Localization in an Inhomogeneous Interacting Quantum Wire

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Collaborations and Acknowledgments

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- Experiments: Hadar Steinberg, Ophir Auslaender, Amir Yacoby
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Papers:

- cond-mat/0501684 (PRB 72, 045315)
- cond-mat/0506812 (PRB 73, 113307)
- arXiv:0707.2992 (PRB 77, 085314)

Luttinger Liquid and Momentum Resolved Tunneling between Quantum Wires

Luttinger liquid model of 1D interacting quantum system

- The Luttinger liquid is a **stable xed point** of 1D quantum systems with gapless excitations
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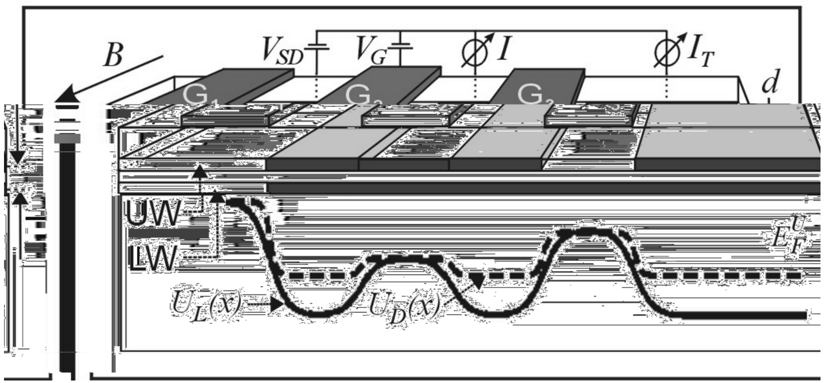
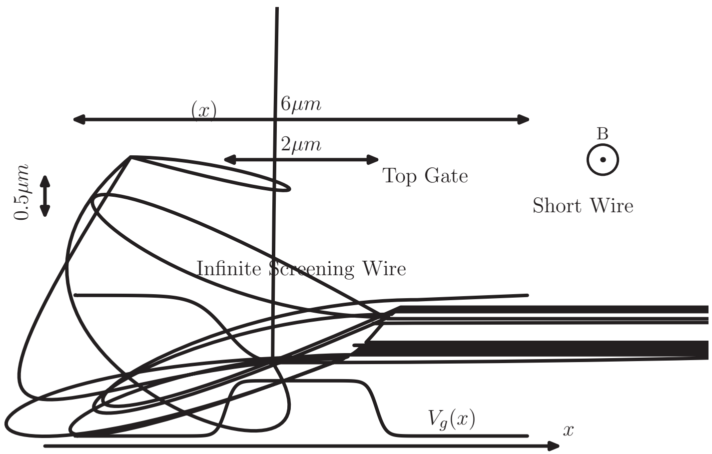
Schematic Diagram of Experimental Setup



Tunneling conductance is $G = dI_T/dV_{SD}$. Experiments measure dG/dV_g to pick out physics sensitive to density.



Model geometry, charge density distribution $r(x)$ and gate voltage V_g

Theory of Tunneling Conductance

At zero temperature, only the tunneling between ground states contributes. The tunneling conductance $G \propto |M(k_+)|^2 + |M(k_-)|^2$, where

$$k_\pm = \pm k_F^{lower} + e\mathbf{B}d/\hbar \ ,$$

and

$$M(k) = \langle \Psi^N | c_k^\dagger | \Psi^{N-1} \rangle \ .$$

It is instructive to de ne a \quasi-wavefunction":

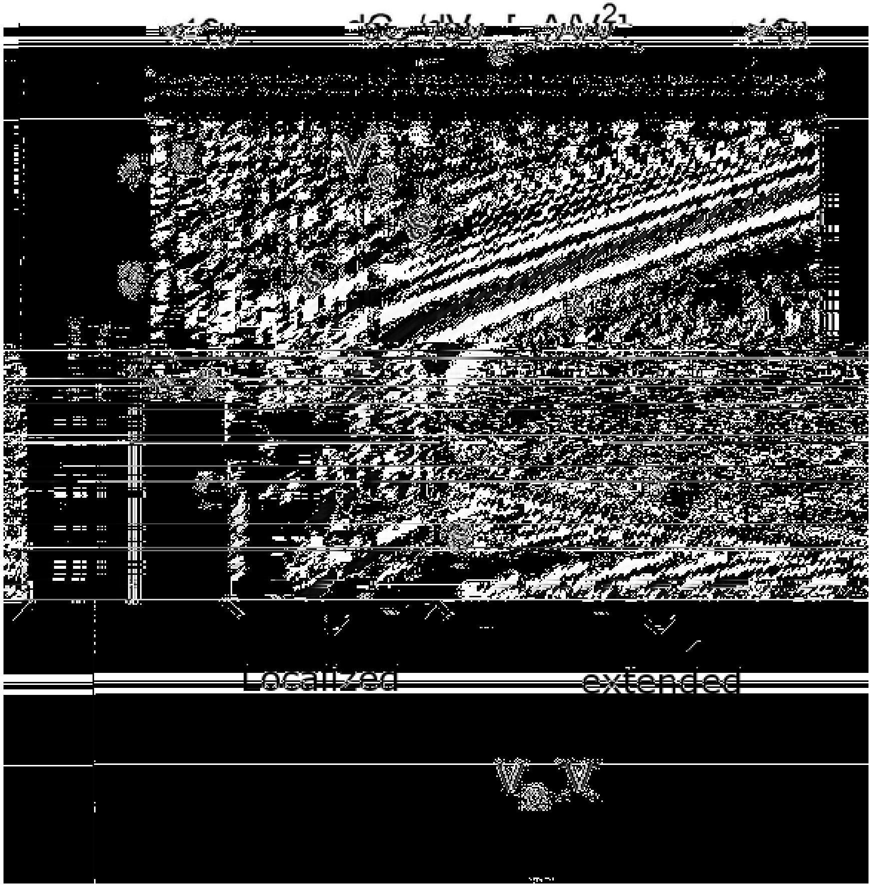
$$\Psi_{\text{eff}}^N(x) \equiv \langle \Psi_g^{N-1} | \psi_s(x) | \Psi_a^N \rangle \ ,$$

