Electronic transport theory of Dirac fermions in graphene

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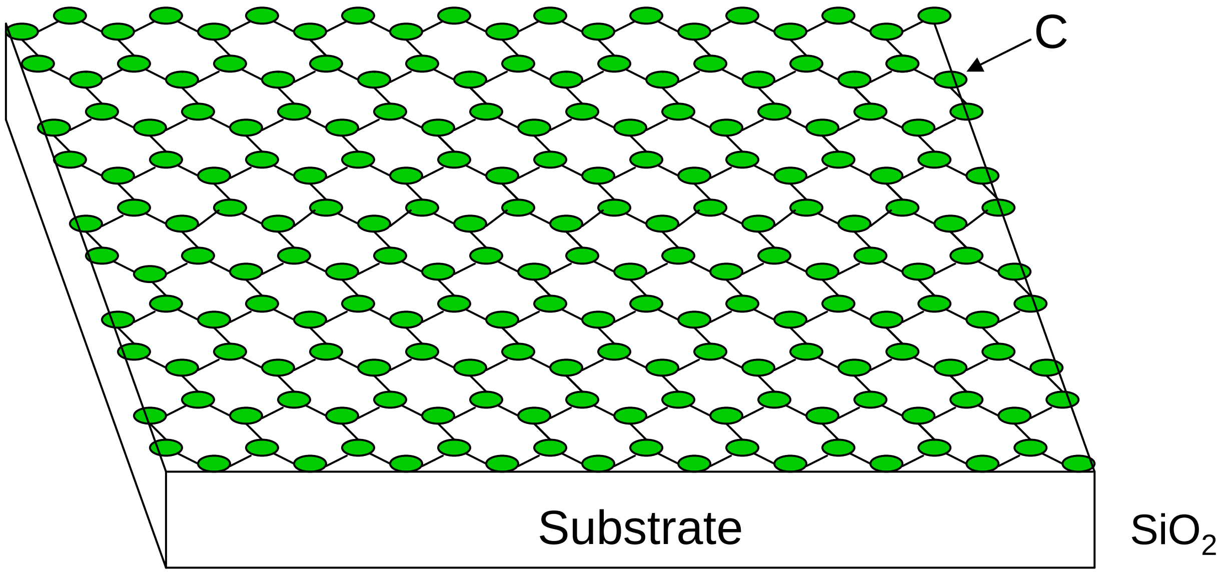
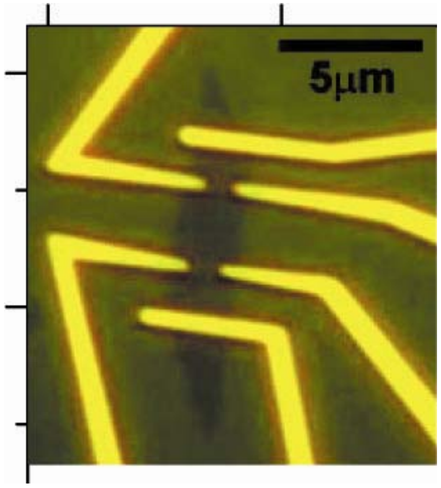
- Introduction of graphene
- Why are the electrons in graphene Dirac fermions?
- Electric transport theory

Graphene





Y. Zhang et al.,
PRL 96, 136806 (2006)

- High optical transmittance
  low resistivity
  high chemical stability and mechanical strength

- The single-particle electronic states around the Dirac point is identical to that of the massless Dirac fermions
  >> graphene experimentalists claim that they are doing high-energy experiments on table-top equipment

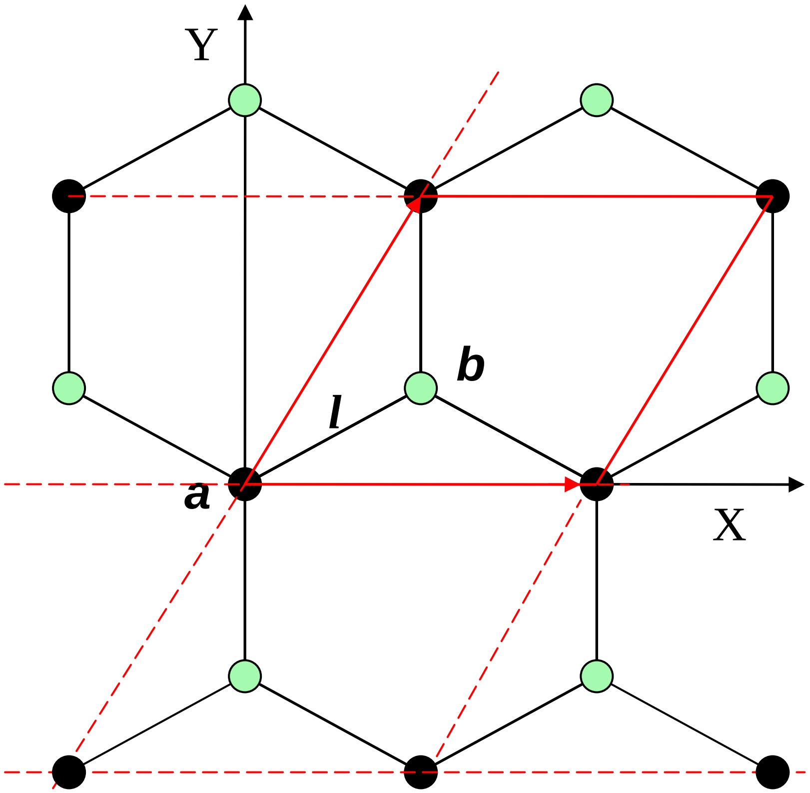
arXiv:0803.3031

Dirac fermions

$$H = -t \sum_{<ij>\alpha} c_{i\alpha}^{+} c_{j\alpha} = \sum_{<ll'>\alpha} \psi_{l\alpha}^{+} h_{ll'} \psi_{l'\alpha}$$

