \frac{\hbar}{4m^2{}_0 c^2} p \notin (\omega \Delta \subseteq V)] \left| nk \right\rangle \mid E_n \left| nk \right\rangle,$$

$$where \quad \phi \mid p \, 2 \frac{\hbar}{4m^2{}_0 c^2} \omega \Delta \subseteq V_0,$$

$$\left| nk \right\rangle \mid \underset{\tau,\omega \mid \Rightarrow \Leftrightarrow}{—} c_{\tau,\omega'} \left| nk_0 \right\rangle \cup \left| \omega' \right\rangle$$

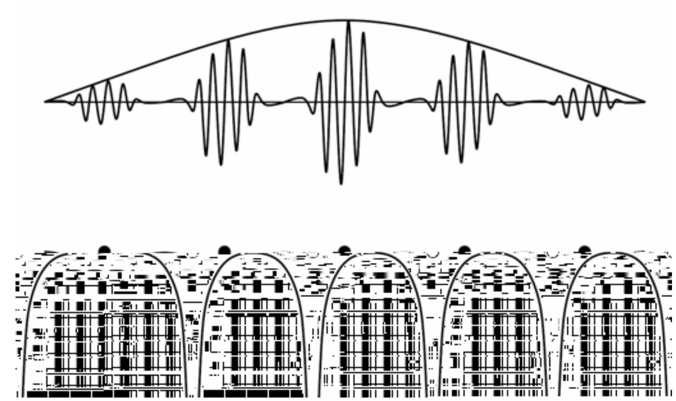
Effective-mass theory

Effective mass k·p theory

$$—_{\tau'\omega'}\left[\left(E_{\tau'}(0)\;2\;\frac{k^{2}}{2m_{0}}\right\}\iota_{\tau\tau'}\,\iota_{\omega\omega'}\;2\;\frac{\hbar}{m_{0}}\vec{k}\notin\vec{P}_{\tau\tau'\omega\omega'}\;2\;\div_{\tau\tau'\omega\omega'}\right] c_{n\tau'\omega'}(k)\mid E_{n}(k)c_{n\tau'\omega'}(k),$$

$$where\quad P_{\tau\tau'\omega\omega'}\mid\langle\tau\omega|\phi|\tau'\omega'\rangle,$$

$$\div_{\tau\tau'\omega\omega'}\mid\frac{\hbar}{4m_{0}^{2}c^{2}}\langle\tau\omega|p\notin(\omega\Delta\subseteq V)|\tau'\omega'\rangle,$$

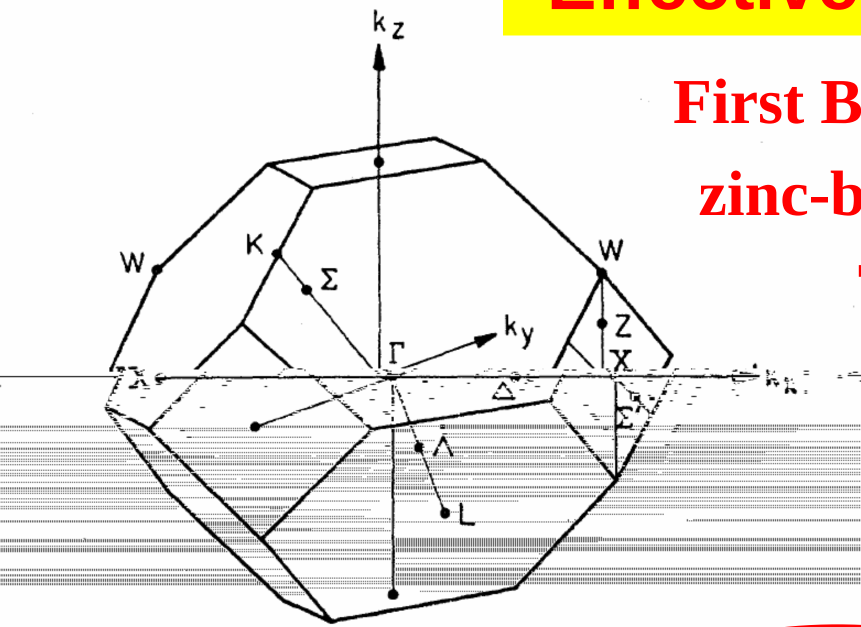


The envelope function approximation

$$\Phi(r)\mid—_{\tau'\omega'}\ldots_{\tau'\omega'}(r)u_{\tau'0}(r)|\omega'\rangle$$

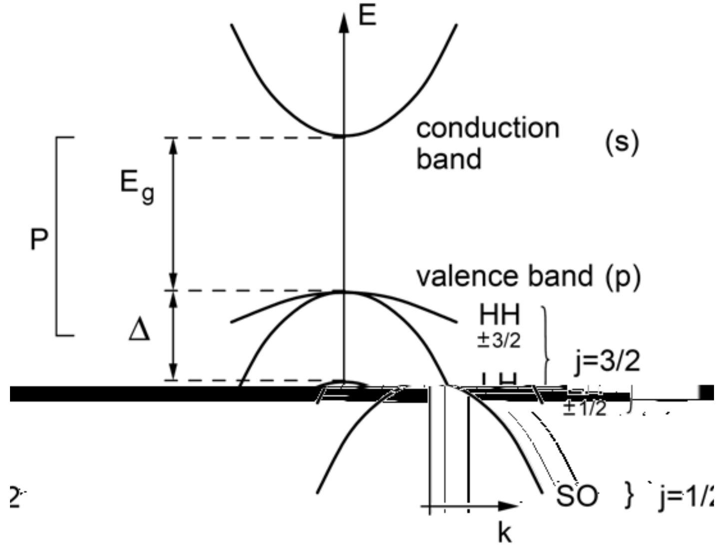
$$—_{\tau'\omega'}\left[\left(E_{\tau'}(0)\;2\;\frac{k^{2}}{2m_{0}}\right\}\iota_{\tau\tau'}\,\iota_{\omega\omega'}\;2\;\frac{\hbar}{m_{0}}\vec{k}\notin\vec{P}_{\tau\tau'\omega\omega'}\;2\;\div_{\tau\tau'\omega\omega'}\right]\ldots_{\tau'\omega'}(r)\mid E_{n}(k)\ldots_{\tau\omega}(r),$$

Effective-mass theory



First Brillouin Zone of zinc-blende structure

T_d group



Basis

$\Phi_k \mid u_k(r)e^{ikr}$

$\Phi_1 \; || \, S \Rightarrow \}$

$\Phi_2 \; || \, S \Leftrightarrow \}$

$\Phi_3 \; || 3/2, 3/2 \, \} \, | \, 1/\sqrt{2} \, | \, X \; 2 \; iY, \Rightarrow \}$

$\Phi_4 \; || 3/2, 1/2 \, \} \, | \, i/\sqrt{6}[| \, X \; 2 \; iY, \Leftrightarrow \} \; 42 \, | \, Z, \Rightarrow \}]$

$\Phi_5 \; || 3/2, 41/2 \, \} \, | \, i/\sqrt{6}[| \, X \; 4 \; iY, \Rightarrow \} \; 22 \, | \, Z, \Leftrightarrow \}]$

$\Phi_6 \; || 3/2, 43/2 \, \} \, | \, 1/\sqrt{2} \, | \, X \; 4 \; iY, \Leftrightarrow \}$

$\Phi_7 \; || 1/2, 1/2 \, \} \, | \, 1/\sqrt{3}[| \, X \; 2 \; iY, \Leftrightarrow \} \; 2 \, | \, Z, \Rightarrow \}]$

$\Phi_8 \; || 1/2, 41/2 \, \} \, | \, i/\sqrt{3}[4 \, | \, X \; 4 \; iY, \Rightarrow \} \; 4 \, | \, Z, \Leftrightarrow \}]$

$$H \mid p^2/2m_0 \; 2 \; V \; 2 \; \frac{\hbar}{4m^2c^2}(\subseteq V \, \Delta \, p) \lceil \omega$$

H_{SO}

Origin of Rashba spin-orbit coupling

$$H_k = \begin{bmatrix} kA_ck & 0 & iP_0k_+/\sqrt{2} & \sqrt{2/3}P_0k_z & iP_0k_-/\sqrt{6} & 0 & iP_0k_z/\sqrt{3} & P_0k_-/\sqrt{3} \\ & kA_ck & 0 & -P_0k_+/\sqrt{6} & i\sqrt{2/3}P_0k_z & -P_0k_-/\sqrt{2} & iP_0k_+/\sqrt{3} & -P_0k_z/\sqrt{3} \\ & & P+Q & -L-i\sqrt{6}S & M & 0 & iL/\sqrt{2}+\sqrt{3}S & -i\sqrt{2}M \\ & & & P-Q & -2\sqrt{2}iS & M & -i\sqrt{2}Q & i\sqrt{3/2}L-S \\ & & & & P-Q & -L-i\sqrt{6}S & -i\sqrt{3/2}L^+ + S^+ & -i\sqrt{2}Q \\ & & & & & P-Q & -i\sqrt{2}M^+ & -iL^+/\sqrt{2}-\sqrt{3}S^+ \\ & & & & & & P & 2\sqrt{2}iS \\ & & & & & & & P \end{bmatrix}$$

$$H_{\rm eff}(\mathbf{k}_{//}) = E_c(z)+V(z)+\mathbf{k}\frac{\hbar^2}{2m^*(z)}\mathbf{k}+\alpha_0(z)(\mathbf{k}_{//}\times\mathbf{e}_z)\cdot\sigma,$$

$$\frac{m_0}{m^*(z)} = \gamma_c + \frac{2E_p}{3U_{\rm lh}} + \frac{E_p}{3U_{\rm so}},$$

[illegible]

$V(z) \mid E_p \lceil (1/U_{lh}\ 4\ 1/U_{so});$

$U_{lh} \mid E\ 4\ H_{lh};$

$U_{so} \mid E\ 4\ H_{so};$

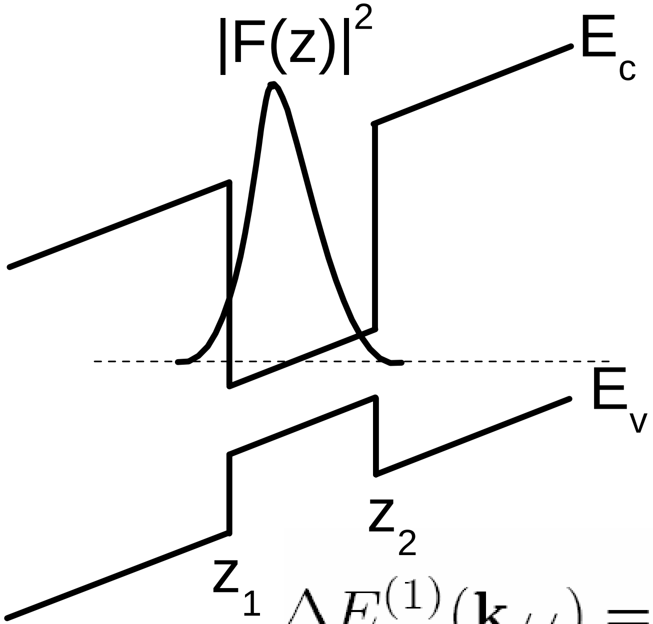
$H_{lh} \mid E_v\ 4\ V\ 2\ P\ 2\ Q;$

$H_{so} \mid E_v\ 4 \div 2\ V\ 2\ P.$

PHYS. REV. B 73, 113303 (2006)

W. Yang and Kai Chang

Zero-field Rashba spin splitting



$$\Delta E_n(\mathbf{k}_{//}) = \Delta E_n^{(1)}(\mathbf{k}_{//}) + \Delta E_n^{(2)}(\mathbf{k}_{//}),$$

$$\Delta E_n^{(1)}(\mathbf{k}_{//}) = \frac{\hbar^2}{3m_0} k_{//} \sum_j |F_n(z_j)|^2 \left[\gamma(z_j^+) - \gamma(z_j^-)\right],$$

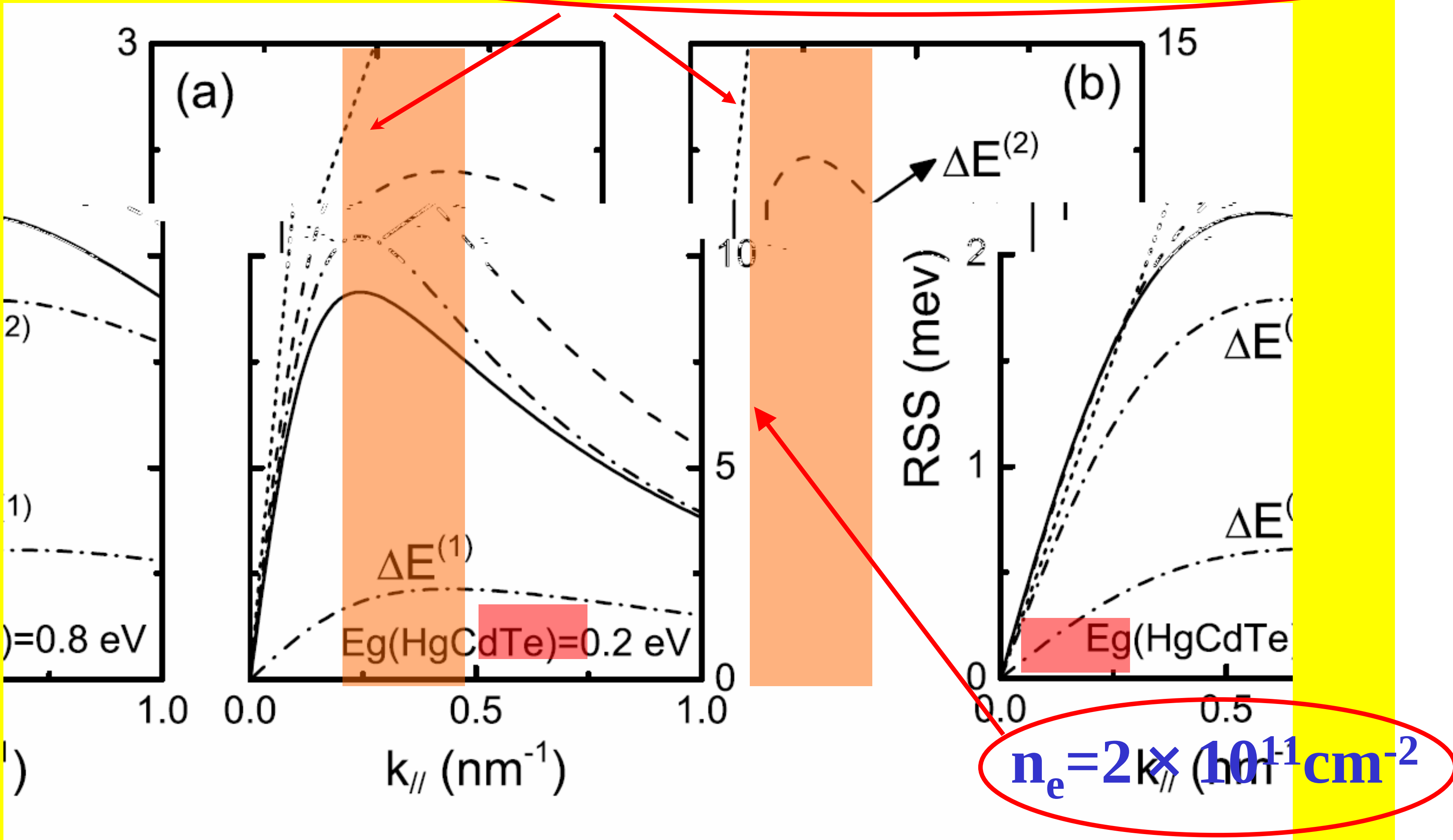
$$= \frac{\hbar^2}{3m_0} E_n e F k_{//} \int dz\, |F_n(z)|^2 \left(U_{lh}^{-2} - U_{so}^{-2}\right), \quad \Delta E_n^{(2)}(\mathbf{k}_{//})$$