then

$$M(k) = \int dx e^{ikx} \Psi_{\text{eff}}^{N*}(x).$$

For non-interacting wire Ψ_{eff}^N is simply the wavefunction of last occupied electron.

Overall Features of Experimental Data

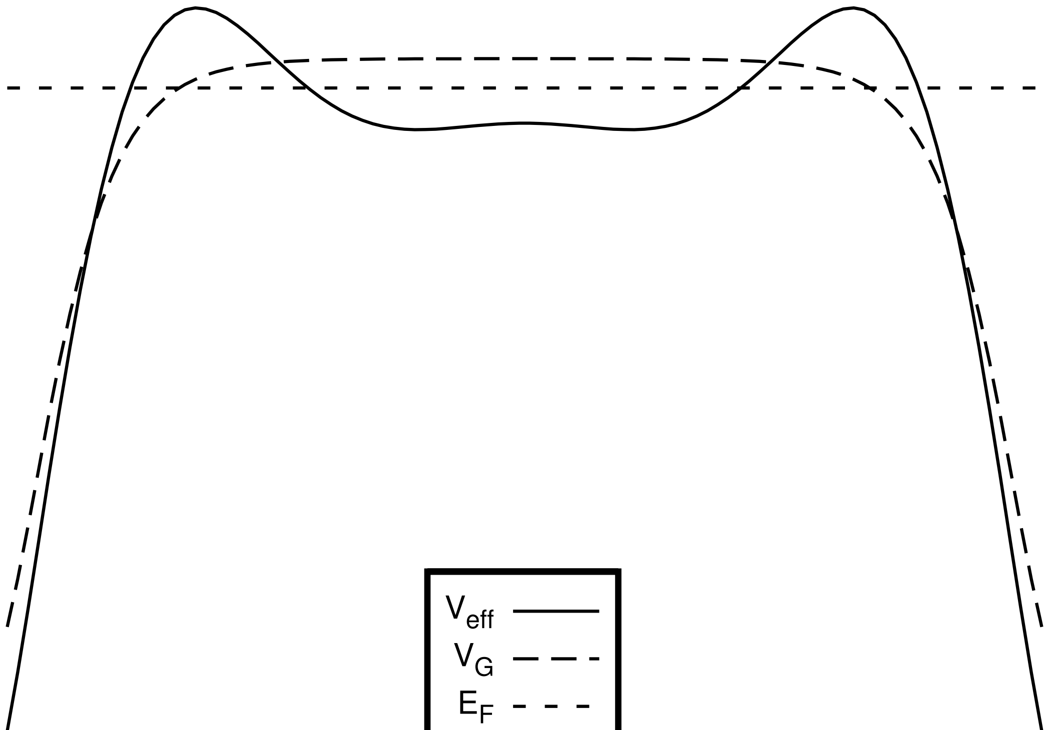


Key Features:

- Extended state: density n changes continuously with gate voltage V_g, momentum dependence of tunneling sharply peaked at $k = \pm k_F(n)$.

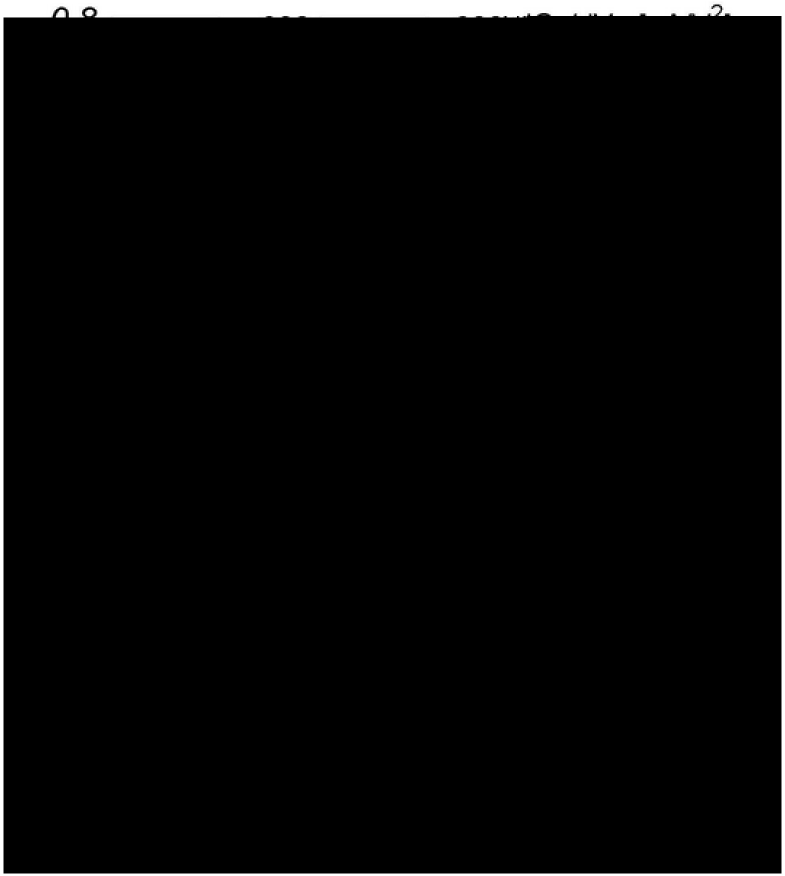
- Localized state: density n changes discretely with gate voltage V_g, momentum dependence of tunneling extended over a wide range of k.