$$\psi_{l\alpha}^{+} = (c_{la}^{+}, c_{lb}^{+})_{\alpha}$$

A. Honeycomb lattice
= the tilted quadrilateral lattice consisting of the unit diamond cells



B. Fourier transform

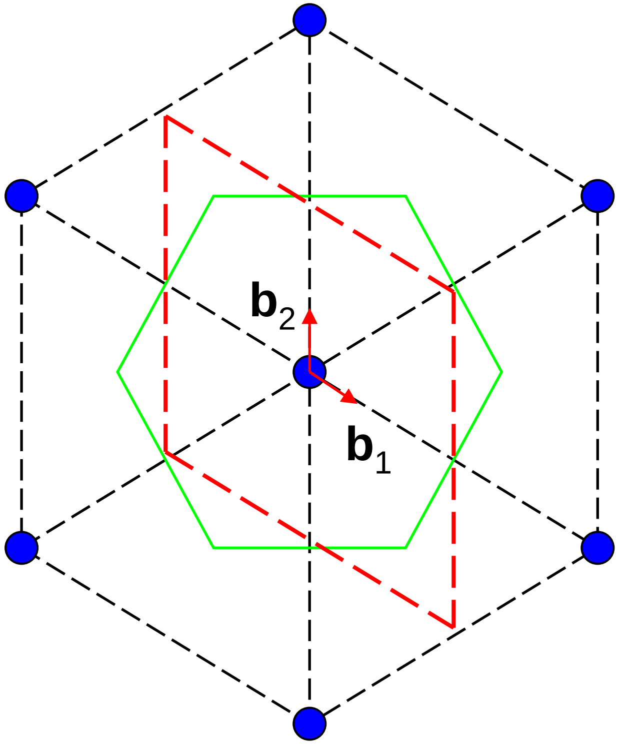
$$\psi_{l\alpha} = \frac{1}{\sqrt{N}} \sum_k \psi_{k\alpha} \exp(i\vec{k} \cdot \vec{l})$$

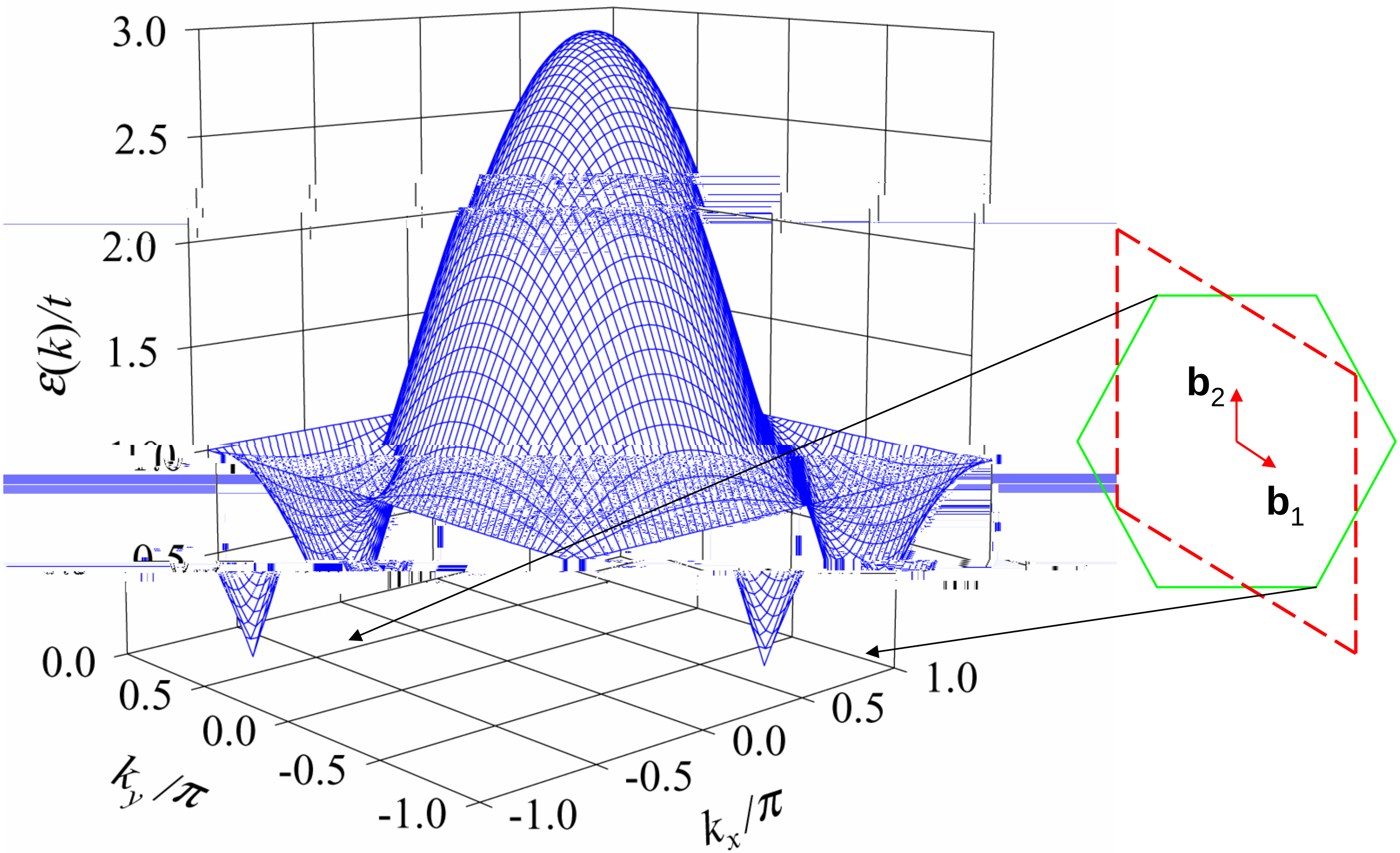
$$H = \sum_{k\alpha} \psi^+_{k\alpha} h_k \psi_{k\alpha}$$