Interface contribution

Interband coupling

(a) HgCdTe QW

Linear Rashba model

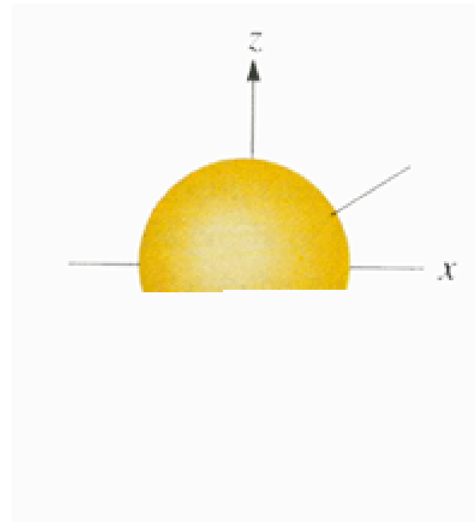


lowest subband as a function of $k_{//}$ for
8 and (b) 0.2 eV. Solid lines denote the
lines are obtained from Eqs. (3) and (5)

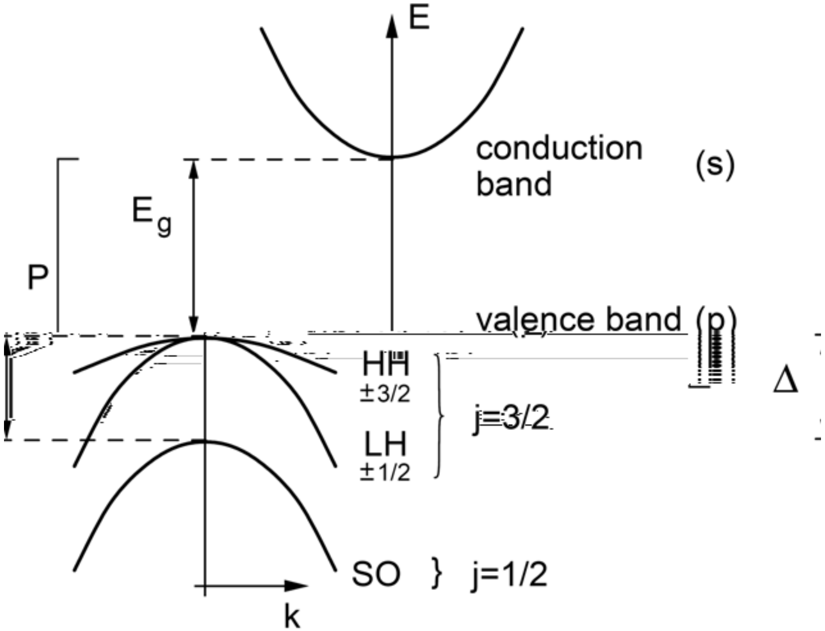
FIG. 1: RSS of the
E_g(HgCdTe)= (a) 0.
exact results, dashed

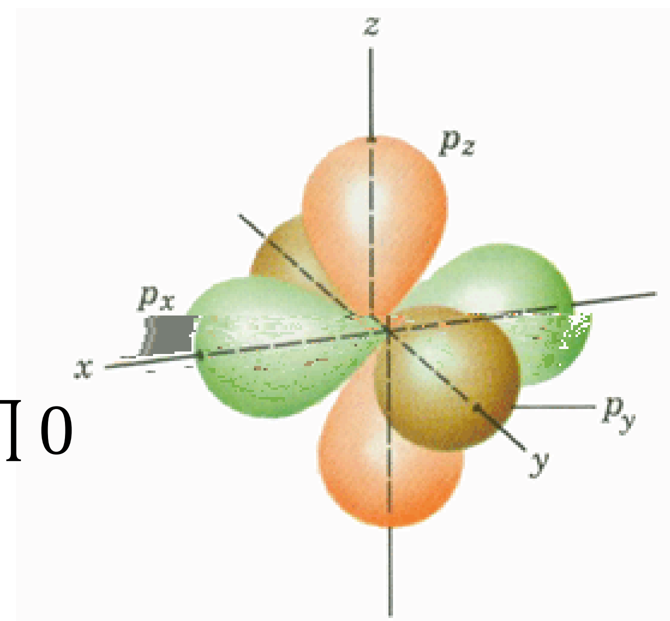
Origin of Rashba spin splitting

$$H_{SO} = \frac{\hbar}{4m^2c^2}(\nabla V \times p)\cdot\sigma$$



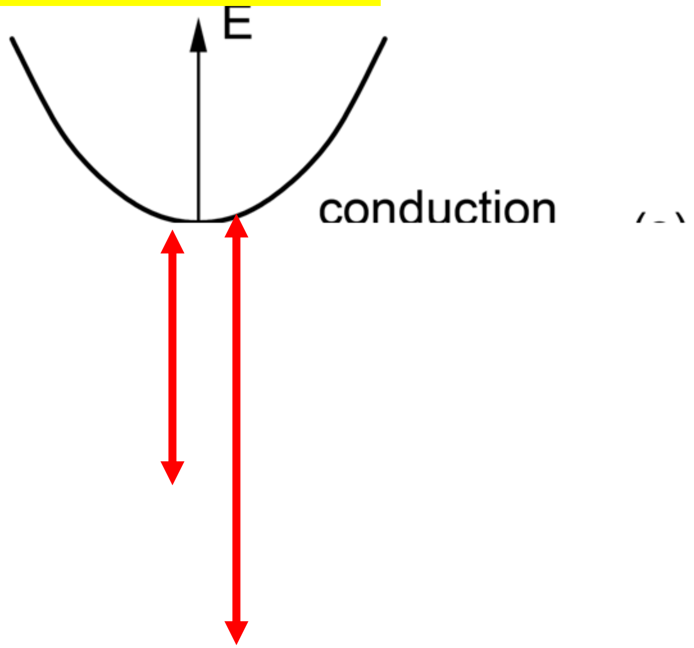
For s-like conduction band $\langle H_{SO} \rangle_S = 0$





For p-like conduction band $\langle H_{SO} \rangle_P \neq 0$

Origin of Rashba spin splitting



Large ÷

Heavy atom

+

Interband coupling

Narrow bandgap

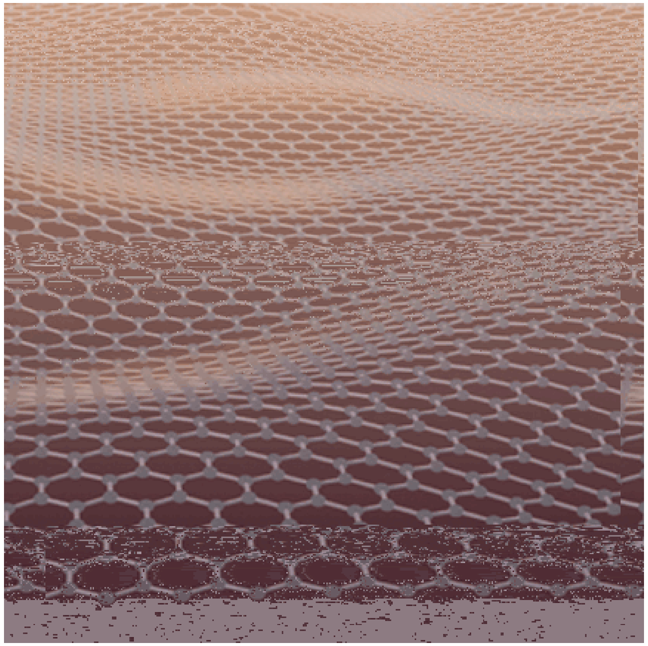
Large Rashba ζ

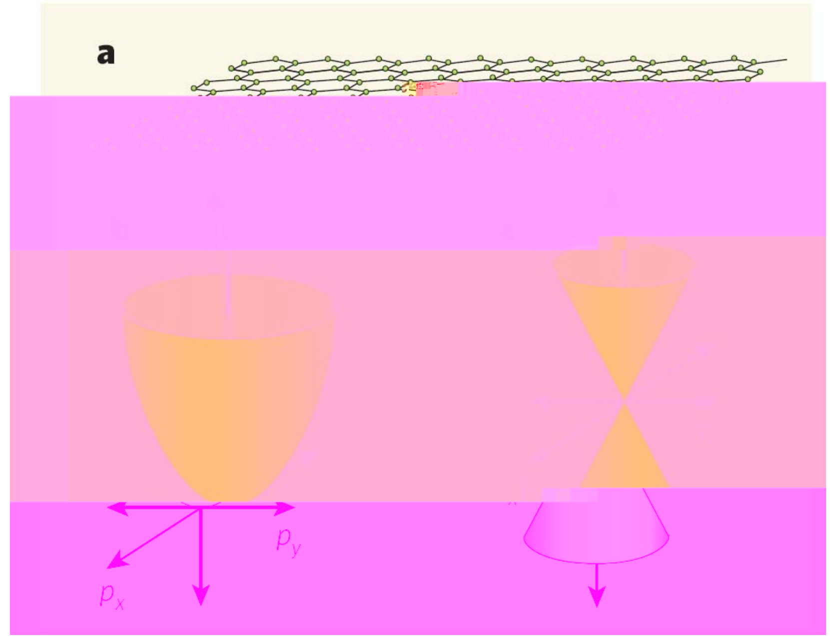
$$= \frac{\hbar^2}{3m_0} E_p e F k_{//} \int dz\, |F_n(z)|^2 \left(U_{lh}^{-2} - U_{so}^{-2}\right).\ \Delta E_n^{(2)}(\mathbf{k}_{//})$$

The formulism can explain why large RSS in HgTe, but small in graphene!

Origin of the nonlinear behavior of RSOI!

SOI in Graphene





The spin-orbit interaction near the K point

1, C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005)(**Q**

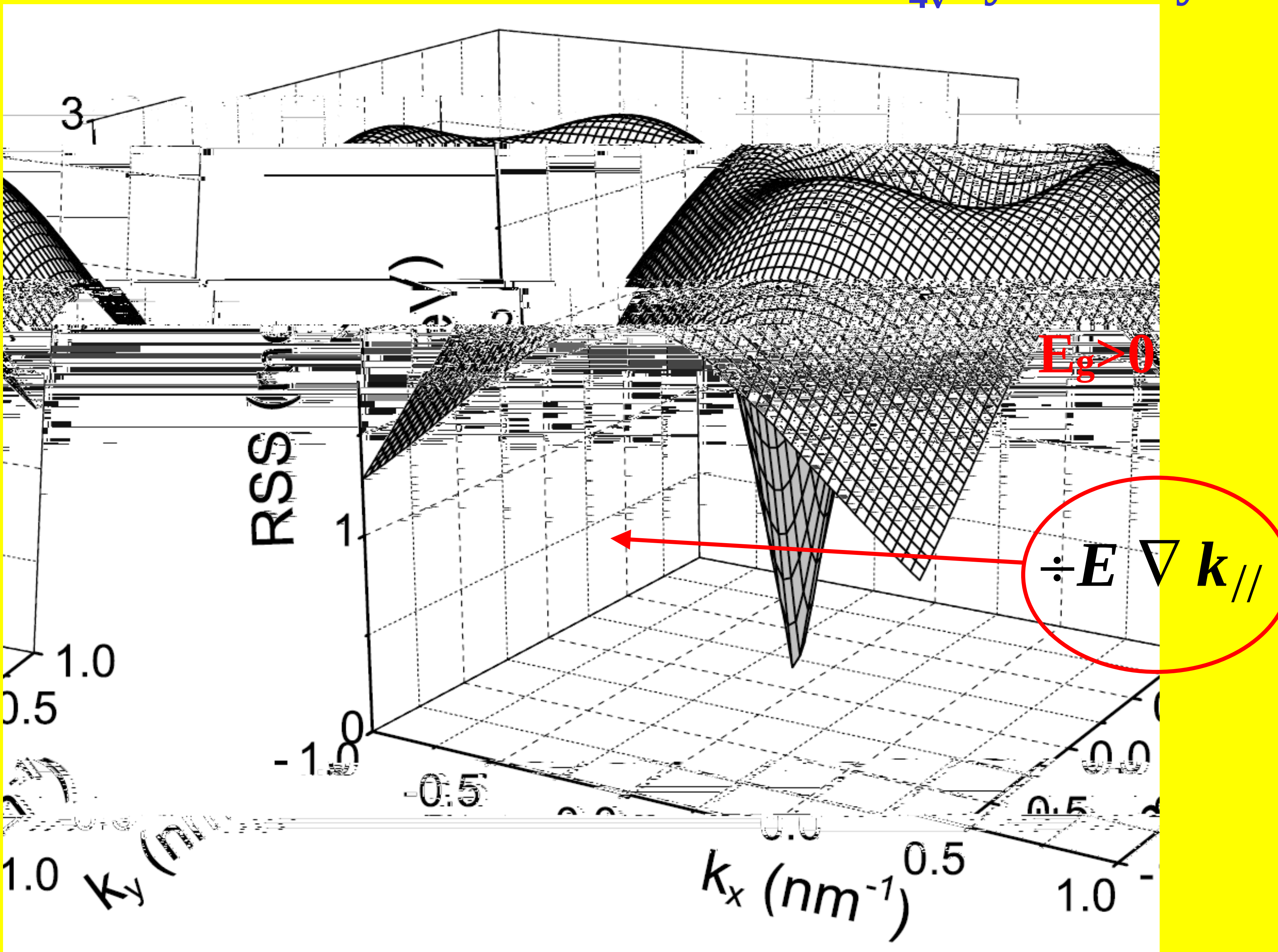
2, Phys. Rev. B 74, 165310 (2006) From MacDonald'group

ς_R –0.0111meV under F=50V/300nm –**1667kV/cm!**

SOI depends on the bandgap and the atomic mass!