Self-consistent Con ning Potential Generated by Interaction?



hypothetical self-consistent potential in the localized phase

Part I: Momentum Dependence of Tunneling Conductance



Basic features of momentum distribution:

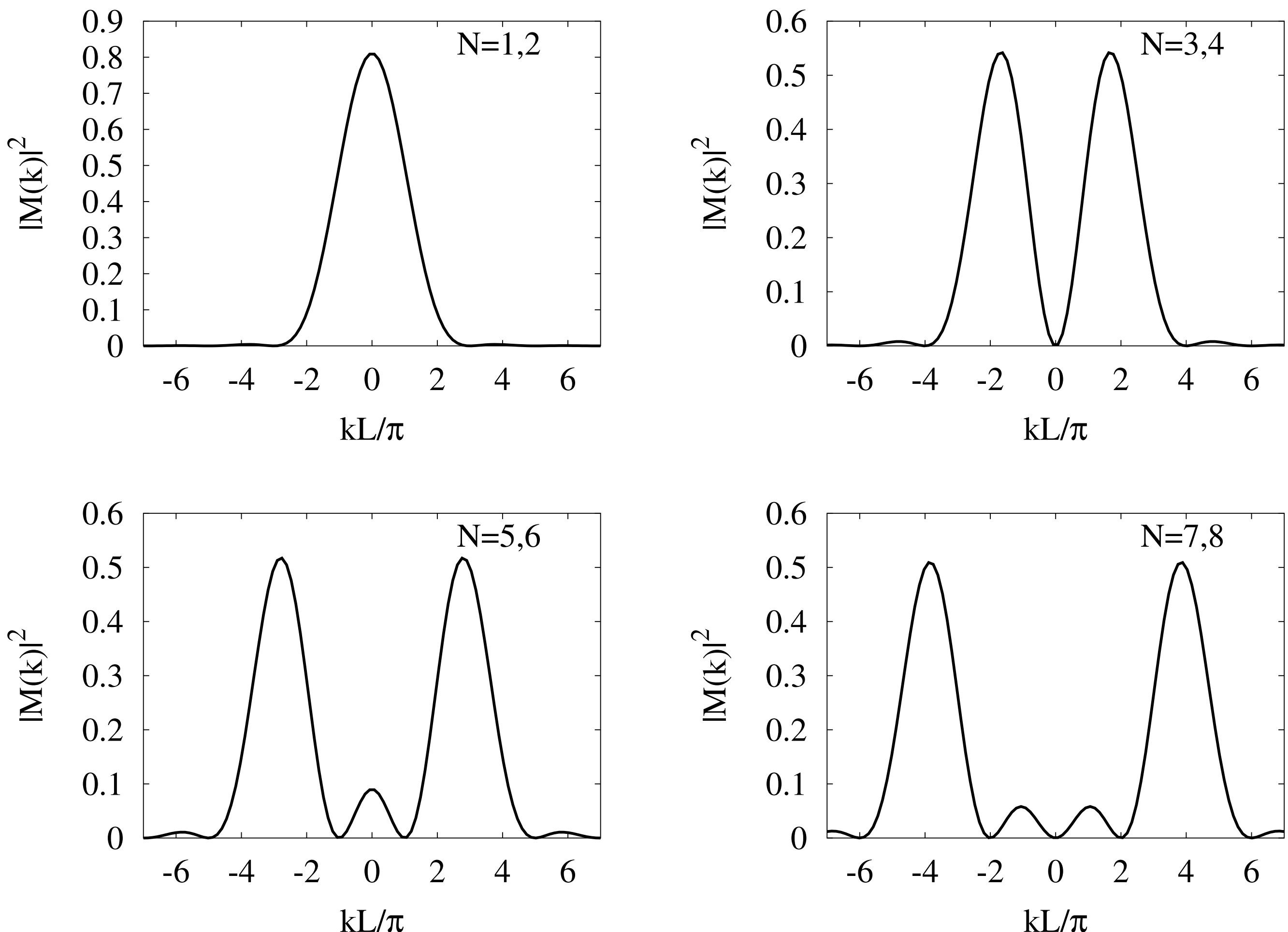
- Broad momentum distribution, implying localized electrons

- Typically two broad peaks, the separation between which widens with increasing particle number N

- Last Coulomb blockade peak has single peak in momentum distribution

Non-interacting Electrons, $T = 0$, Box with Hard Wall

As large N, $|M(k)|^2$ becomes peaked at $k_N = Np/L$ with width $dk = 2p/L$.