$$h_k = \varepsilon_{k1}\sigma_1 + \varepsilon_{k2}\sigma_2$$

$$\varepsilon_{k1} = -t(1 + \cos k_x + \cos k_y)$$

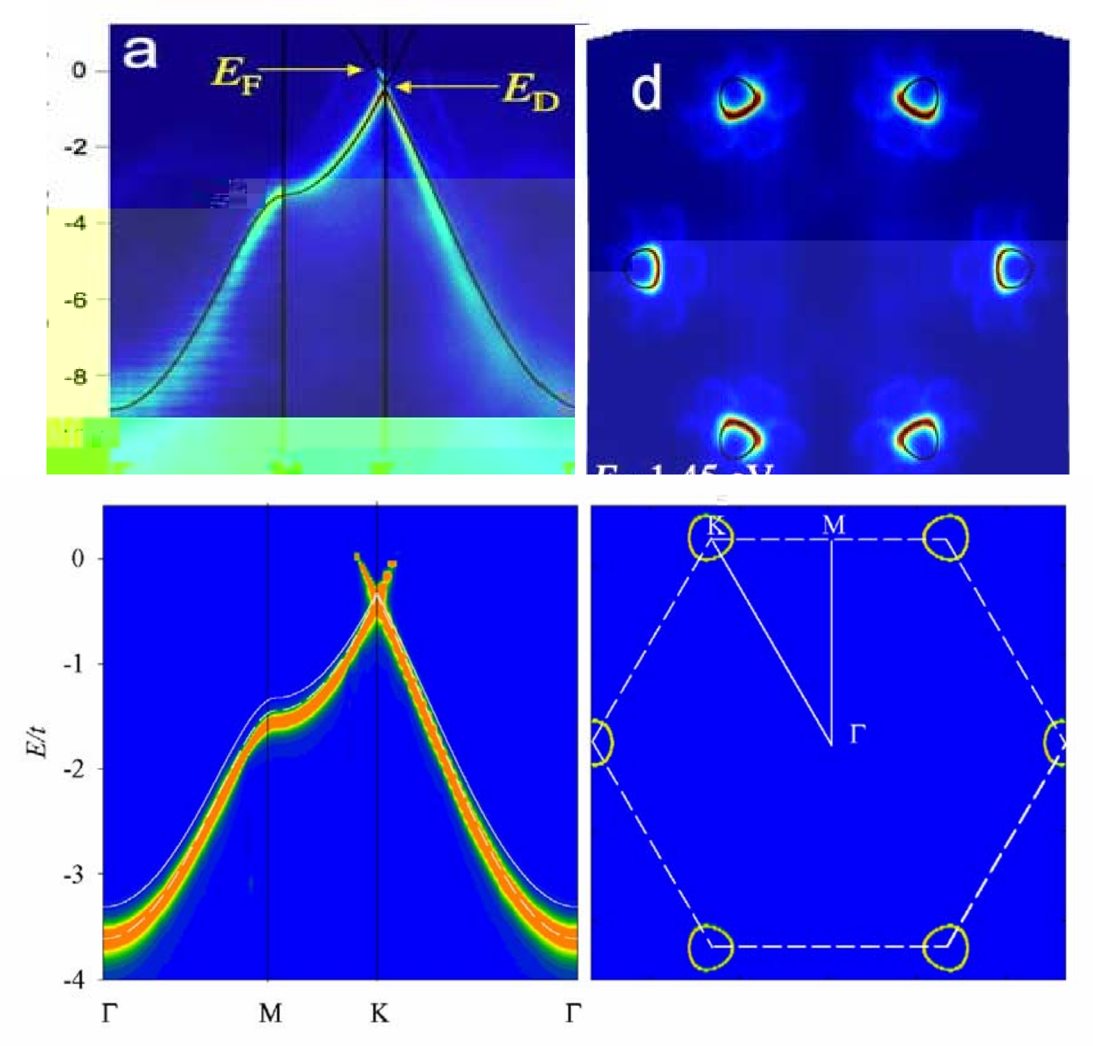
$$\varepsilon_{k2} = -t(\sin k_x + \sin k_y)$$





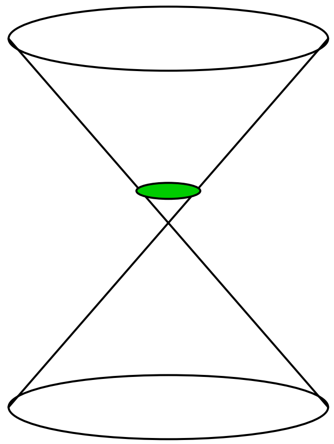
3.0
2.5
2.0
1.5
1.0
0.5
0.0
$\mathcal{E}(k)/t$
k_y/π
k_x/π
-1.0
-0.5
0.0
0.5
1.0
$\mathbf{b}_2$
$\mathbf{b}_1$

Expt: A. Bostwick *et al*., Nat. Phys. 3, 36 (2007)



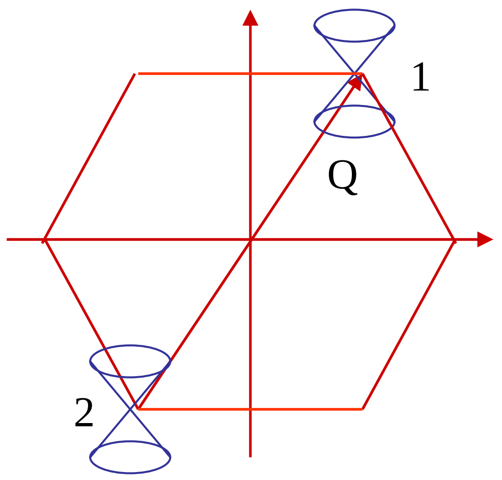
Theory: X. –Z. Yan & C. S. Ting, PRB 76, 155401 (2007)

C. For low energy excitations, expanding h_k around $h_k = 0$ to the order of linear *k,* and returning to the orthogonal system



$$h_k = v\vec{\sigma}\cdot\vec{k}$$

D. There are two Dirac points in the Brillouin zone



$$h_k \Rightarrow v\vec{\sigma}\cdot\vec{k}\tau_3$$

$$\psi^+_{k\alpha} \Rightarrow (c^+_{ka1}, c^+_{kb1}, c^+_{kb2}, c^+_{ka2})_\alpha$$

Dirac particle

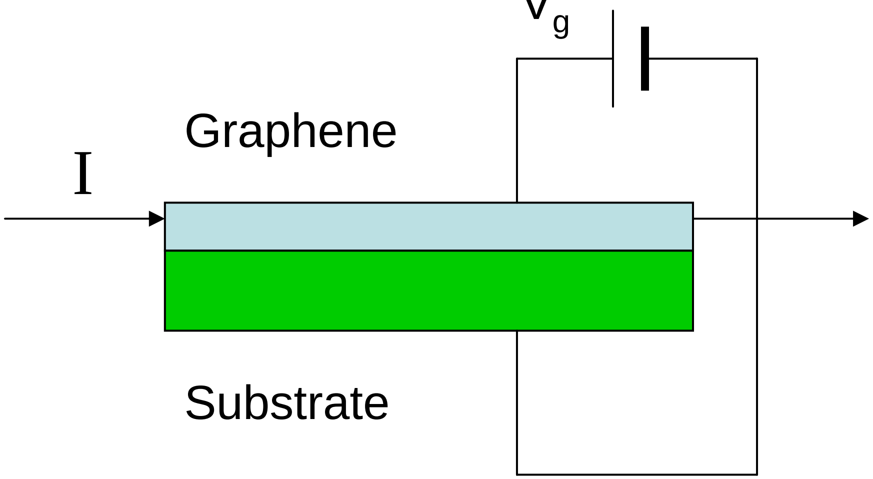
$$H = c\vec{\sigma}\cdot\vec{k}\tau_1 + mc^2\tau_3$$