Anisotropy of Rashba spin splitting

C_{4v} symmetry



Two-coefficient nonlinear Rashba model

$$\Delta E = 2\alpha k_{//}$$

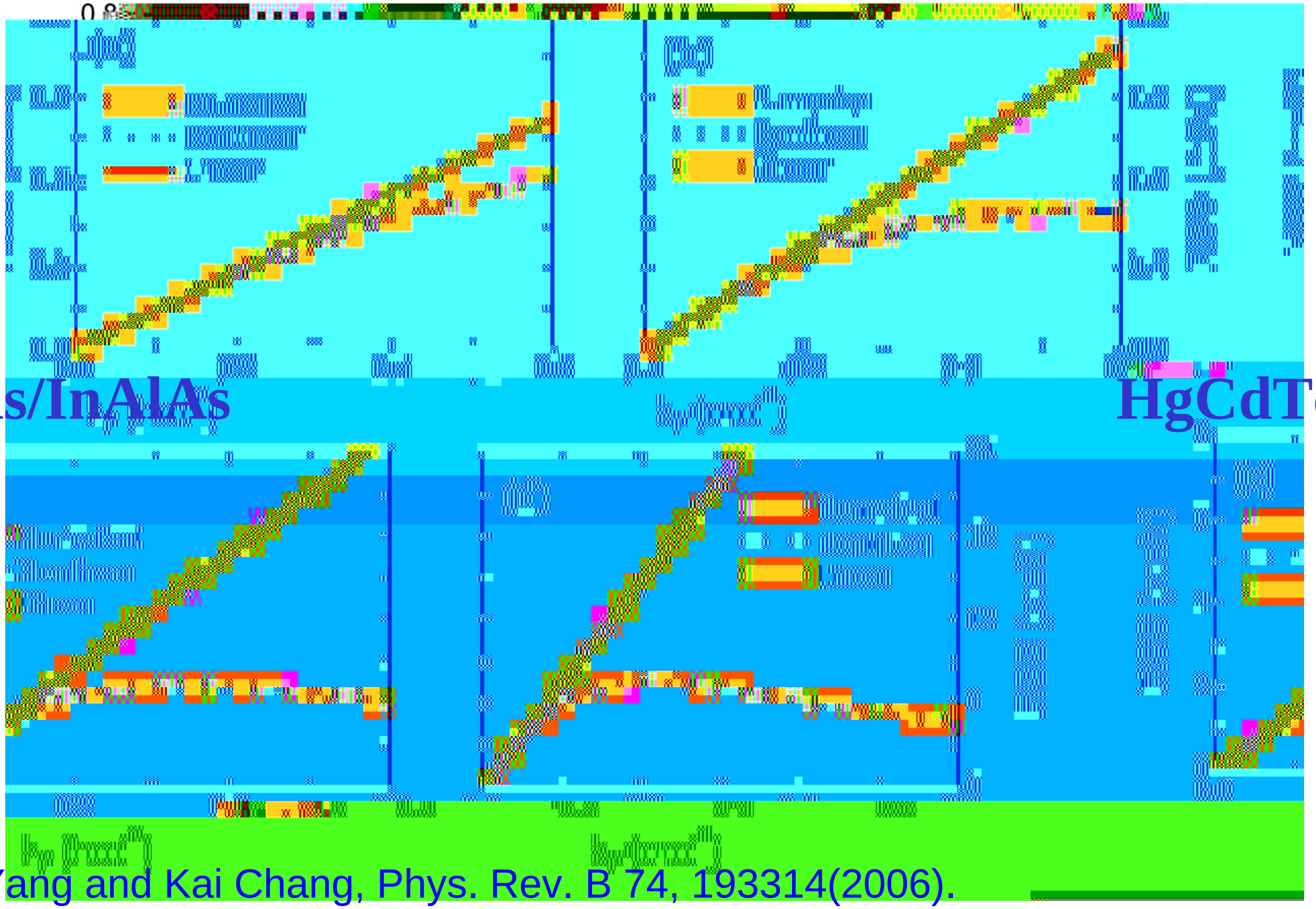
$$\Delta E = \frac{2\alpha k_{//}}{1+\beta k_{//}^2}$$

GaAs/AlGaAs

InGaAs/GaAs

InGaAs/InAlAs

HgCdTe/CdTe



W. Yang and Kai Chang, Phys. Rev. B 74, 193314(2006).

- **Many-body effect**

$$H\Phi(z) \mid E\Phi(z)$$

$$H \mid H_K \, 2 \, V_{conf}(z) \, 2 \, V_H(z)$$

$$\langle \mathcal{E}_z^{\mathrm{ext}} \rangle = \frac{e}{\epsilon\epsilon_0}\left[N_{\mathrm{A}}(z_{\mathrm{d}} - \langle z \rangle) + \frac{N_{\mathrm{s}}}{2} \right]$$

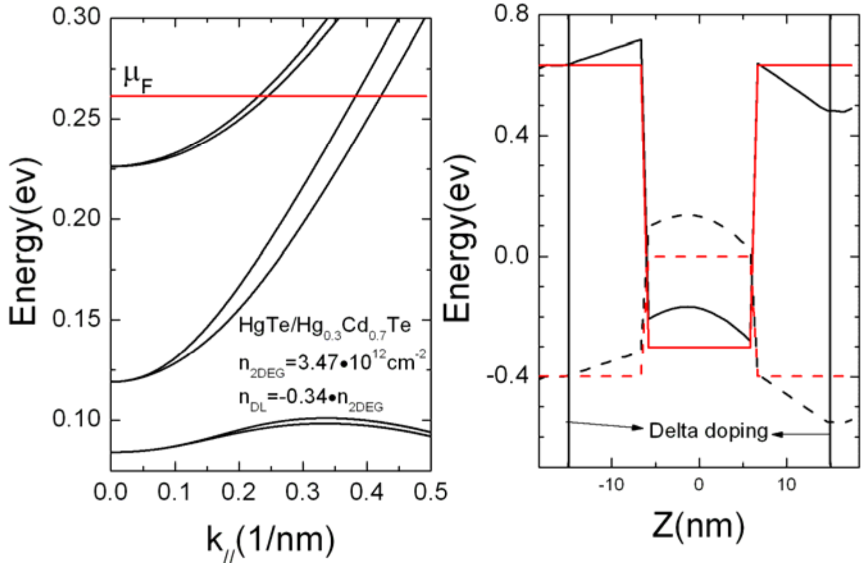
$$\mathcal{E}_z^{\mathrm{ext}}(z) = (1/e)\,\partial_z V_{\mathrm{H}}(z) = \frac{e}{\epsilon\epsilon_0}\left[N_{\mathrm{A}}(z_{\mathrm{d}} - z) + N_{\mathrm{s}} - \int_{-\infty}^{z} \mathrm{d}z' \, \rho(z') \right]$$

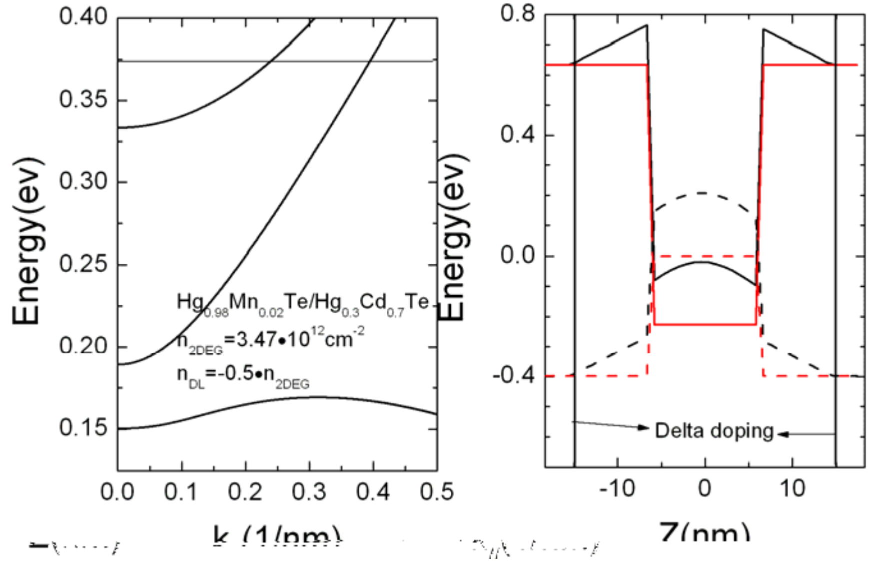
Self-consistent eight-band calculation including Hatree potential

We try to include the exchange-correlation interaction in the next step.

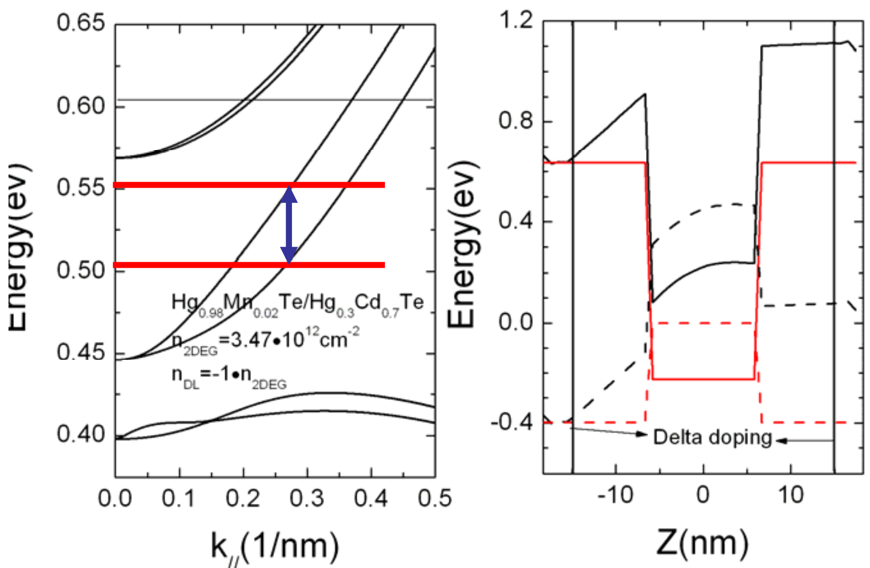
• Delta Doping In Quantum Well

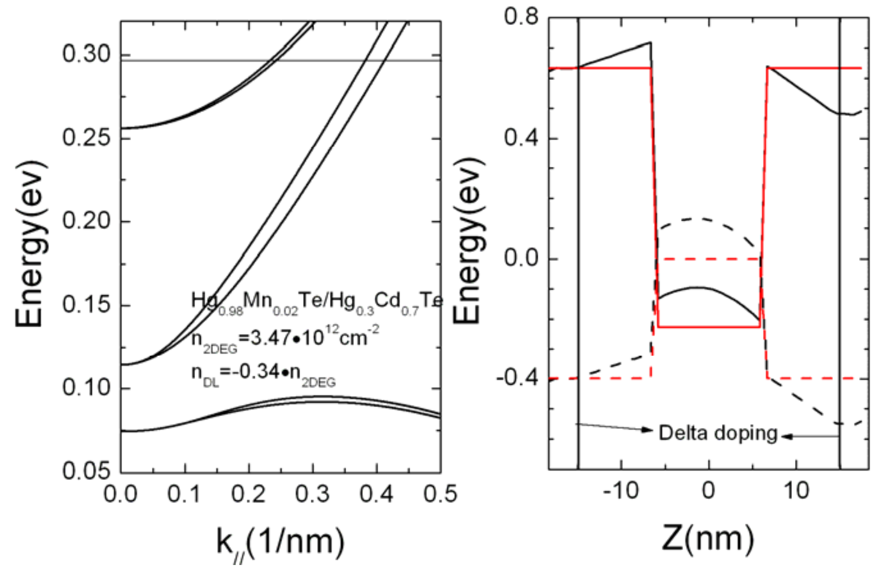
Lz=12.2nm,
Ne=3.47 × 10^{12}/cm^2





~50meV, Giant splitting!!!