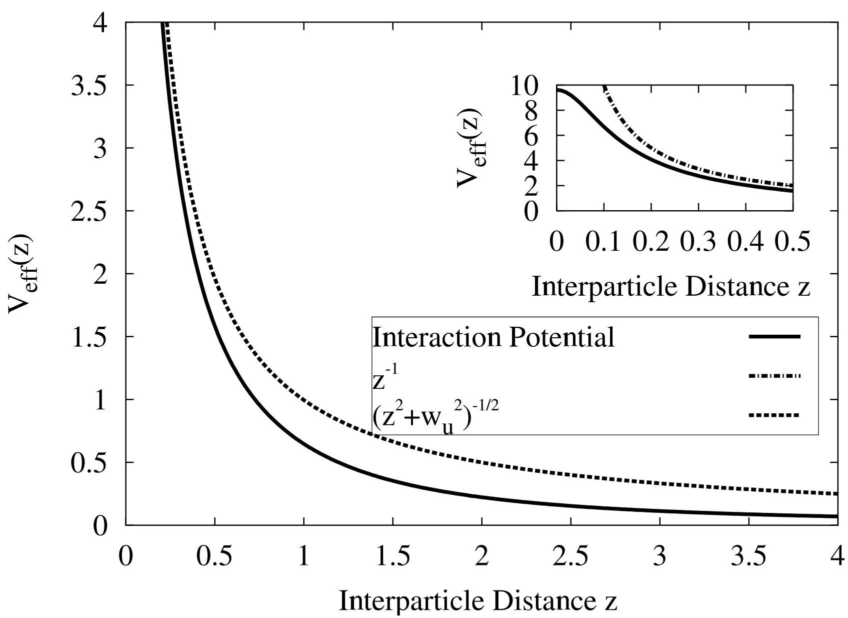


Screened Coulomb Interaction Potential

The Coulomb interaction in the short upper wire is assumed to have a **short-range** cuto due to the nite width of the wire and **long-range** cuto due to screening by the more conducting lower wire.

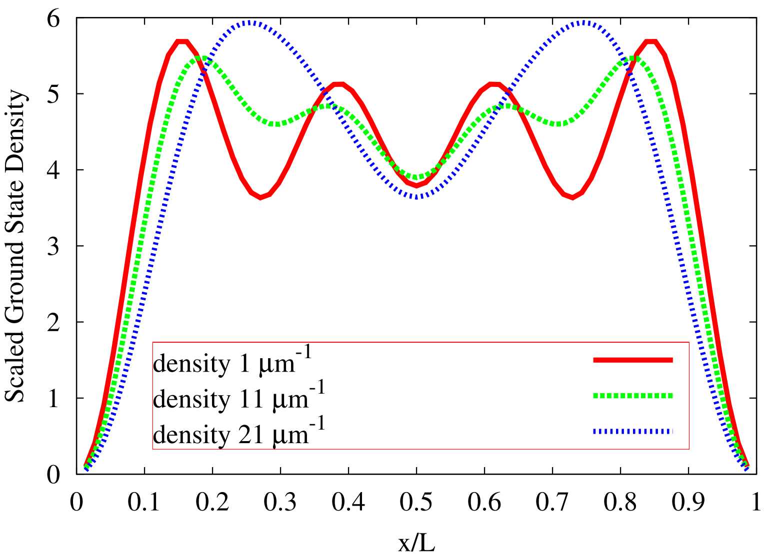
$$\tilde{V}_{\text{eff}}(q) = \tilde{V}_0(q, W_u) - \frac{\tilde{V}_0^2(q,d)}{\tilde{V}_0(q, W_l)} \,,$$

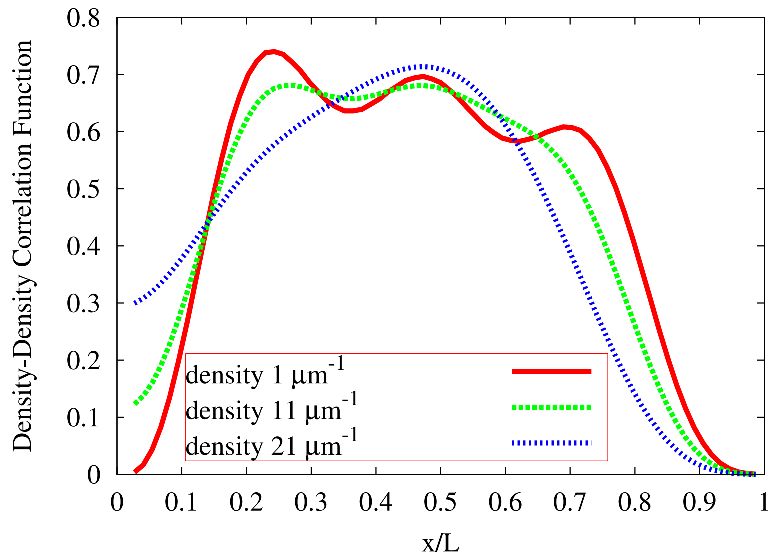
where $\tilde{V}_0(q,W) = \int_{-\infty}^{\infty} dx \frac{e^{iqx}}{\sqrt{x^2+W^2}} = 2K_0(Wq)$. K_0 is modi ed Bessel function.