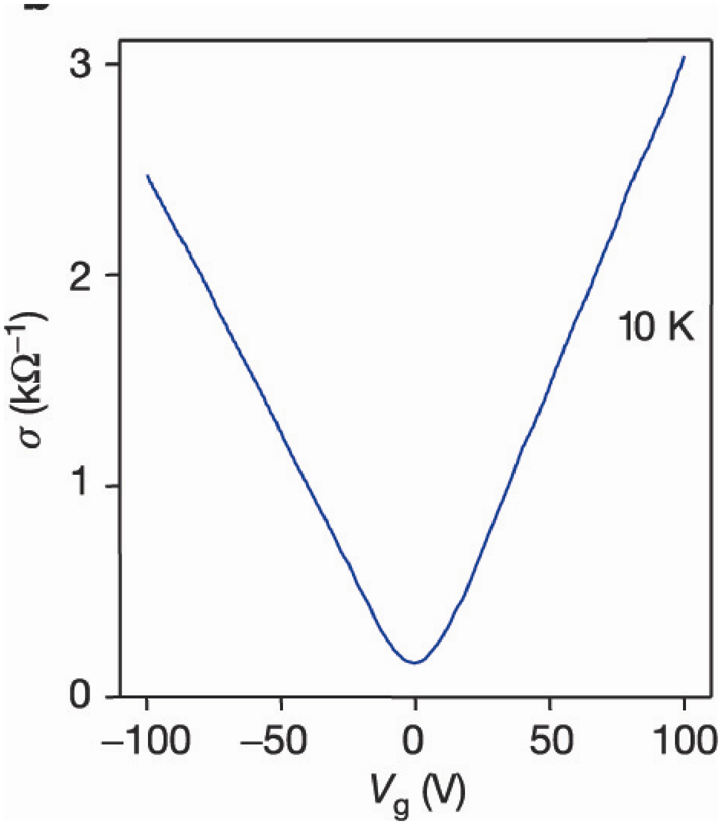
By rotating 90^0 of the τ axes around the τ_2 axis

$$T = \exp(-i\tau_2\pi/4) = (1 - i\tau_2)/\sqrt{2}$$

$$T^+HT = c\vec{\sigma}\cdot\vec{k}\tau_3 - mc^2\tau_1$$

Electric conductivity





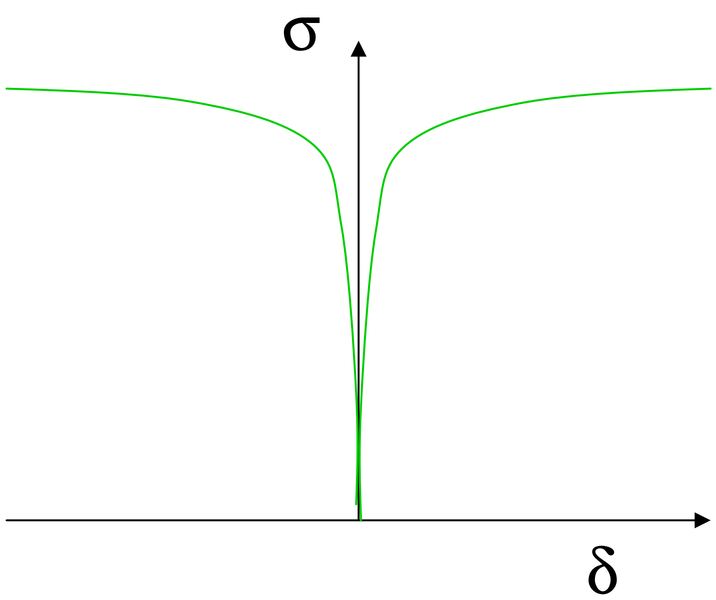
K. S. Novoselov et al., Nature 438, 197 (2005)

Theoretical model

Hamiltonian in the presence of impurities

$$H = \sum_k \psi_k^+ h_k \psi_k + \sum_j \int d\vec{R}\, n(\vec{r}_j) v_i(|\vec{r}_j - \vec{R}|) n_i(\vec{R})$$

For δ-type potential, σ is given as in the figure

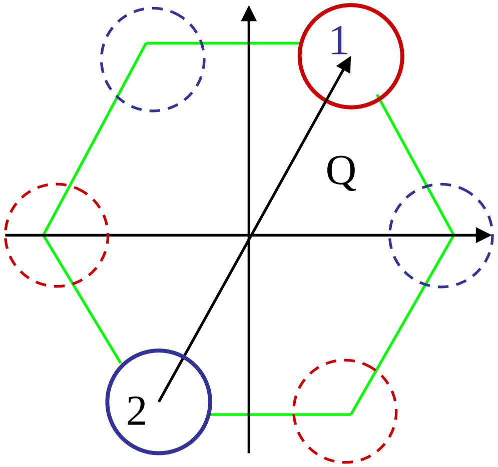


In momentum space

- For low carrier concentration, consider only low energy states close to the Dirac points.

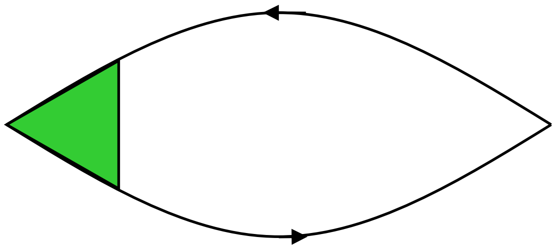
$$H = \sum_k \psi_k^+ h_k \psi_k + \frac{1}{V}\sum_{kq} \psi_{k-q}^+ V_i(q)\psi_k$$

$$V_i(q) = \begin{matrix} n_i(-q)v_0(q)\sigma_0 & n_i(Q-q)v_1\sigma_1 \\ n_i(-Q-q)v_1\sigma_1 & n_i(-q)v_0(q)\sigma_0 \end{matrix}$$