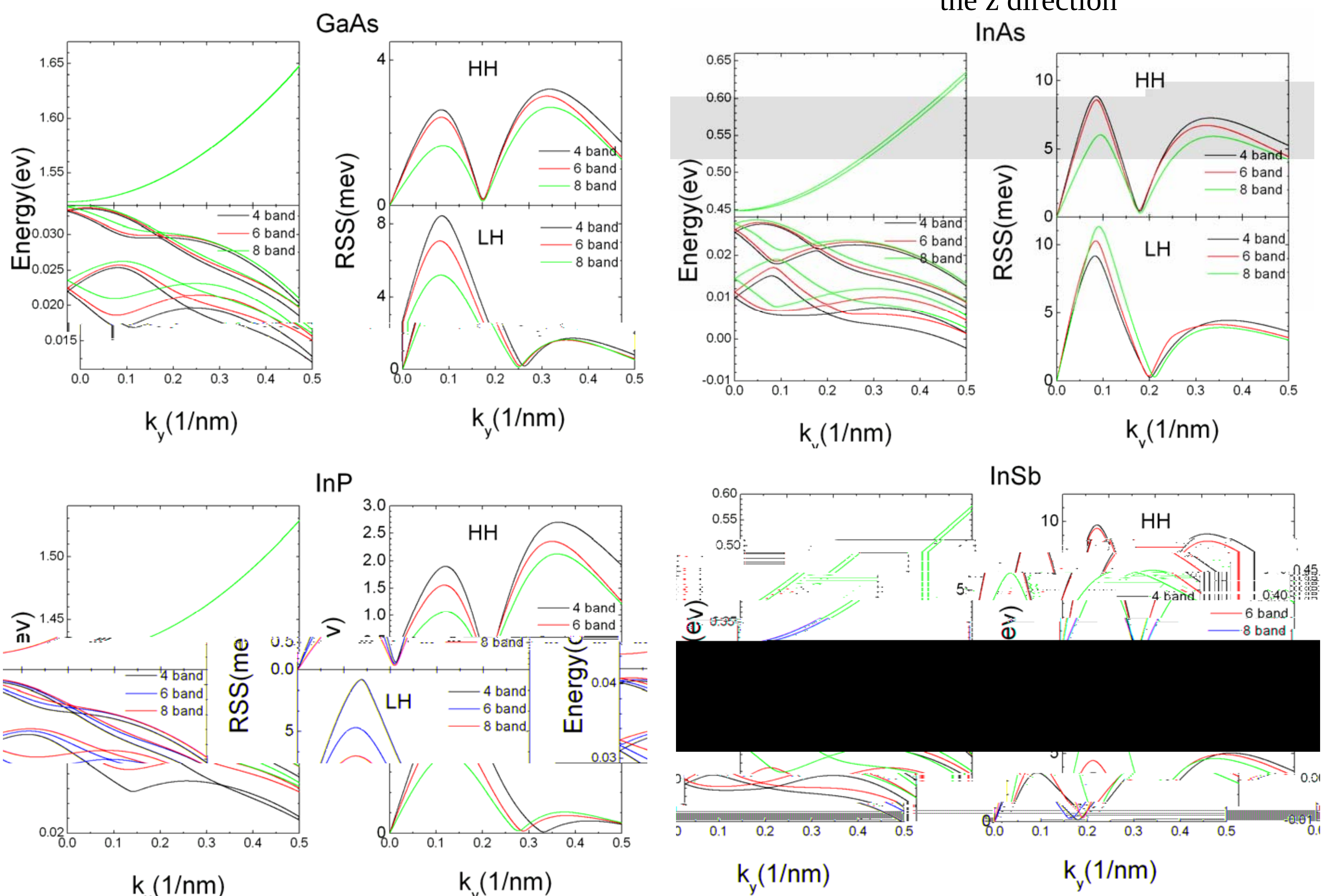


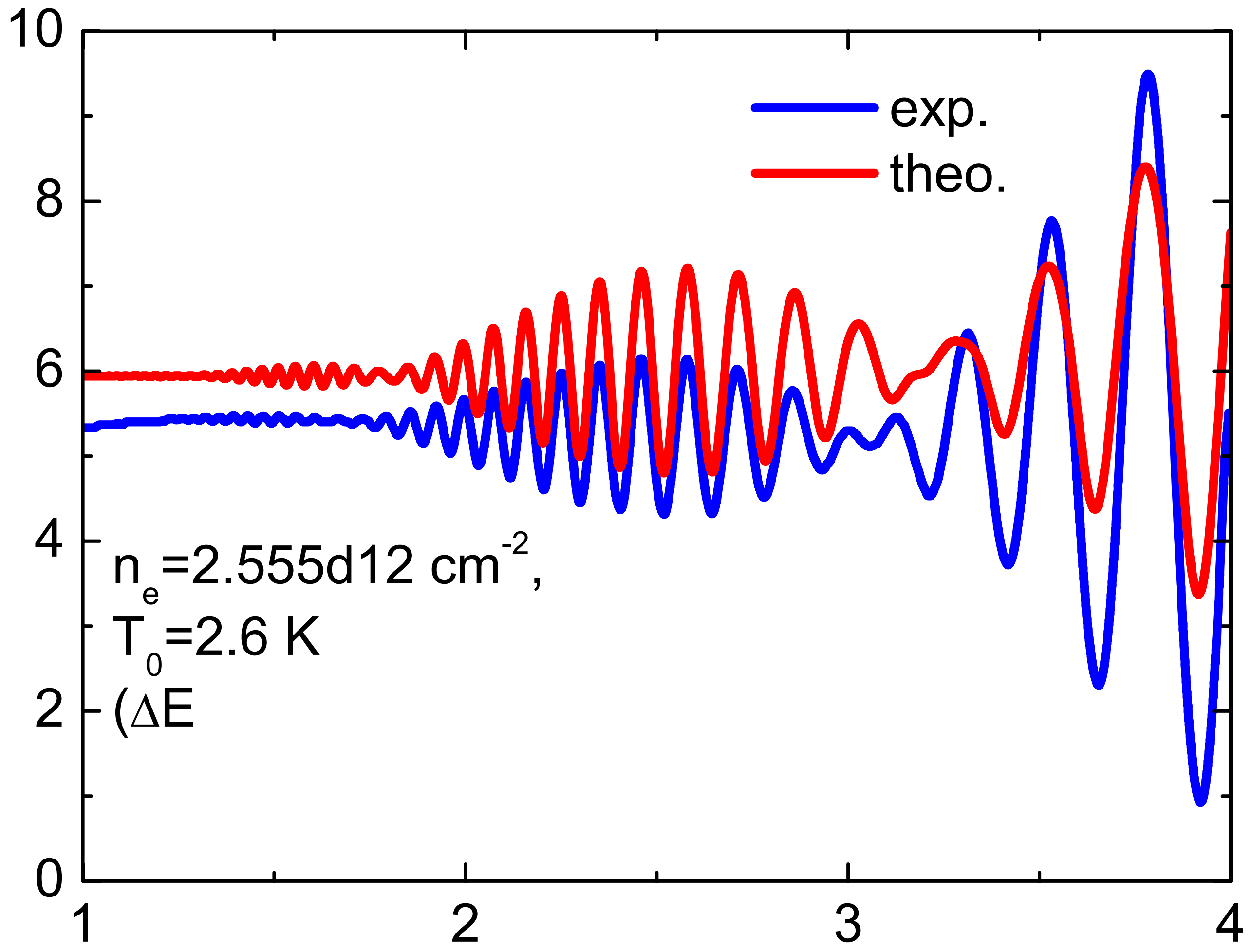


- # Free standing Quantum Wire

Lz=20(nm)Lx=20(nm)

E=100(kv/cm) is added in the z direction



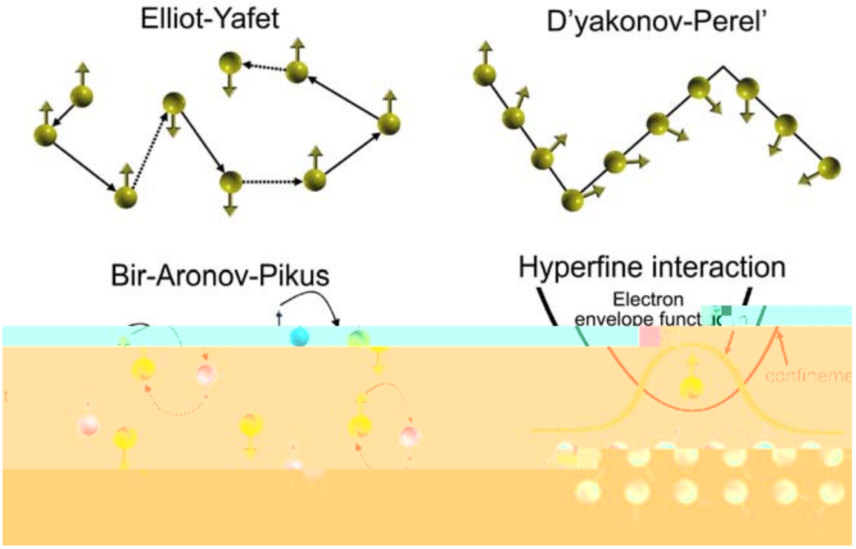


exp.
theo.
n_e=2.555d12 cm^-2,
T_0=2.6 K
(ΔE
0
2
4
6
8
10
1
2
3
4

II Spin Relaxation

Family of spin relaxation mechanisms:

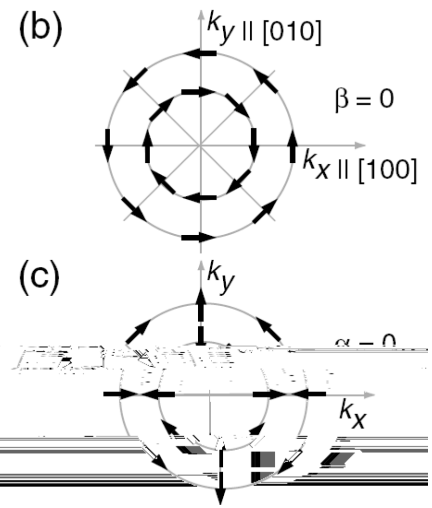
- **D'yakonov-Perel' (DP) mechanism(SOI)**
- **Elliot-Yafet (EY) mechanism(SOI)**
- **Bir-Aronov-Pikus (BAP) mechanism(exchange)**
- **Hyperfine (s-d) mechanism(nuclear, magnetic ions)**



II Spin Relaxation

D'yakonov Perel' (DP) mechanism:

- **Origin: spin-orbit interaction: RSOI and DSOI**



$$\mathbf{B_R} = \zeta\,(\mathbf{k} \times \mathbf{z})$$

$$\mathcal{H}_{SO} \sim \sigma \cdot \mathbf{B}(\mathbf{k})$$

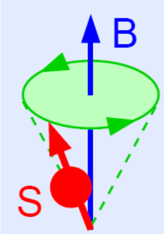
Motional narrowing

$$\tau_s \sim \tau_p^{-1}$$

Electrons ($S = 1/2$):

$$\hat{H} = H_0 + \tfrac{1}{2}\mathbf{S}\cdot\boldsymbol{\mathcal{B}}$$

$$\frac{d\mathbf{S}}{dt} = \frac{i}{\hbar}[\hat{H}, \mathbf{S}]$$



$$\langle\dot{\mathbf{S}}\rangle = \tfrac{1}{\hbar}\langle\mathbf{S}\rangle \times \boldsymbol{\mathcal{B}}$$

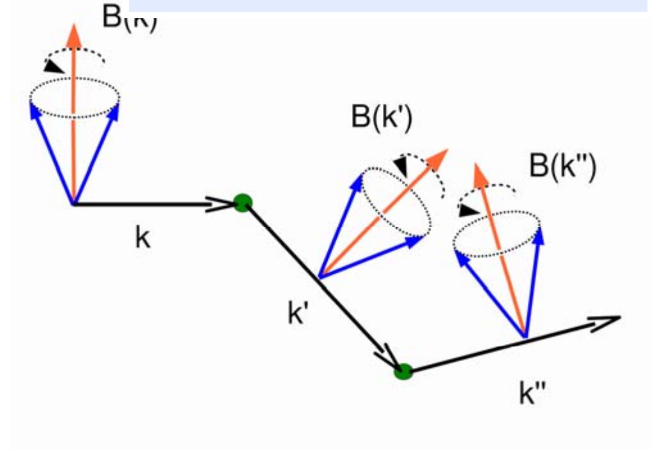
$$\frac{d\,|\langle\mathbf{S}\rangle|}{dt} = 0$$

$\langle\mathbf{S}\rangle$ = Bloch vector

Scattering effect:

1, change the in-plane momentum *k* of electron

2, change the effective magnetic field



II Spin Relaxation

Equation of motion given by *Liouville equation*:

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[H, \rho],$$

Density matrix:

$$\mathbf{k}) := \begin{pmatrix} \varrho_{\uparrow\uparrow}(\mathbf{k}) & \varrho_{\uparrow\downarrow}(\mathbf{k}) \\ \varrho_{\downarrow\uparrow}(\mathbf{k}) & \varrho_{\downarrow\downarrow}(\mathbf{k}) \end{pmatrix} \quad \varrho(\mathbf{k}$$

Decay of the components:

1, diagonal elements (occupation number) $T_{1;}$

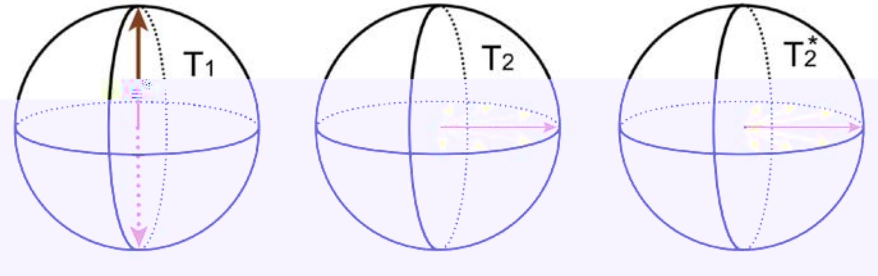
2, off-diagonal elements (decoherence) T_2

J. Kainz, U. Rössler and R. Winkler, Phys. Rev. B 70, 195322 (2004);
N. S. Averkiev and L. E. Golub, Phys. Rev. B 60, 15582 (1999)