Formation of Quasi-Wigner Crystal Order with Lowering Density

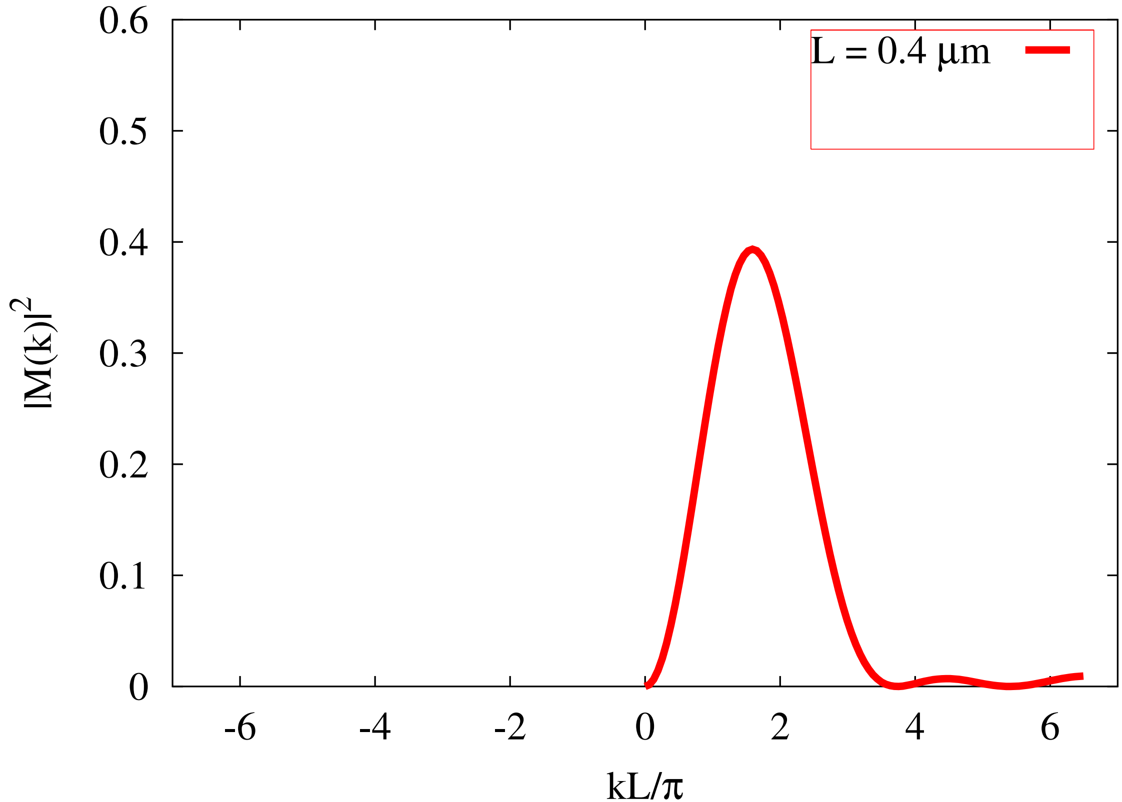
Instead of the Friedel oscillation of frequency **$2k_F$**, clear oscillations of frequency **$4k_F$** show up, both in density and in density-density-correlation, at low density. Here density-density correlation function is de ned as $\frac{1}{1-x}\int_0^{1-x} r(x')\, r(x'+x)\, dx'$.





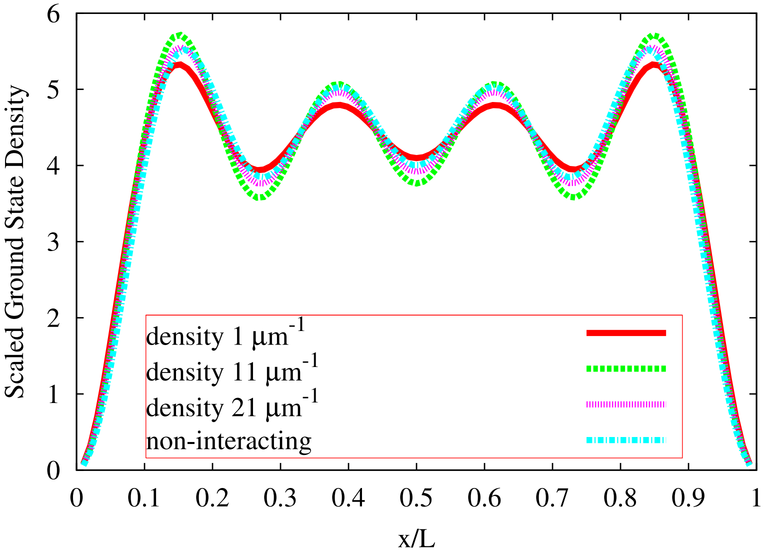
Ground State Tunneling: Exact-Diagonalization

$|M(k)|^2$ is insensitive to interactions. Following plot show $|M(k)|^2$ for tunneling from $N = 3$ to $N = 4$ state.

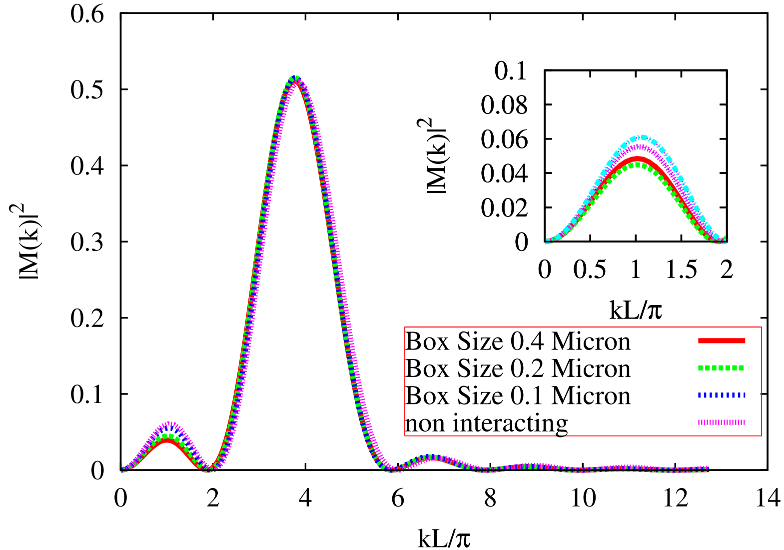


Spinless/Polarized Electrons

Under the experimental parameters, spinless electrons are essentially non-interacting for both high and low density.