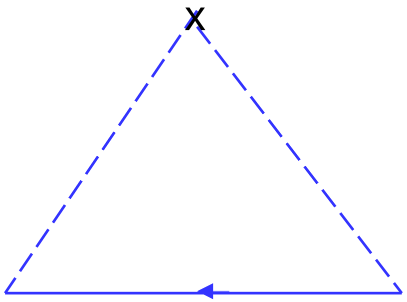
arXiv: 0810.4197

Electric conductivity



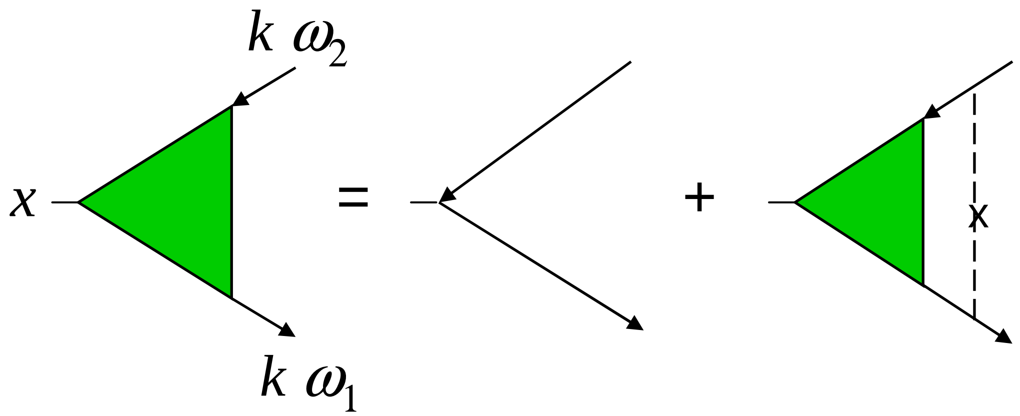
(a) Self-consistent Born approximation

$$G(\vec{k},\omega)=\frac{\tilde{\omega}+h_k\tau_3\vec{\sigma}\cdot\hat{k}}{\tilde{\omega}^2-h_k^2}$$



$$\tilde{\omega}=\omega+\mu-\Sigma_0(k,\omega)$$
$$h_k=vk+\Sigma_c(k,\omega)$$

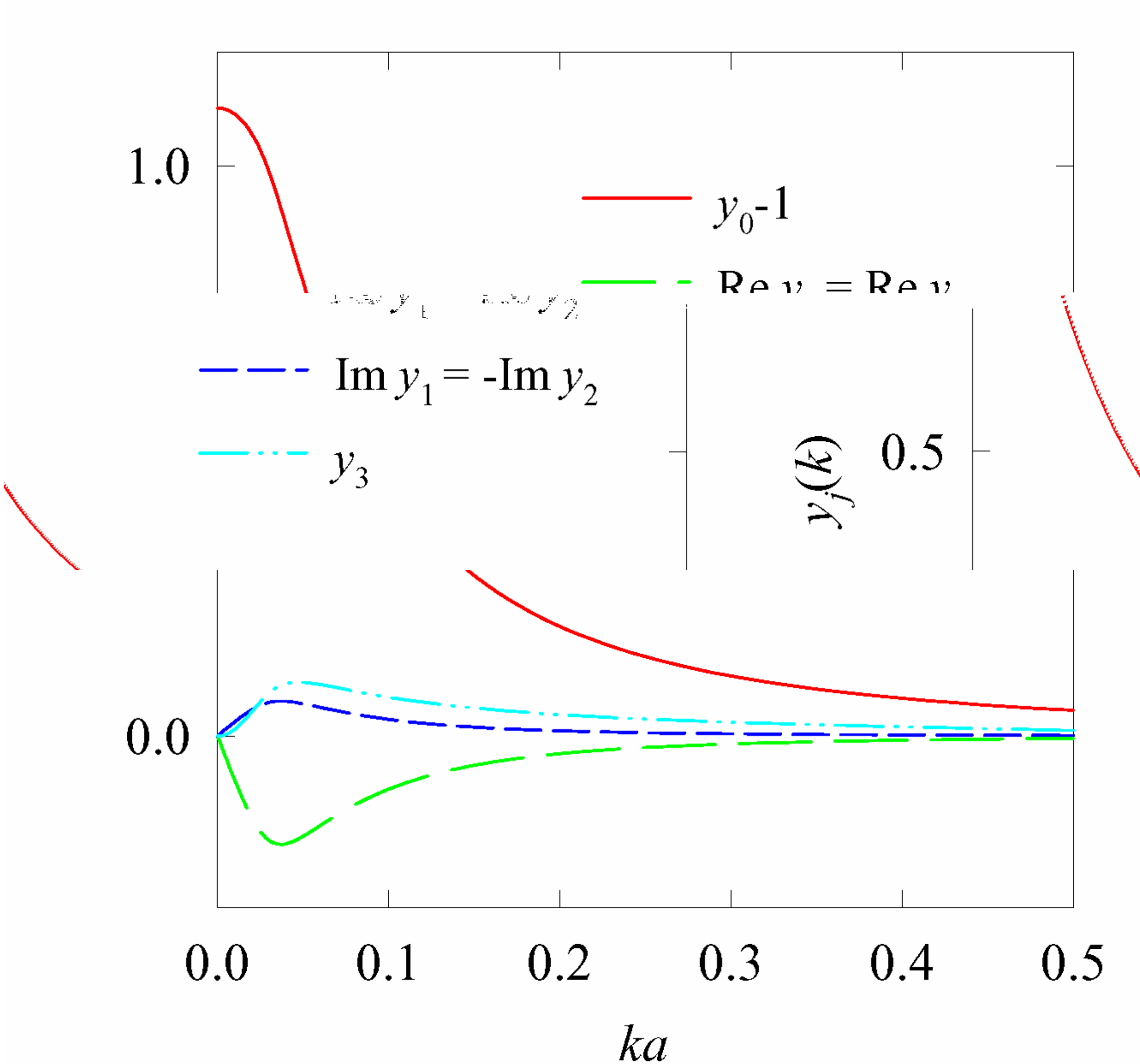
(b) Current vertex, integral 4x4 matrix equation



$$\Gamma_x(\vec{k},\omega_1,\omega_2) = \tau_3\sigma_x + \frac{1}{V^2}\sum_{k'}\langle V_i(\vec{k}-\vec{k}')G(\vec{k}',\omega_1)\Gamma_x(\vec{k}',\omega_1,\omega_2)G(\vec{k}',\omega_2)V_i(\vec{k}'-\vec{k})\rangle,$$

$$\Gamma_x(\vec{k},\omega_1,\omega_2) = \sum_j y_j(k,\omega_1,\omega_2)A_j^x(\hat{k}),$$

$$\begin{aligned}
A_0^x(\hat{k}) &= \tau_3\sigma_x, \\
A_1^x(\hat{k}) &= \sigma_x\vec{\sigma}\cdot\hat{k}, \\
A_2^x(\hat{k}) &= \vec{\sigma}\cdot\hat{k}\sigma_x, \\
A_3^x(\hat{k}) &= \tau_3\vec{\sigma}\cdot\hat{k}\sigma_x\vec{\sigma}\cdot\hat{k}
\end{aligned}$$