$H=H_0+H_{im}+H'$

$H'=H_{SO}$

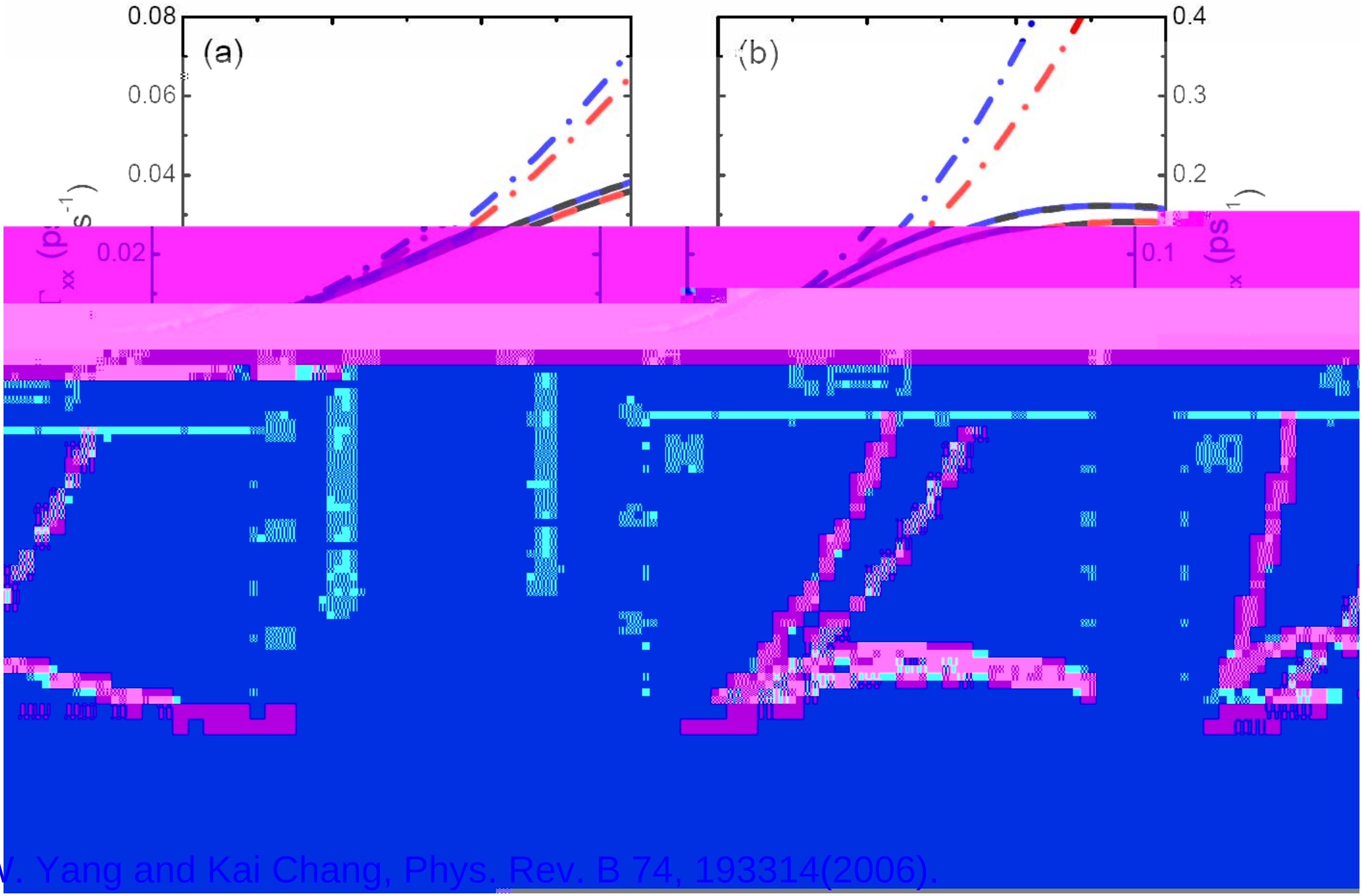
scattering



II Spin Relaxation

Comparing between widely used linear Rashba model

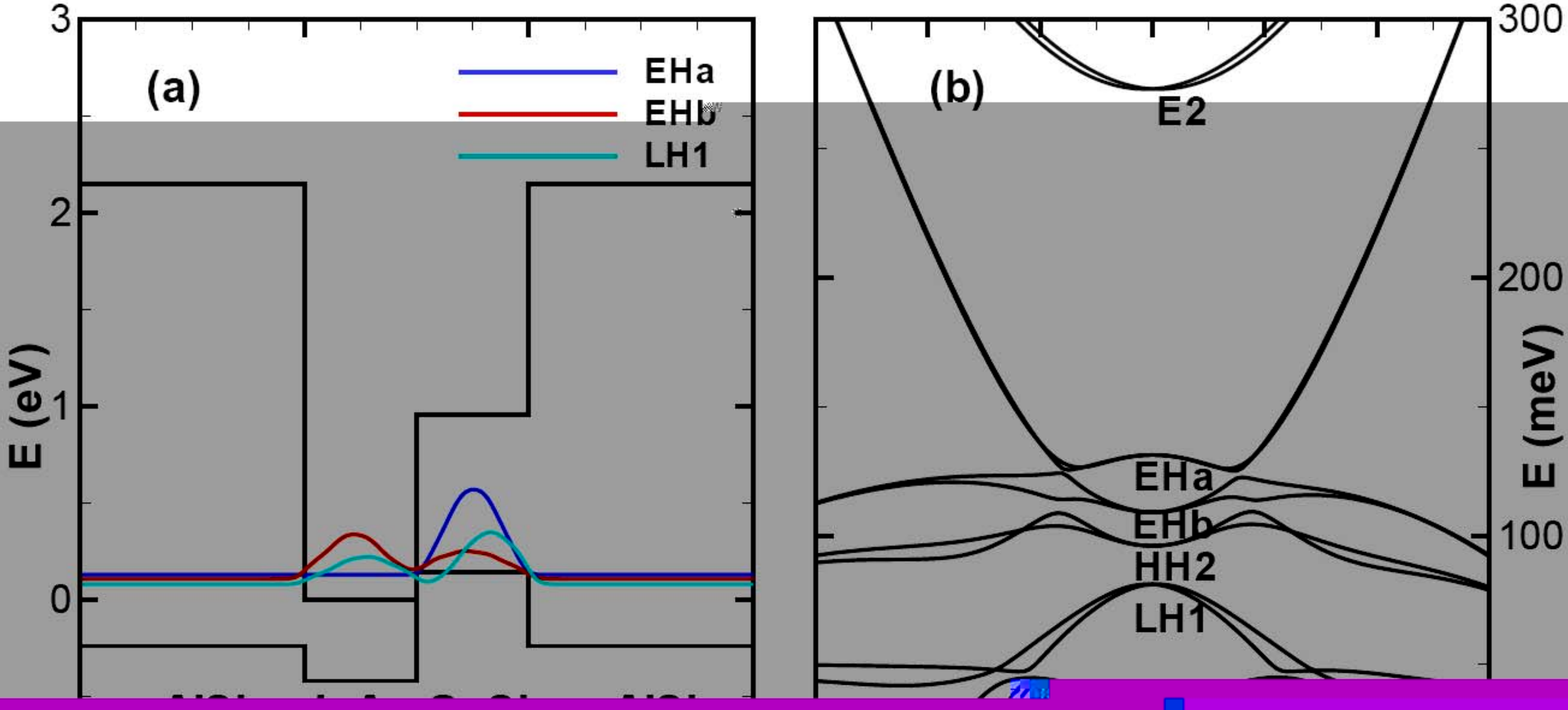
And our nonlinear Rashba model



W. Yang and Kai Chang, Phys. Rev. B 74, 193314(2006).

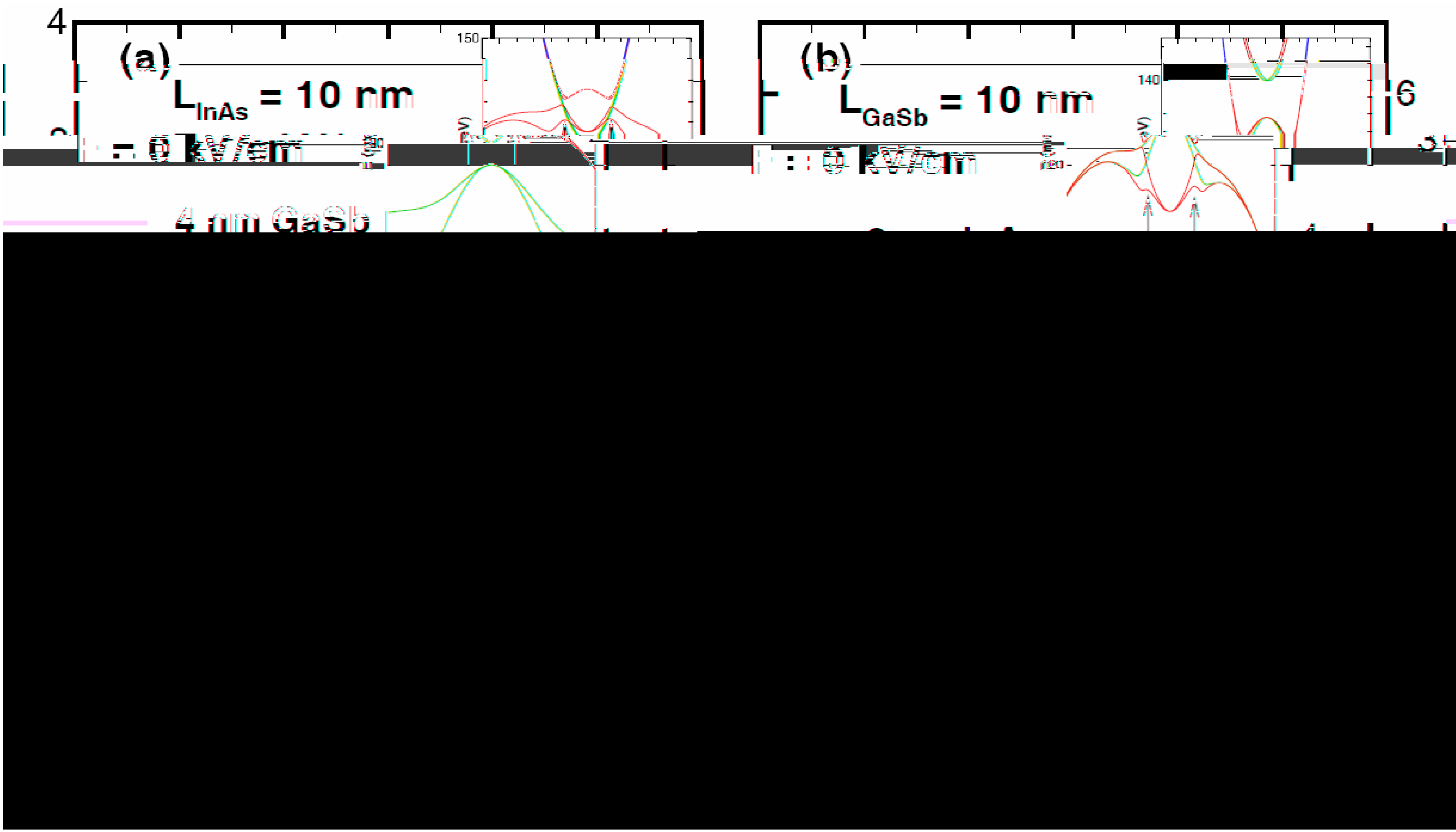
Anomalous SOI in InAs/GaSb QW

obtained from self-consistent calculation



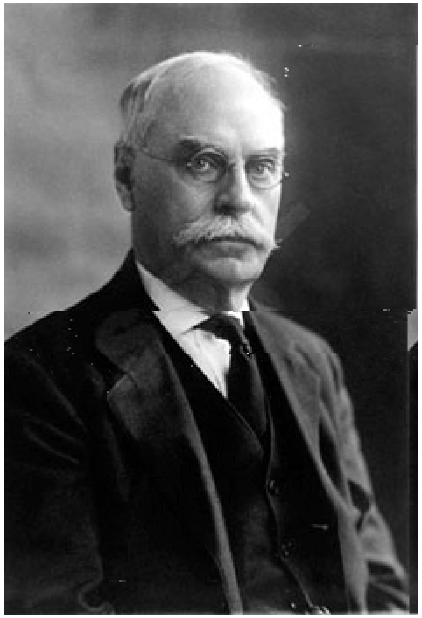
J. Li, Kai Chang, APL (in press)

Anomalous SOI in InAs/GaSb QW



Conclusion:

Nonlinear Rashba model can give correct behavior at large in-plane momentum. Nonlinear behavior of RSS is universal phenomenon in semiconductor nanostructures; can lead to surprising consequences, e.g., spin relaxation.



E. H. Hall, (1855-1938)

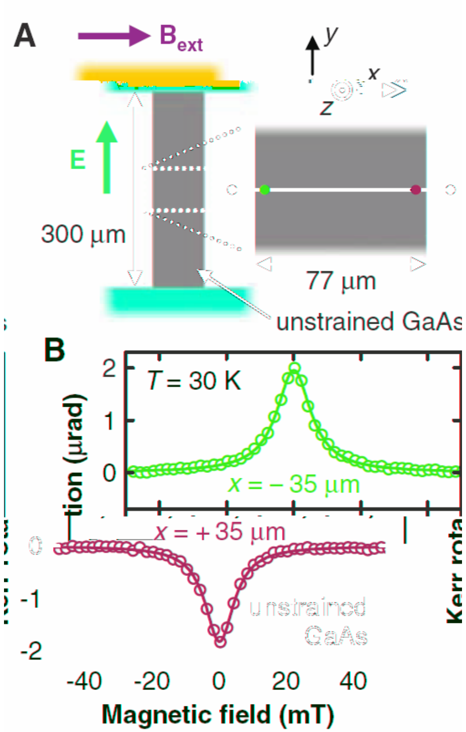
The Hall Effect

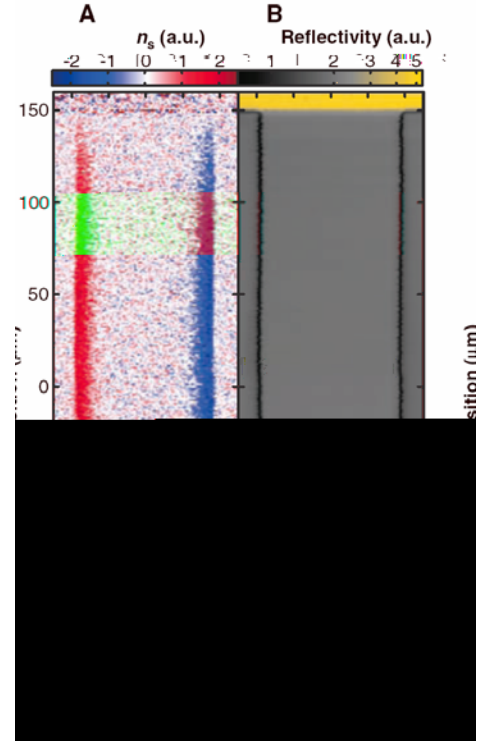
Classic Hall Effect (1879);
Anomalous Hall Effect (1881);
Quantum Hall Effect(1978;1982);
Spin Hall Effect (1971, 2004).

What is the spin Hall effect?

Electric field induces transverse spin current due to spin-orbit coupling

Spin Hall effect





Extrinsic Spin Hall Effect: impurity scattering

D'yakonov and Perel' /JETP 8[>80_

Hirsch /PRL 8[[[03#Zhang /PRL 97770

Intrinsic Spin Hall Effect: band effect (SOI)

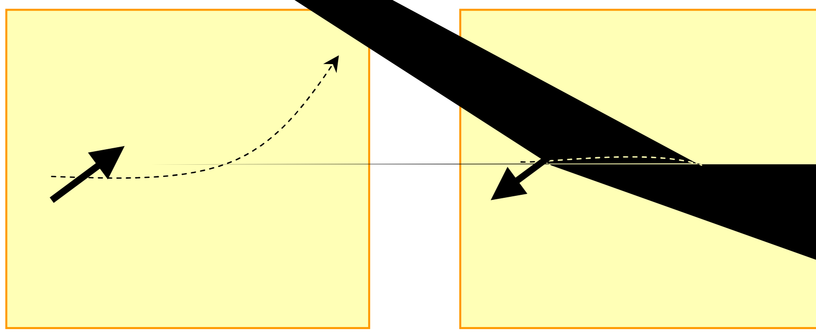
Murakami, Nagaosa, Zhang, Science (2003);

J. Sinova et al (PRL 2004)

The Extrinsic Spin Hall effect

(arising from impurity scattering with spin-orbit coupling)

impurity scattering = skew scattering + side-jump effect



The Intrinsic Spin Hall Effect

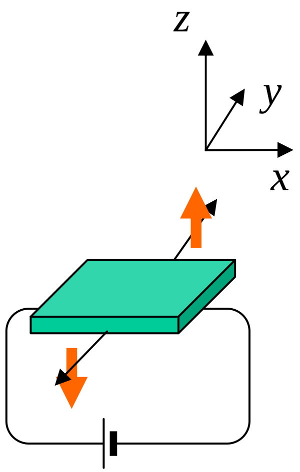
Berry phase in momentum space

Independent of impurities

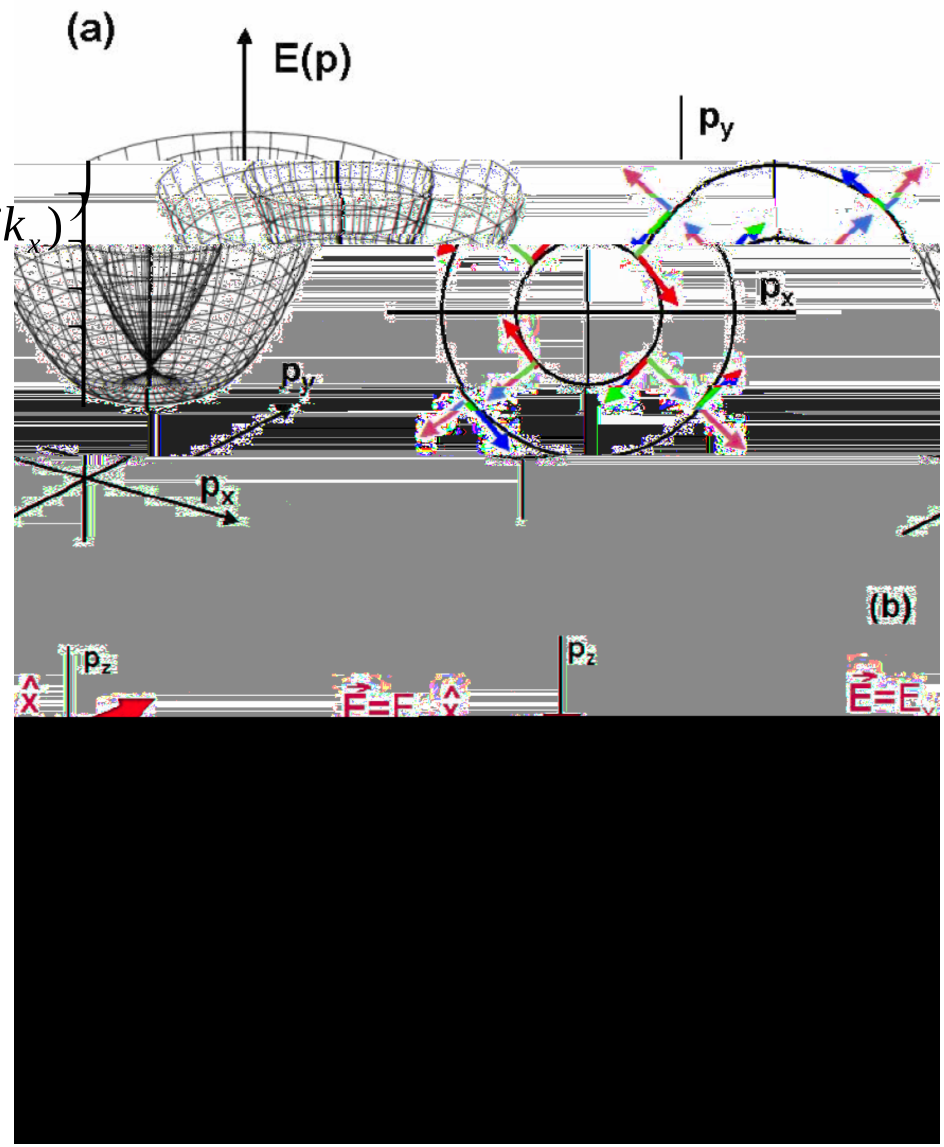
The intrinsic Spin Hall effect

In clean 2DEG

$$H \mid \frac{k^2}{2m} 2\, \varsigma (\vec{\omega} \Delta \vec{k})_z \mid \begin{pmatrix} \frac{k^2}{2m} & \varsigma(k_y\, 2\, ik_x) \\ \varsigma(k_y\, 4\, ik_x) & \frac{k^2}{2m} \end{pmatrix}$$