(i) Density of Four Interaction Spinless Electrons



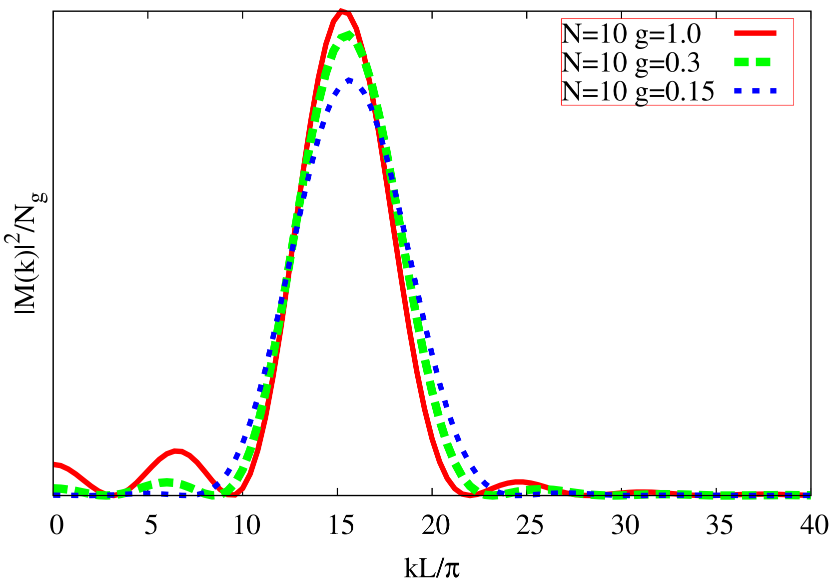
(j) Tunneling of Three to Four spinless Electron

Large-N limit: Ground State Tunneling

For large but finite N and not too close to wall, Luttinger liquid theory gives an estimate of the ground state quasi-wavefunction as defined before:

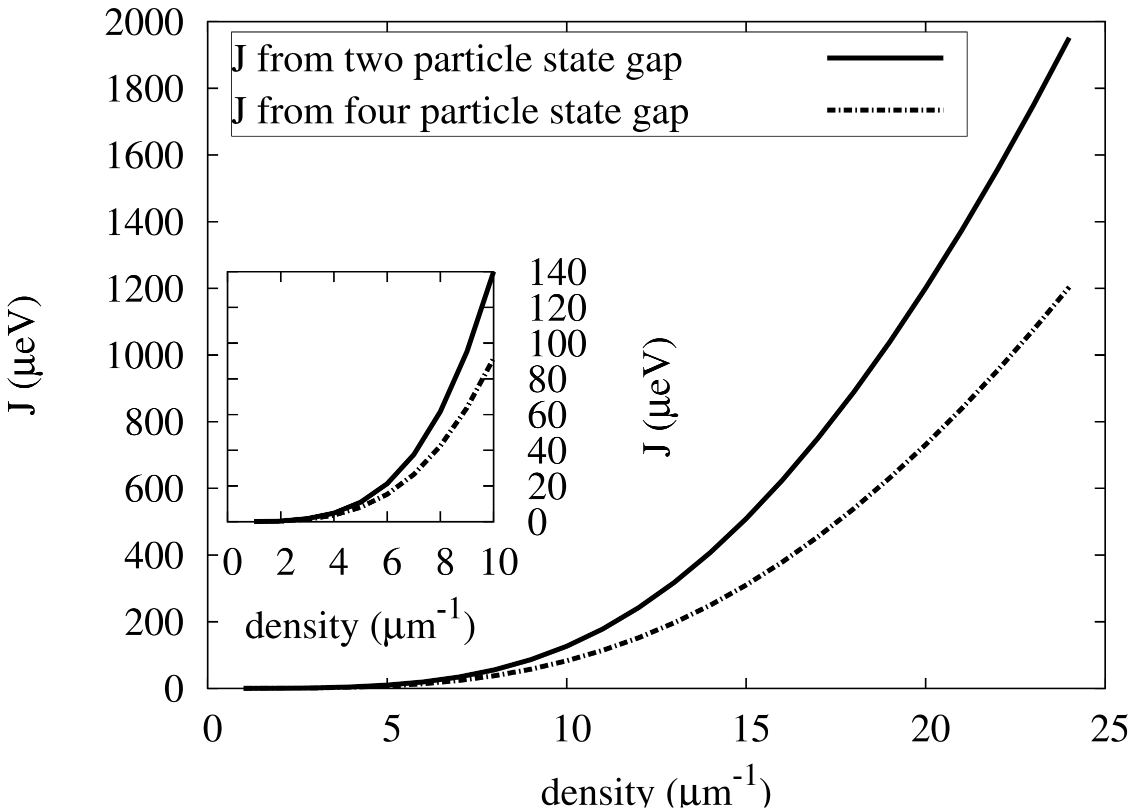
$$\Psi_{\rm eff}^{N}(x) \sim \frac{1}{\sqrt{LN^{\alpha}}}\left[\sin\left(\frac{\pi x}{L}\right)\right]^{\frac{1}{2}(\alpha_{\rm end}-\alpha)} \sin(k_F x) \,,$$

where tunneling exponent for bulk and end is given by Luttinger liquid interaction parameter g as $\alpha = (g+g^{-1}-2)/4$ and $\alpha_{end} = (g^{-1}-1)/2$, respectively. A normalization factor N_g is used so that the integrated areas under the three curves are the same.