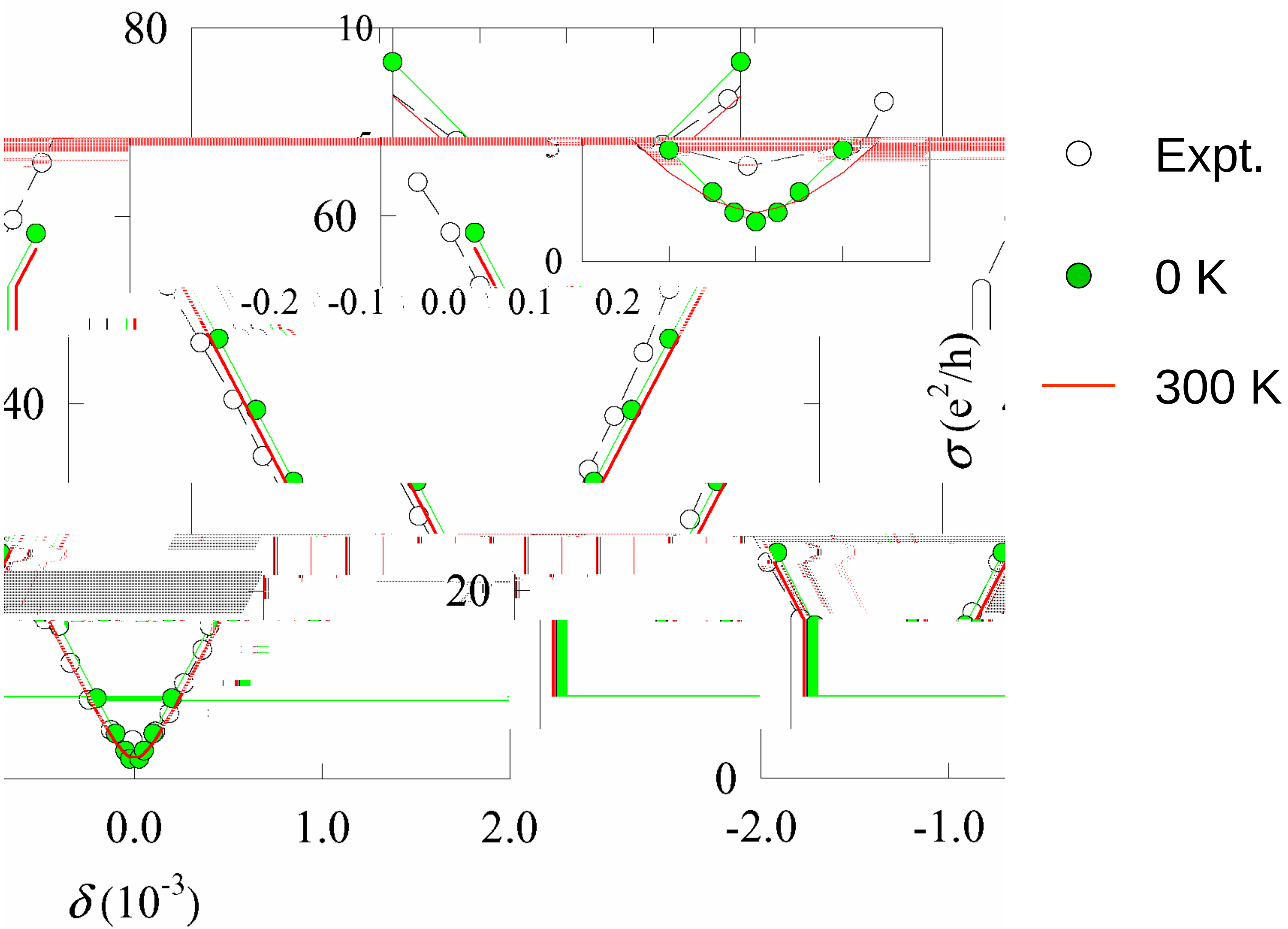


$$y_j(k,-i0,i0)$$

$$T = 0$$

$$\delta = 1.0\times10^{-4}$$

For details, see X. –Z. Yan *et al.* PRB, 77, 125409 (2008)



80
60
40
20
0
10
5
0
-0.2
-0.1
0.0
0.1
0.2
0.0
1.0
2.0
-2.0
-1.0
δ (10^-3)
σ (e^2/h)
Expt.
0 K
300 K

Summary

1. Using SCBA, we have presented electric transport theory for Dirac fermions under finite-range impurity scatterings in graphene.

   4 integral equations for determing the vertex correction

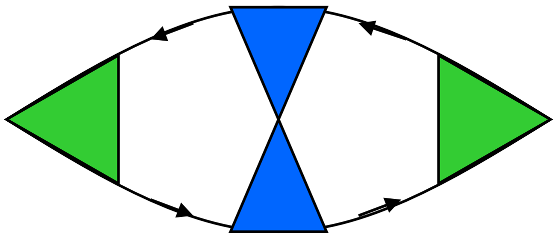
2. The theory is in good agreement with experiment.

Weak Localization of Dirac Fermions in Graphene

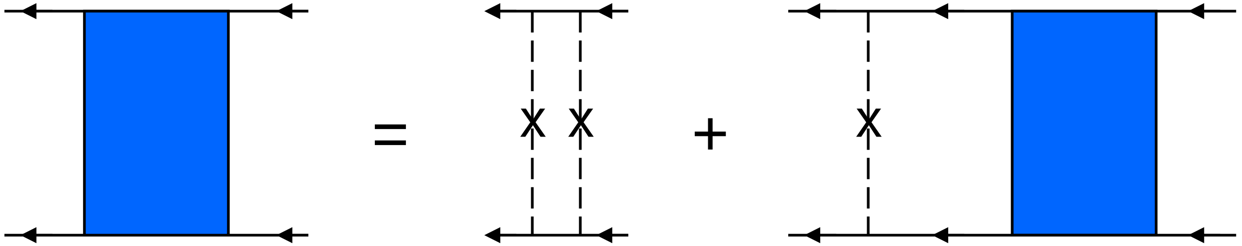
- No WL was observed, contradictory with the conventional theory
- Existing theories, $\delta(r)$-potential impurities
- Charged impurities— $\sigma \propto$ concentration of doped electrons
- For charged impurity, what is the theoretical prediction for WL of DF in graphene?