2D heterostructure



PRL(2003) J.Sinova, D.Culcer, Q. Niu, A. H. MacDonald

Spin Hall effect

Kubo linear response theory:

$$\sigma_{xy}^{\mathrm{sH}}(\omega) = \frac{e\hbar}{V} \sum_{\mathbf{k}, n \neq n'} (f_{n',k} - f_{n,k}) \times \frac{\mathrm{Im}[\langle n'k | \hat{j}^{z}_{\mathrm{spin}\,x} | nk \rangle \langle nk | v_y | n'k \rangle]}{(E_{nk} - E_{n'k})(E_{nk} - E_{n'k} - \hbar\omega - i\eta)}$$

Linear Rashba Model: (PRL(2004))

$$\sigma_{\mathrm{sH}} \equiv -\frac{j_{s,y}}{E_x} = \frac{e}{8\pi}, \quad n_{2\mathrm{D}} > n_{2\mathrm{D}}^*$$

$$\sigma_{\mathrm{sH}} = \frac{e}{8\pi} \frac{n_{2\mathrm{D}}}{n_{2\mathrm{D}}^*}. \qquad n_{2\mathrm{D}} < n_{2\mathrm{D}}^*$$

$$n_{2\mathrm{D}}^* \equiv m^2 \lambda^2 / \pi \hbar^4$$

J. Inoue, G. E. Bauer and L. W. Molenkamp PRB (2004)

1, single parabolic band,

2, linear response theory,

3, linear Rashba model,

4, short-range potential, Born approximation.

Conclusions:

1, vanishing spin Hall effect including the vertex correction

2, dominant forward scattering leads to a nonvanishing SHE.

- Sinova [PRL, 2003]

- Inoue [PRB, 2004] ι ( )

  Inoue (ι)

- S. Murakami [PRL (2004)]

- Raimondi [PRB, 2005]
- Dimitrova [PRB,2005] Rashba

- Krotkov [PRB, 2006] Inoue Rashba

- Khaetskii [PRL 2006] Rashba Born

SHE

Controversy　Does Intrinsic SHE vanish in 2DEG and/or 2DHG

Conclusions　SHE vanishes at

1, Linear Rashba model,

2, parabolic band,

3, short-range impurity potential, Born approximation.

1, **Nonlinear behavior of Rashba SOI**

2, Non-parabolic band

Narrow band gap semiconductors

PRL 100, 056602(2008)

Unsolved:

1, Long-range potential? (Khaetskii 2006 SHE still vanishes!)

2, Beyond Born approxiamtion

The operators in the eight band Kane model

$$\phi_1 = S\uparrow,$$

$$\phi_2 = S\downarrow,$$

$$\phi_3 = \left|\frac{3}{2},\frac{3}{2}\right\rangle = \frac{1}{\sqrt{2}}(X+iY)\uparrow,$$

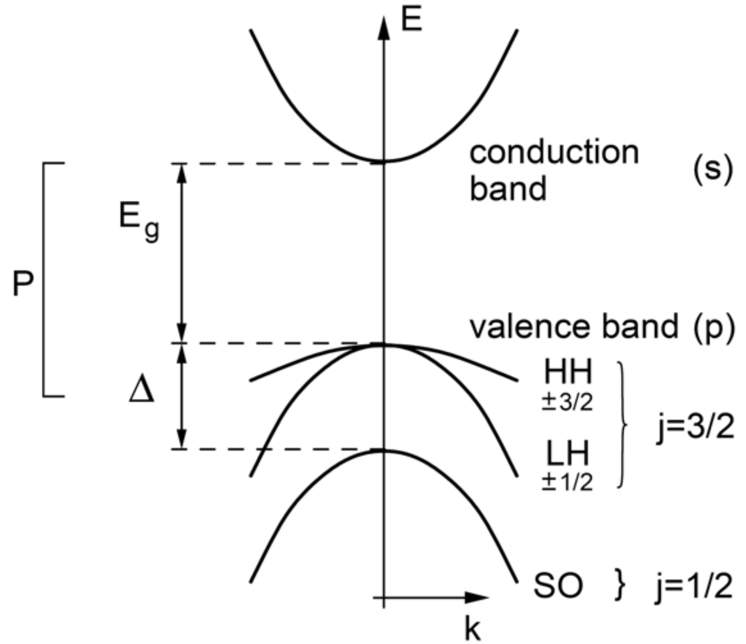
$$\phi_4 = \left|\frac{3}{2},\frac{1}{2}\right\rangle = \frac{i}{\sqrt{6}}\left[(X+iY)\downarrow -2Z\uparrow\right],$$

$$\phi_5 = \left|\frac{3}{2},-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{6}}\left[(X-iY)\uparrow +2Z\downarrow\right],$$

$$\phi_6 = \left|\frac{3}{2},-\frac{3}{2}\right\rangle = \frac{i}{\sqrt{2}}(X-iY)\downarrow,$$

$$\phi_7 = \left|\frac{1}{2},\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}\left[(X+iY)\downarrow +Z\uparrow\right],$$

$$\phi_8 = \left|\frac{1}{2},-\frac{1}{2}\right\rangle = \frac{i}{\sqrt{3}}\left[(X-iY)\uparrow -Z\downarrow\right]$$



PRL 100, 056602(2008)

For the operators O:

$$\tilde{\mathbf{O}}_{j\mathbf{k},j'\mathbf{k}'} = \mathbf{O}_{j\mathbf{k},j'\mathbf{k}'} + \sum_{l\mathbf{k}''}\left[\mathbf{O}_{j\mathbf{k},l\mathbf{k}''}(E-E_l)^{-1}(H_{kp})_{l\mathbf{k}'',j'\mathbf{k}'} + (H_{kp})_{j\mathbf{k},l\mathbf{k}''}(E-E_l)^{-1}\mathbf{O}_{l\mathbf{k}'',j'\mathbf{k}'}\right].$$

The operators in the eight band Kane model

$$[\tilde{v}_\alpha(\mathbf{k})]_{jj'} = \frac{p^\alpha_{jj'}}{m_0} + \delta_{jj'}\frac{\hbar k_\alpha}{m_0} + \frac{\hbar}{4m_0^2c^2}(\boldsymbol{\sigma}\times\nabla V)_{\mu\mu'}$$
$$+ \frac{\hbar}{m_0^2}\sum_{\beta,l}\left(\frac{p^\alpha_{jl}p^\beta_{lj'}}{E-E_l}k_\beta + k_\beta\frac{p^\beta_{jl}p^\alpha_{lj'}}{E-E_l}\right).$$

二维电子气的二维本征自旋霍尔电导率 $\sigma^{\alpha\beta\gamma}_{2D} = \langle J^\alpha_\beta\rangle / E^\gamma$

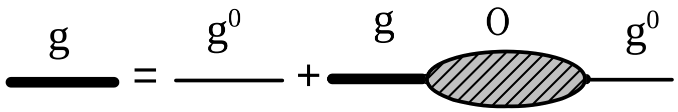
$$\sigma^{3D}_{\alpha\beta\gamma} = \frac{1}{\hbar V}\lim_{\omega\to 0}\frac{e}{i\omega}\left[G^r_{AB}(\omega) - G^r_{AB}(0)\right]$$

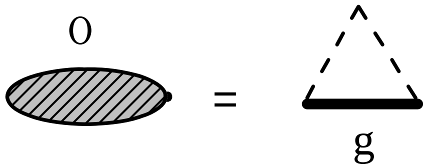
PRL 100, 056602(2008)

We adopt:

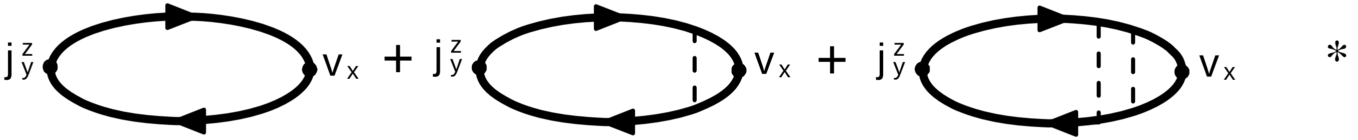
1 **8band Hamiltonian in axial approximation ($\nu_2 = \nu_3$)**