Estimates of E ective Heisenberg Exchange Constant J

At strong interaction, the dynamics of system can be described by Heisenberg model. The Heisenberg exchange parameter J can be extracted from gap D between ground state and rst excited state. For $N = 2$ $J = D$ and for $N = 4$ $J = 1.5178D$.



Tunneling Conductance at Finite Temperature

Total Conductance $G = C(\mathscr{B}(k_+) + \mathscr{B}(k_-))$, where

$$\mathscr{B}(k) = \sum_{\alpha\gamma\sigma} |\langle \Psi_\alpha^N | c_{k\sigma}^\dagger | \Psi_\gamma^{N-1} \rangle|^2 w_{\alpha\gamma} \ ,$$

$$\begin{aligned} w_{\alpha\gamma} &= e^{-\beta[E_\gamma^{N-1} - \mu(N-1)]} f(\epsilon_{\alpha\gamma}) \\ &= e^{-\beta(E_\alpha^N - \mu N)} [1 - f(\epsilon_{\alpha\gamma})] \ , \end{aligned}$$

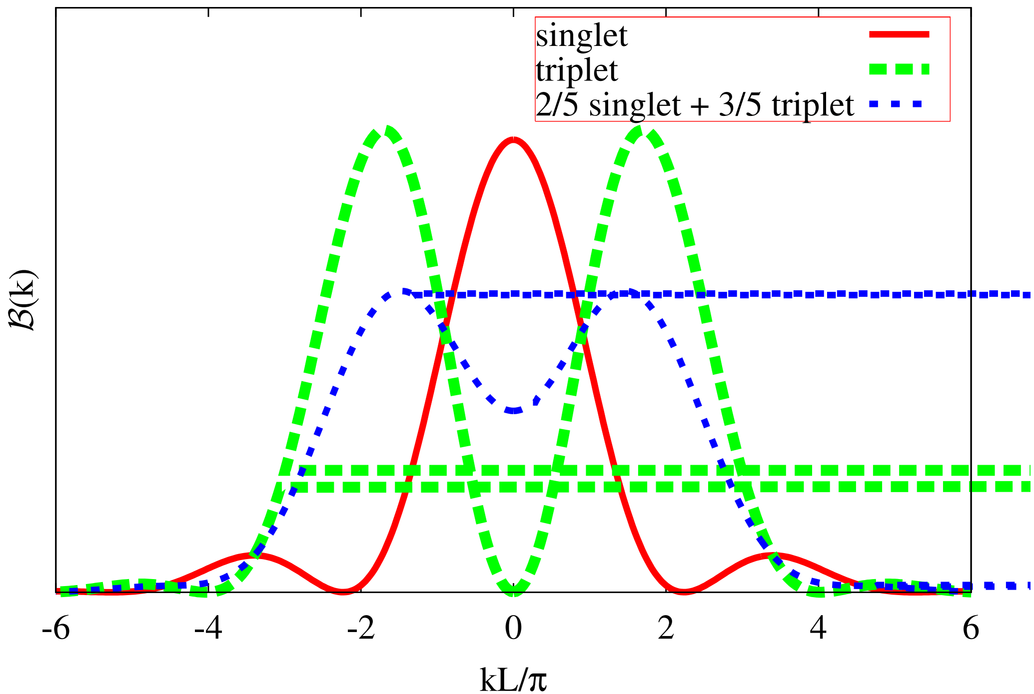
$$k_\pm = \pm k_F^l + eBd/\hbar \ .$$

$$\epsilon_{\alpha\gamma} = E_\alpha^N - E_\gamma^{N-1} \ ,$$

$$C = \frac{\pi e^2}{2\hbar} \lambda^2 \beta \nu L \frac{e^{-\beta\mu N}}{Z_N + e^{-\beta\mu} Z_{N-1}} \ ,$$

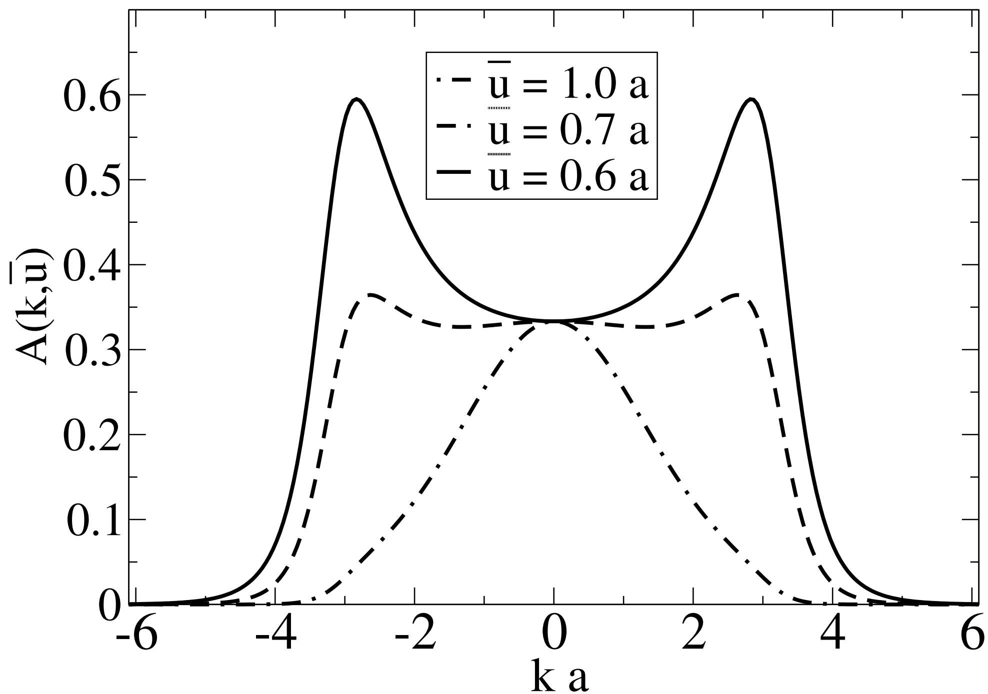
Finite Temperature and Mixing Spins: Exact Diagonalization

For strong repulsive interaction, spin excitation energy scale $D = J/N$ may become very small. Three energies scales are important: spin gap D, Zeeman energy E_Z and thermal energy $k_B T$. Following picture shows tunneling from $N = 1$ to $N = 2$: the case of singlet ground state ($E_Z < D$), triplet ground state ($E_Z > D$) and high temperature mixed state($E_Z, D \ll k_B T \ll D_{charge}$).