- For details, see X. –Z. Yan and C. S. Ting, PRL, 101, 126801 (2008)

Quantum-interference correction (QIC) to the electric conductivity



Cooperon propagator



To solve this integral 4x4 matrix equation, we classify the Cooperon by *pseudospin* and *isospin* according to McCann *et al*. [PRL, 97, 146805 (2006)]

- *singlet* pseudospin channel, to WL effect
- *triplet,* delocalization effect

Isospin

$$\Sigma_0 = \tau_0\sigma_0, \quad \Sigma_1 = \tau_3\sigma_1, \quad \Sigma_2 = \tau_3\sigma_2, \quad \Sigma_3 = \tau_0\sigma_3$$

pseudospin

$$\Lambda_0 = \tau_0\sigma_0, \quad \Lambda_1 = \tau_1\sigma_3, \quad \Lambda_2 = \tau_2\sigma_3, \quad \Lambda_3 = \tau_3\sigma_0$$

$$M_s^l = \Sigma_2\Sigma_s\Lambda_2\Lambda_l$$

Cooperon in isospin-pseudospin representation

$$C_{ss'}^{ll'} = \frac{1}{4}\sum_{\{j,\alpha\}}(M_s^l)_{\alpha_1\alpha_2}^{j_1 j_2} C_{\alpha_1\alpha_2\alpha_3\alpha_4}^{j_1 j_2 j_3 j_4}(M_{s'}^{l'+})_{\alpha_4\alpha_3}^{j_4 j_3}$$

Solution for the Cooperon propagator

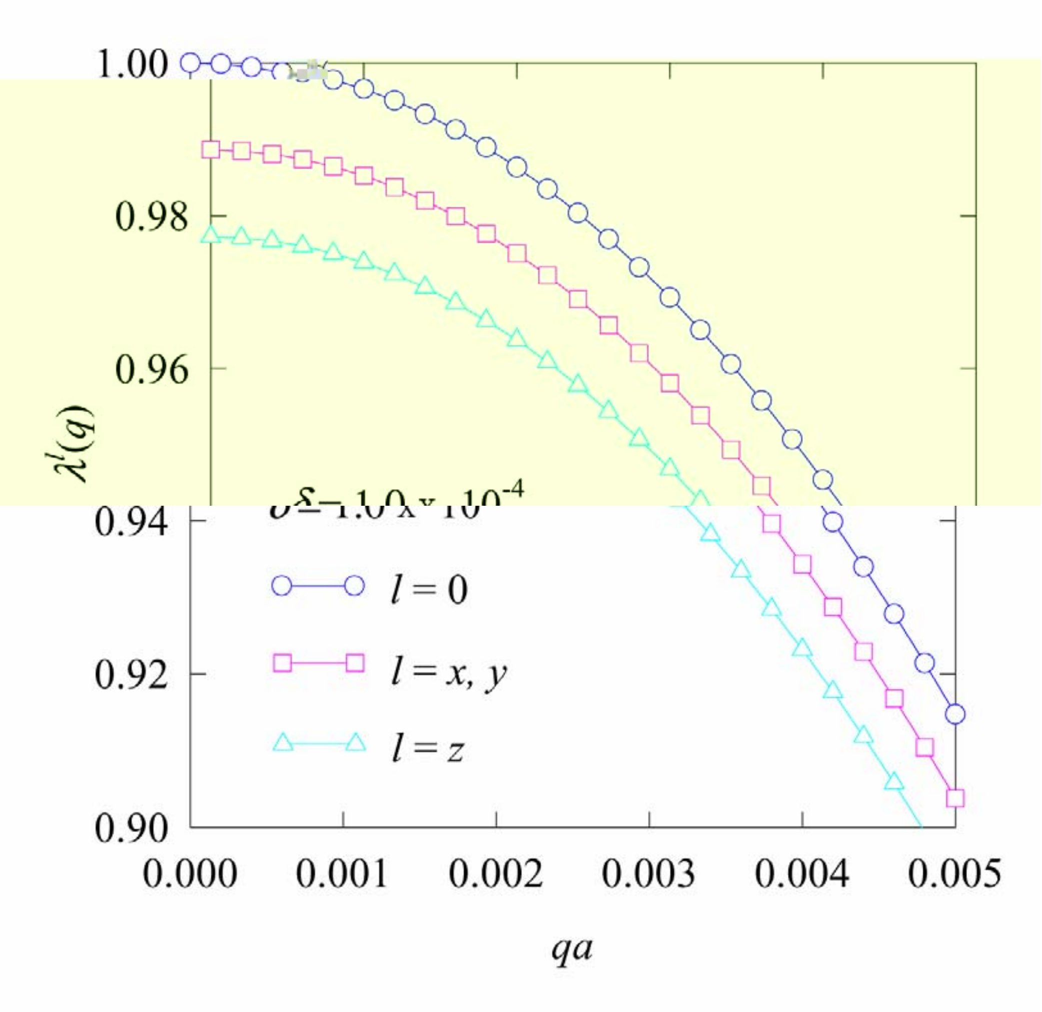
1. For the Cooperon of zero momentum $q = 0$ in the *singlet* pseudospin channel, we obtain an explicit eigen state ψ within the SCBA. The state is the most important one since it gives rise to the logarithmic divergence to QIC.