2 Green function: self-consistent Born approximation

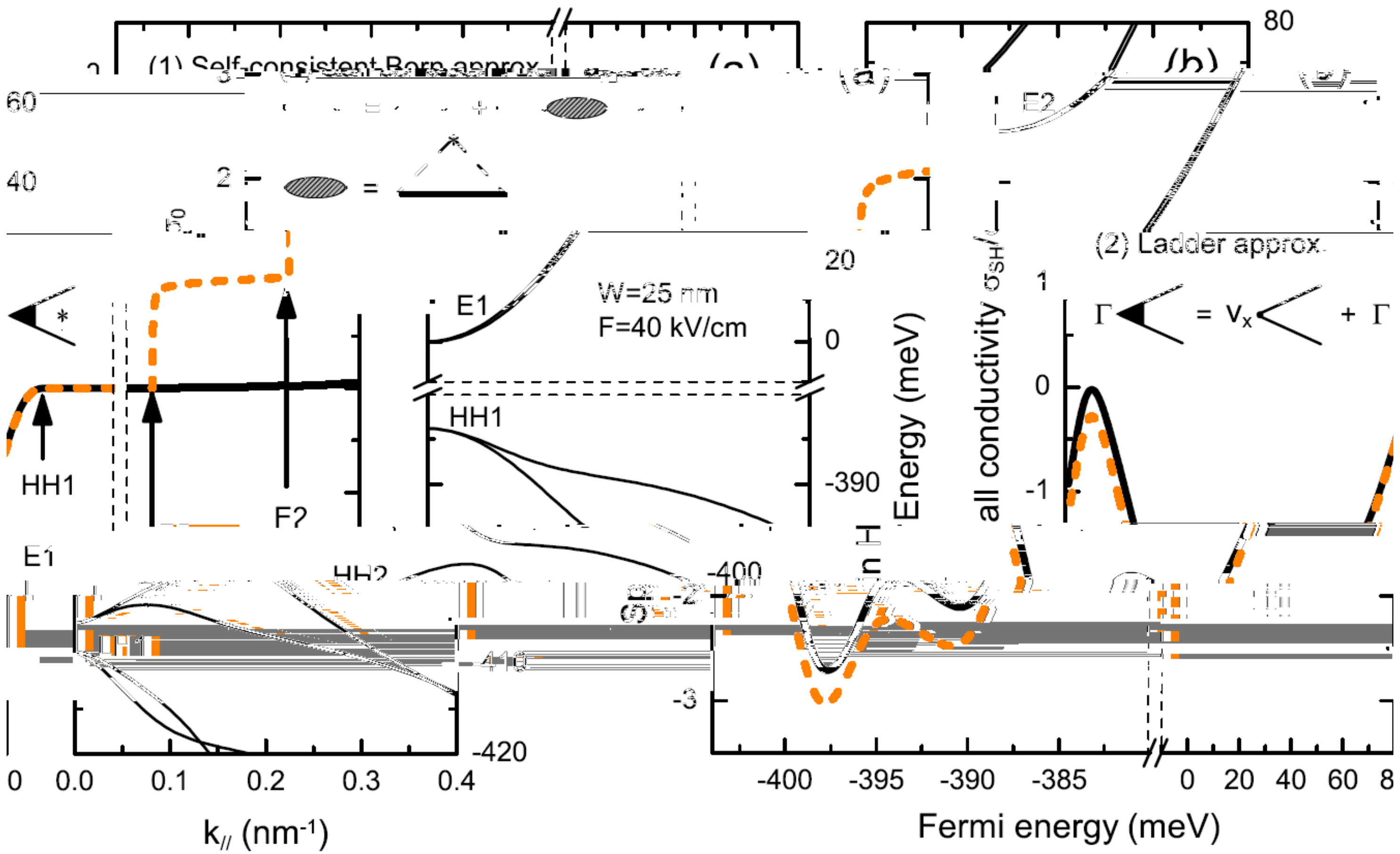




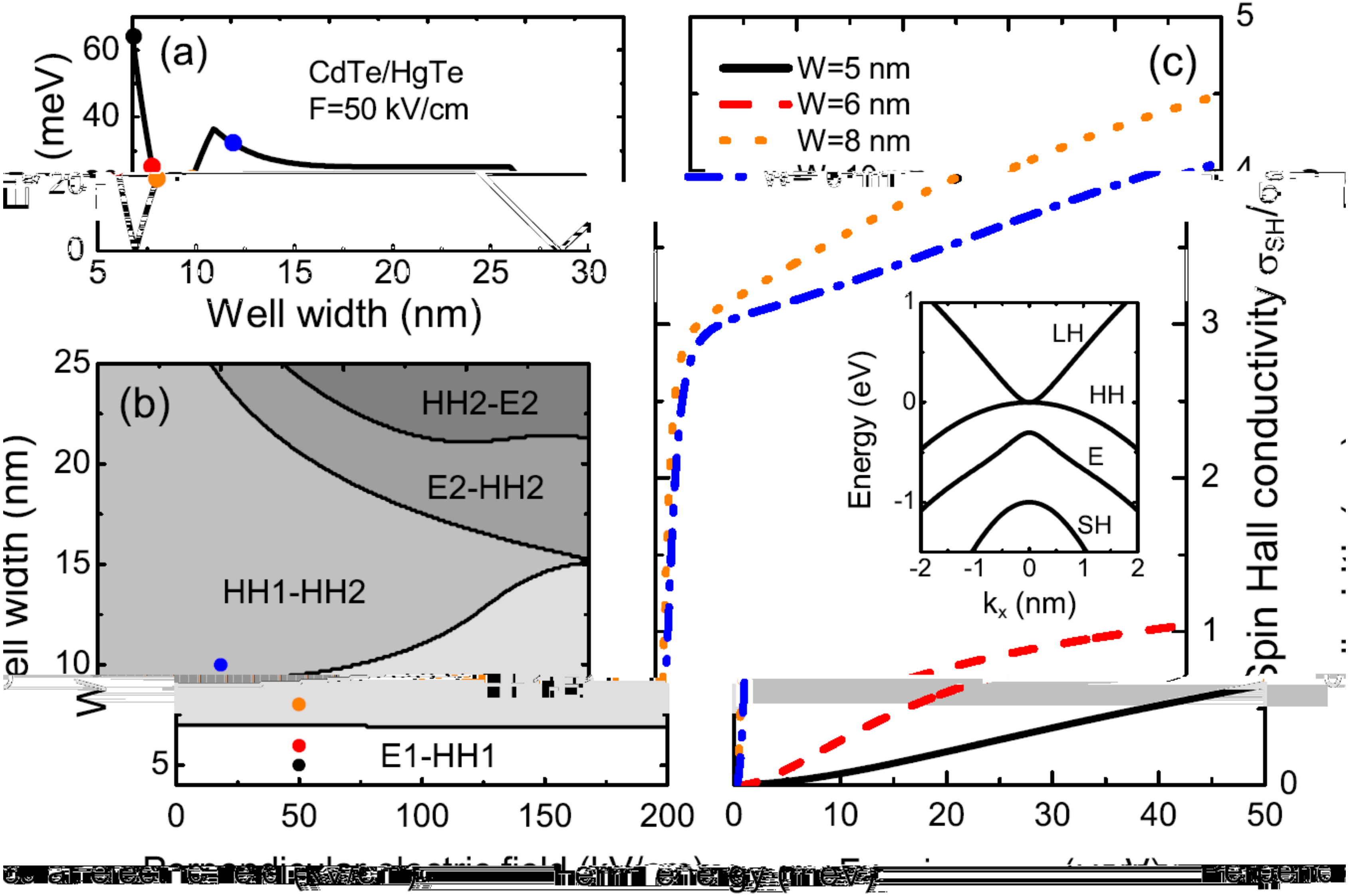
Bethe-Salpeter equation



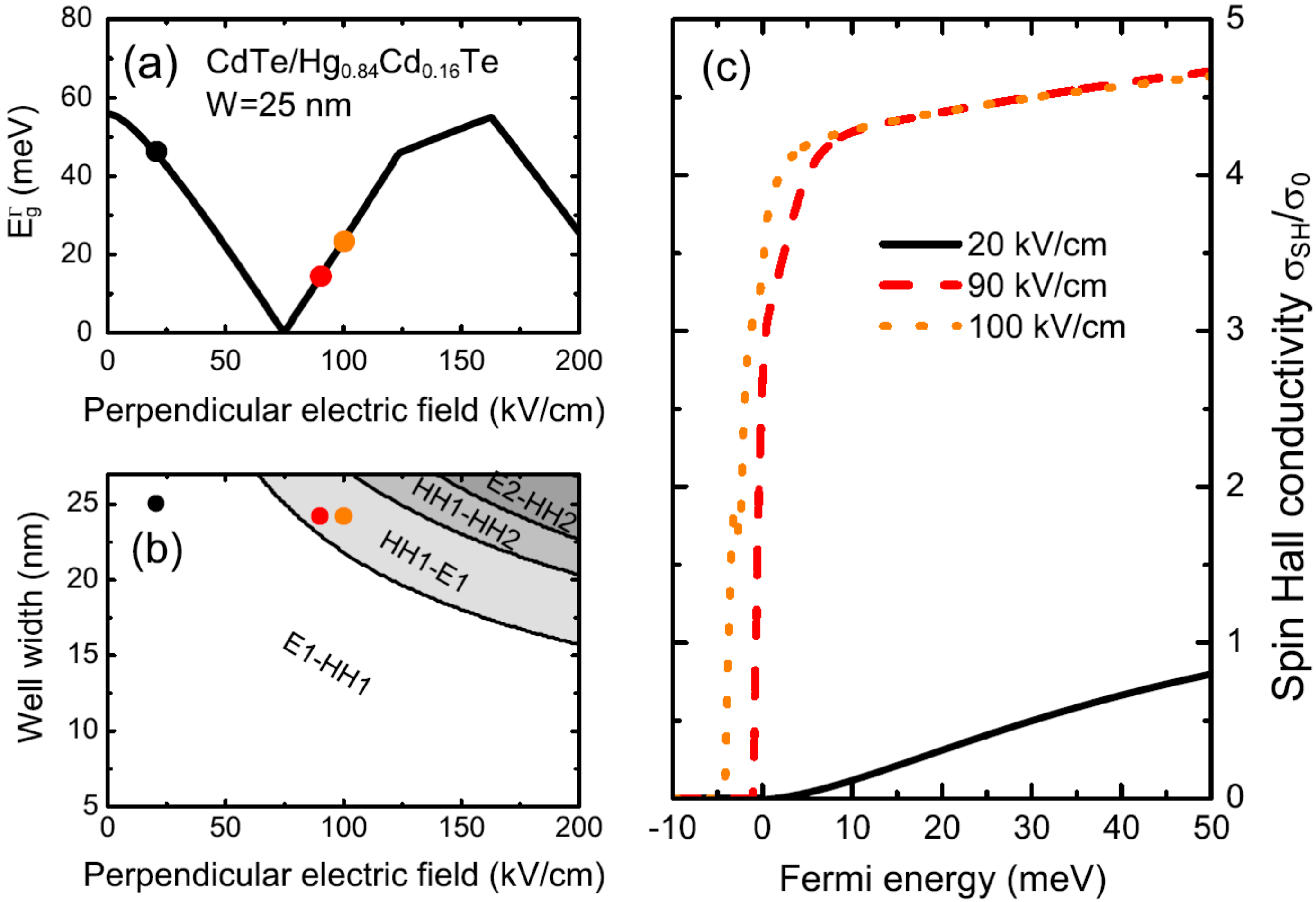
Spin Hall effect

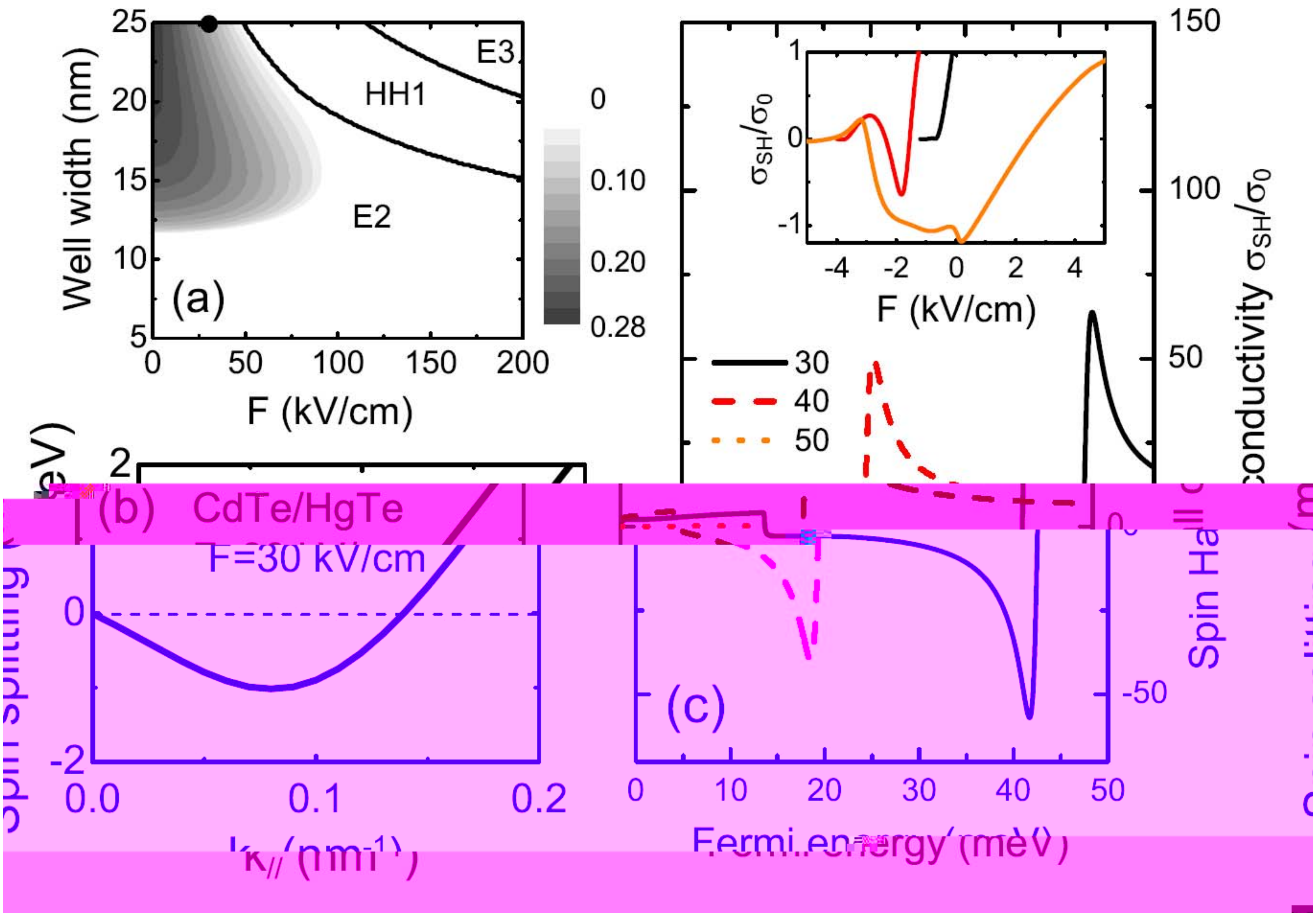


Intrinsic spin Hall conductivity ω_{sH} with vertex correction



Spin Hall effect





(a)
Well width (nm)
F (kV/cm)
HH1
E2
E3
(b)
CdTe/HgTe
F=30 kV/cm
$k_{//}$ (nm^{-1})
(c)
Fermi energy (meV)
Spin Hall conductivity σ_{SH}/σ_0
σ_{SH}/σ_0
30
40
50

Conclusions:

- Unified description of Spin Hall effect;
- Switching of SHE utilizing the external electric field;
- Could provides us a possible way to distinguish extrinsic and intrinsic Spin Hall effects.

Optical detection of spin current

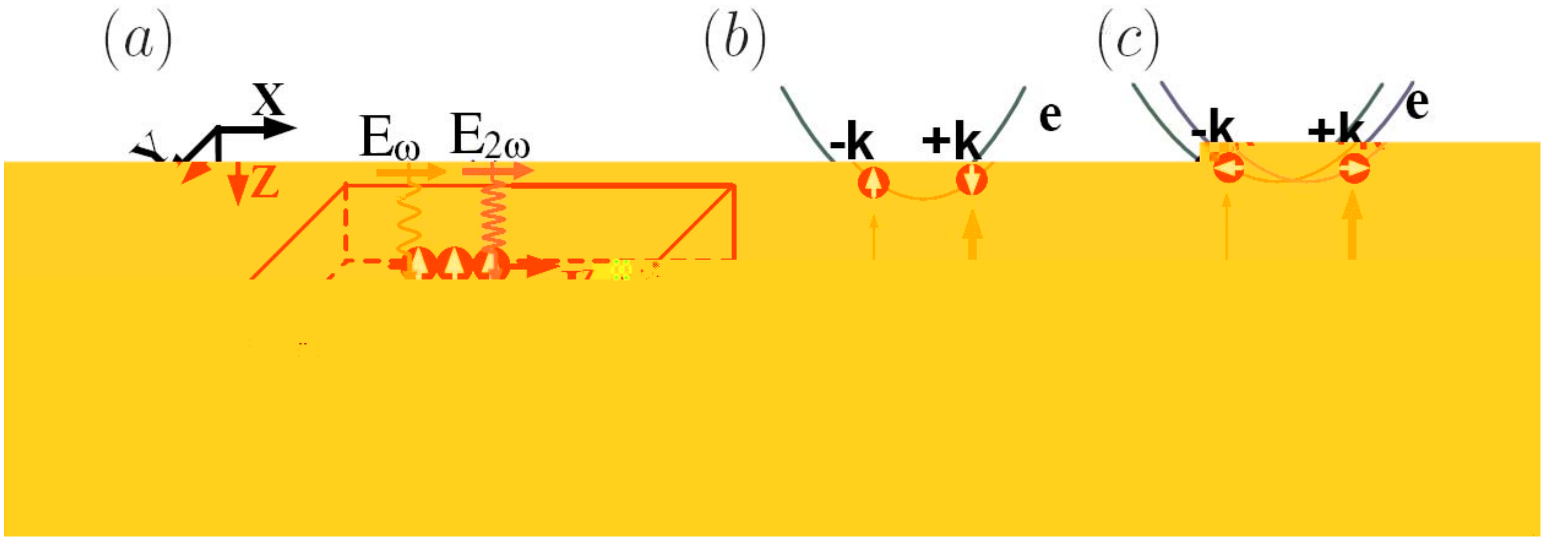
Pure spin current: the spin-up and spin-down electron currents have equal magnitudes but travel in opposite directions

→ vanishing charge current and total spin

→ No conventional magneto-optical effects

Asymmetric distribution in k space
f(-k)=f(+k) ≠ 0

Transition rate of QUIC:
W(-k)=0, W(+k) ≠ 0



Theory:

The Luttinger-Kohn Hamiltonian:

$$H_h(k,\theta,\varphi) = \frac{\hbar^2 k^2}{2m_0}\begin{pmatrix} H_h & L & M & 0 \\ L^* & H_l & 0 & M \\ M^* & 0 & H_l & -L \\ 0 & M^* & -L^* & H_h \end{pmatrix} - \frac{\hbar^2 k^2}{2m_0}\gamma_1$$

where

$$H_h = -\gamma_2 \sin^2\theta + 2\gamma_2\cos^2\theta$$

$$H_l = +\gamma_2 \sin^2\theta - 2\gamma_2\cos^2\theta = -H_h$$

$$M = -\sqrt{3}\gamma_2 \sin^2\theta e^{-2i\varphi}$$

$$L = i2\sqrt{3}\gamma_2 \cos\theta \sin\theta e^{-i\varphi}$$

The Valkov-type solution

$$\ldots_{c,v}(\mathbf{k},r,t) \mid u_{c,v}(\mathbf{k},r)\exp[i\mathbf{k}\notin\mathbf{r} \; 4 \; i\varpi_{c,v}(\mathbf{k})t \; 2 \; \frac{ie}{m_{c,v}}\Big|_0^t \mathbf{k}\notin A(\vartheta)d\vartheta]$$

The transition rate is calculated using Fermi golden rule:

$$S = -\frac{i}{\hbar}\int_{-\infty}^{\infty} dt' \int d^3r\, \psi_c^*(\mathbf{k}, r, t')\left(-\frac{e}{m_0 c}\right) A\cdot P\, \psi_v(\mathbf{k}', r, t')$$