2. For small $q > 0$ or *triplet* channel, the important states are obtained by perturbation from ψ.

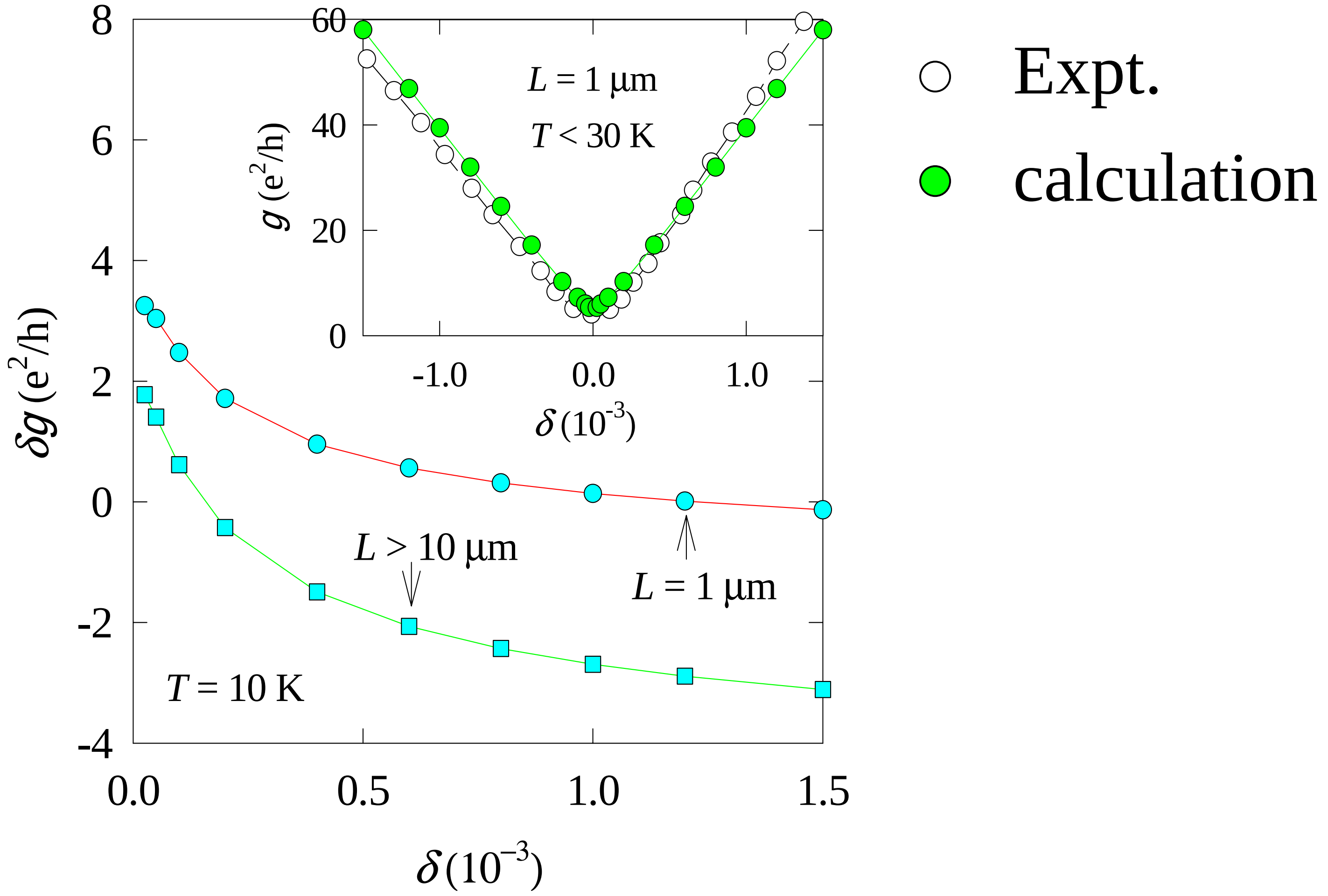
Lower cutoff $q_m = \max(1/L_{in}, 1/L)$.

τ_{in} inelastic collision time, using the result of Yan&Ting, PRB **76**, 155401 (2007).

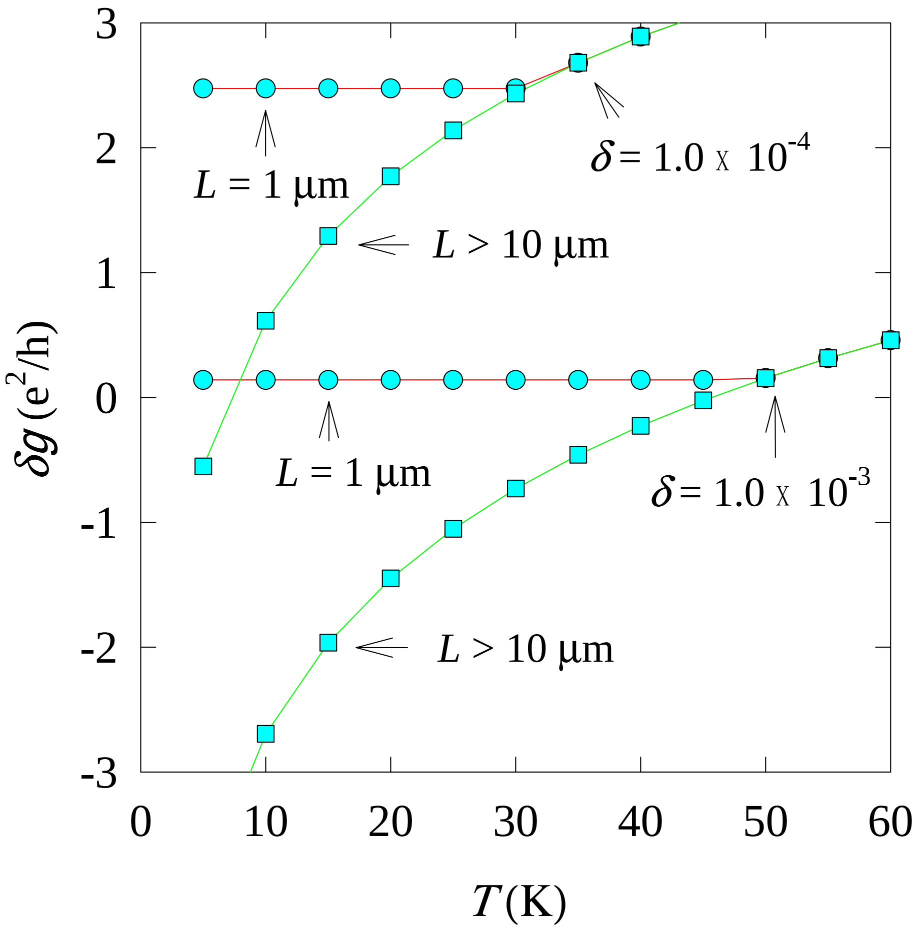
Eigenvalues of Cooperon propagator



QIC & the corrected conductivity *g*



QIC as function of *T* for various sample sizes *L*



Summary

1. Using SCBA, we have investigated WL of Dirac fermions under finite-range impurity scatterings in graphene.
2. The WL is present for large samples at finite carrier concentrations. Close to zero doping, the system may be delocalized.WL is quenched at low T for small size samples.
3. The calculated minimum conductivity is about 4.5, in good agreement with experiment.