The transition rate

$$w(\mathbf{k}) = \lim_{t\to\infty}\frac{d}{dt}|S|^2$$

$$\propto \left(\frac{\xi_1}{2}\right)^2 \Big\{ |\mathbf{p}_{vc}\cdot\mathbf{a}_1|^2 A_1^2 + |\mathbf{p}_{vc}\cdot\mathbf{a}_2|^2 A_2^2 + A_1 A_2 \frac{\xi_1}{2}$$
$$\left[(\mathbf{p}_{vc}\cdot\mathbf{a}_1)^*(\mathbf{p}_{vc}\cdot\mathbf{a}_2)\exp i(2\phi_1 - \phi_2) + (\mathbf{p}_{vc}\cdot\mathbf{a}_1)(\mathbf{p}_{vc}\cdot\mathbf{a}_2)^*\exp i(-2\phi_1 + \phi_2)\right]\Big\}$$

where

$$\xi_1 = \frac{eA_1}{\omega c m_{cv}}\mathbf{k}\cdot\vec{a}_1, \quad \frac{1}{m_{cv}} = \frac{1}{m_c} - \frac{1}{m_v}$$

Consider ω and 2ω beams are polarized along the x direction, the electric fields are given by

$$\mathbf{E}(\omega) = E(\omega)e^{i\phi_1}\hat{x}$$

$$\mathbf{E}(2\omega) = E(2\omega)e^{i\phi_2}\hat{x} = E(2\omega)e^{i\phi_2}\left[(\hat{x} + i\hat{y}) + (\hat{x} - i\hat{y})\right]$$

Magneto-optical Effects: Faraday Rotation

$$\theta_F(\omega) = \frac{\omega}{c} Re(N_+ - N_+)$$

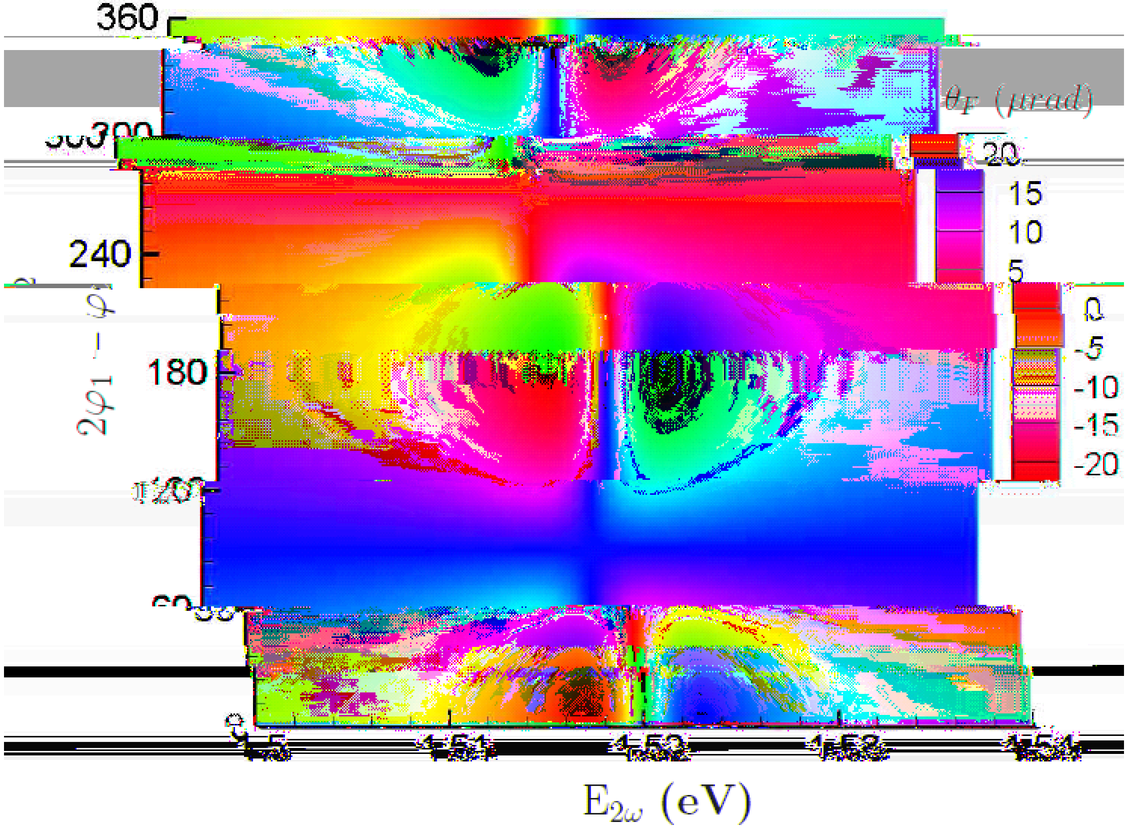
$$N_+ - N_- \propto W_+(+k) + W_+(-k) - W_-(+k) - W_-(-k)$$

$$= -C_0[C_1(f_u + f_d)\cos(2\varphi_1 - \varphi_2) + C_2 \sin(2\varphi_1 - \varphi_2)$$

$$, \quad \times \sin(2\varphi_e)(f_d - f_u) + C_l(f_u - f_d)]$$

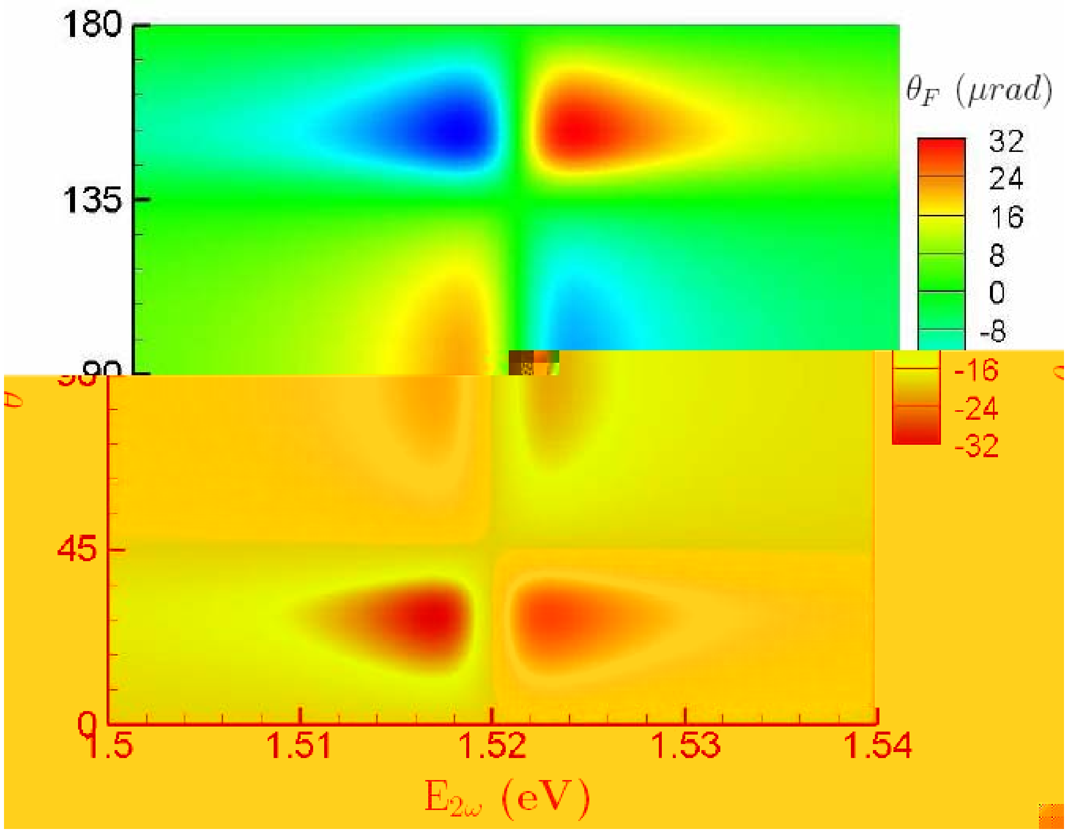
If fd=fu (Pure spin current)
→ Conventional Magneto-optical Effect vanishes,
but FR of QUIP appears

Results and Discussion



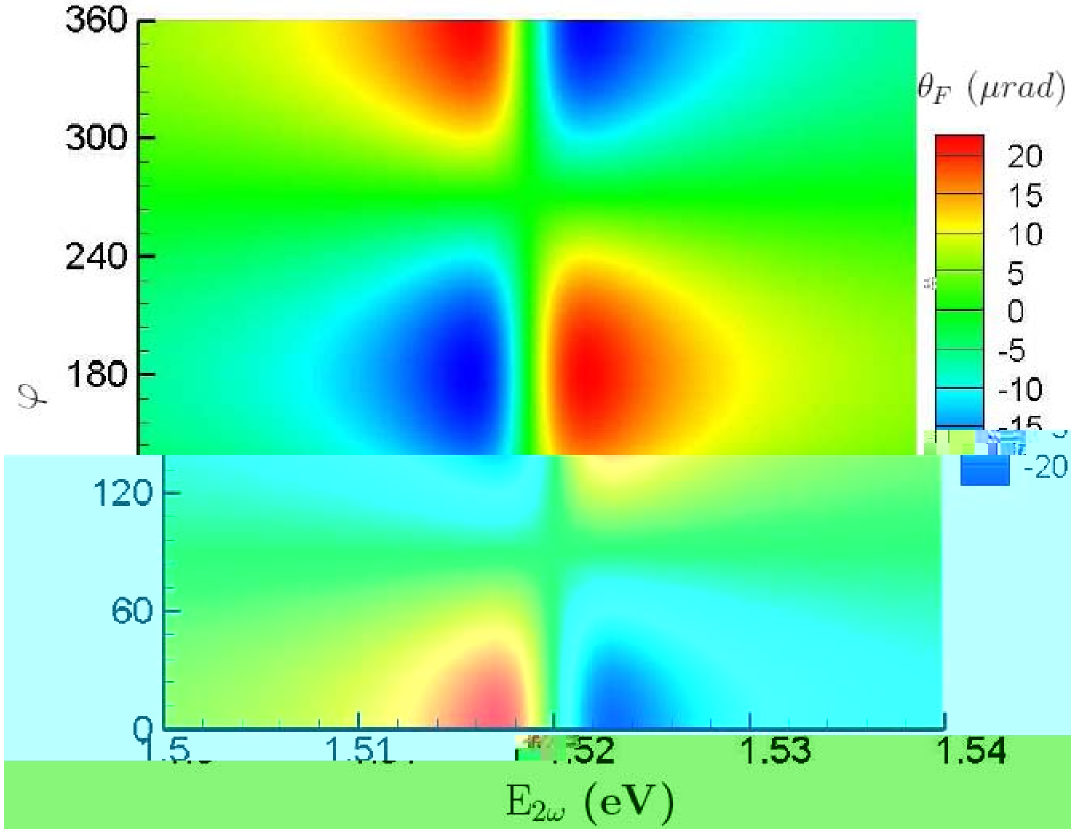
Contour map of the Faraday rotation angle χ_F /σrad0#as a function of transition energy and the relative phase of the two fields $2\pi_8 4\pi_9$. The pure spin carriers is along the k_x direction.

Results and Discussion



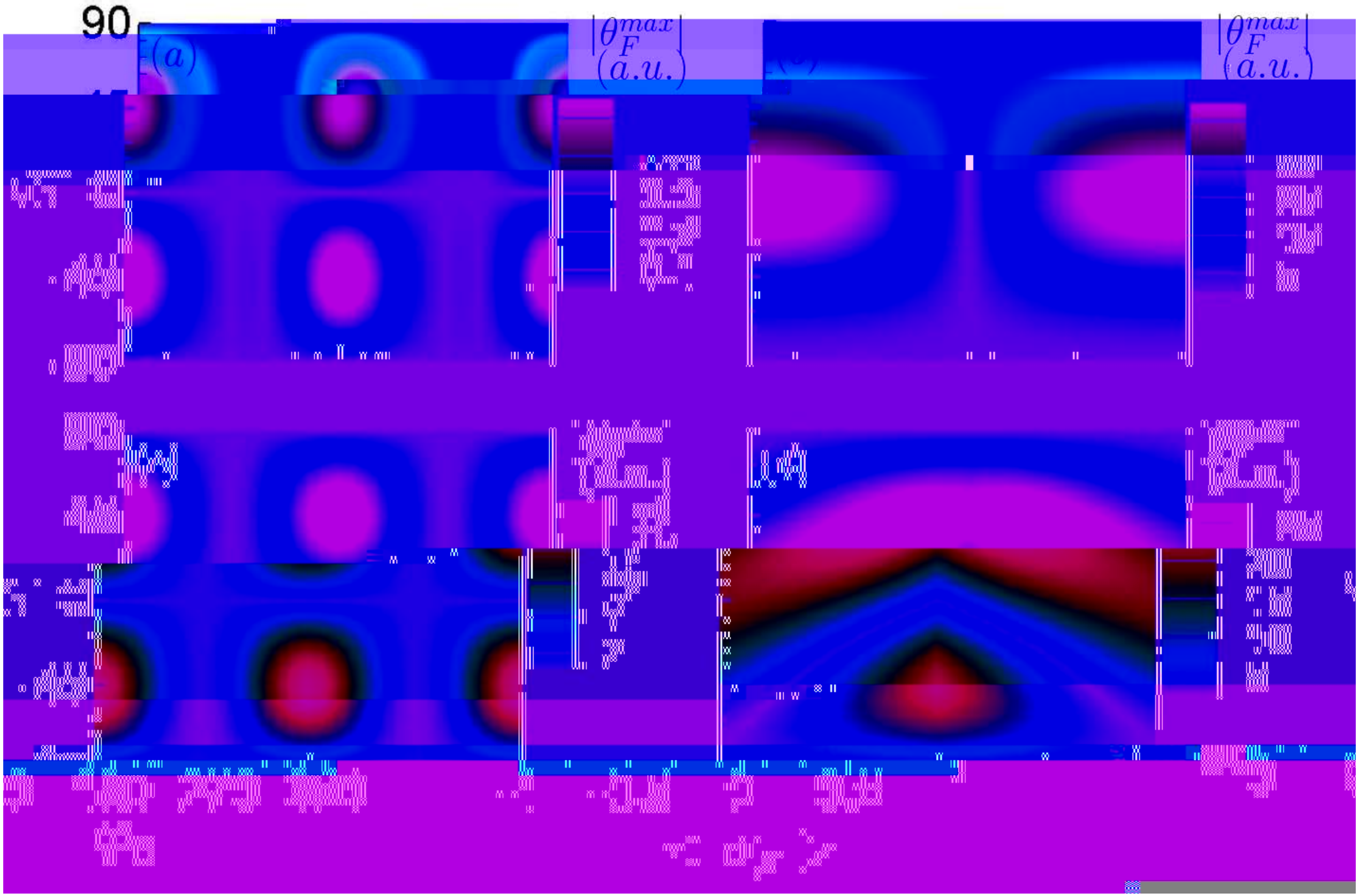
Contour map of the Faraday rotation angle χ_F /σrad0#as a function of transition energy and the polar angle χ of the direction of pure spin carriers for $2\pi_8 4\pi_9$|7.

Results and Discussion



Contour map of the Faraday rotation angle χ_F /σrad0#as a function of transition energy and the polar angle π of the direction of pure spin carriers for $2\pi_8 4\pi_9$ | 7#and χ=90 .

Results and Discussion



Conclusions:

Quantum interference Faraday rotation provides us a possible way to detect spin current directly, and help us to distinguish extrinsic and intrinsic Spin Hall effects.

Thank you for your attention!