Finite Temperature and Mixing Spins: Free Spin Regime and Large N limit

If $J \ll k_B T \ll \hbar v_c k_F$, spin configurations have equal thermal weight but there's no charge excitation, we find a spectral weight as following:



Here $\bar{u} = \frac{a}{\pi}\sqrt{2g\ln(L/a)}$ is the root-mean-square fluctuation of electron position. g is the Luttinger liquid interaction parameter.

Conclusions for Part I

We investigated the momentum dependence of tunneling matrix elements from a in nite wire into short quantum containing interacting electrons.

• For $N \leq 4$ exact diagonalization is carried out, ground state tunneling matrix element $|M(k)|^2$ is computed.

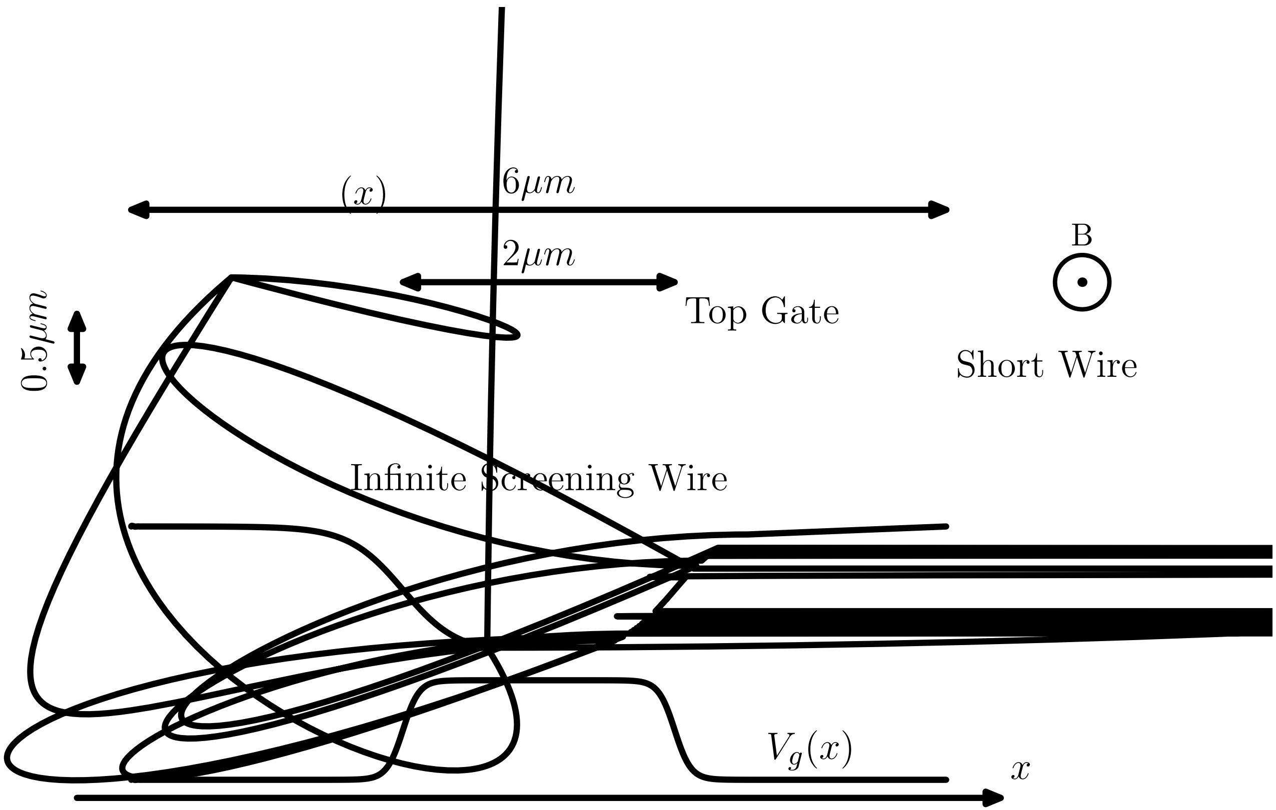
• Large N calculations of tunneling amplitude, both for ground state tunneling and for free spin regime, are carried out using Luttinger liquid theory.

Other Possible Factors in Accounting for Experimental Observation

• Soft instead of hard wall con nement: more spectral weight at center.
• Partial spin polarization
• Asymmetry of con nement potential

Part II: Electronic States of Low Density Region

Model geometry for the electronic density distribution $r(x)$ and gate potential $V_g(x)$



The Restricted Hartree-Fock Hamiltonian

Assumptions and simplifications of the Hartree-Fock model:

- Spin restricted to be either aligned or anti-aligned with magnetic field B
- Two subbands corresponding to different transverse modes in the quantum wire
- Electrons in different subbands interact only through Hartree terms

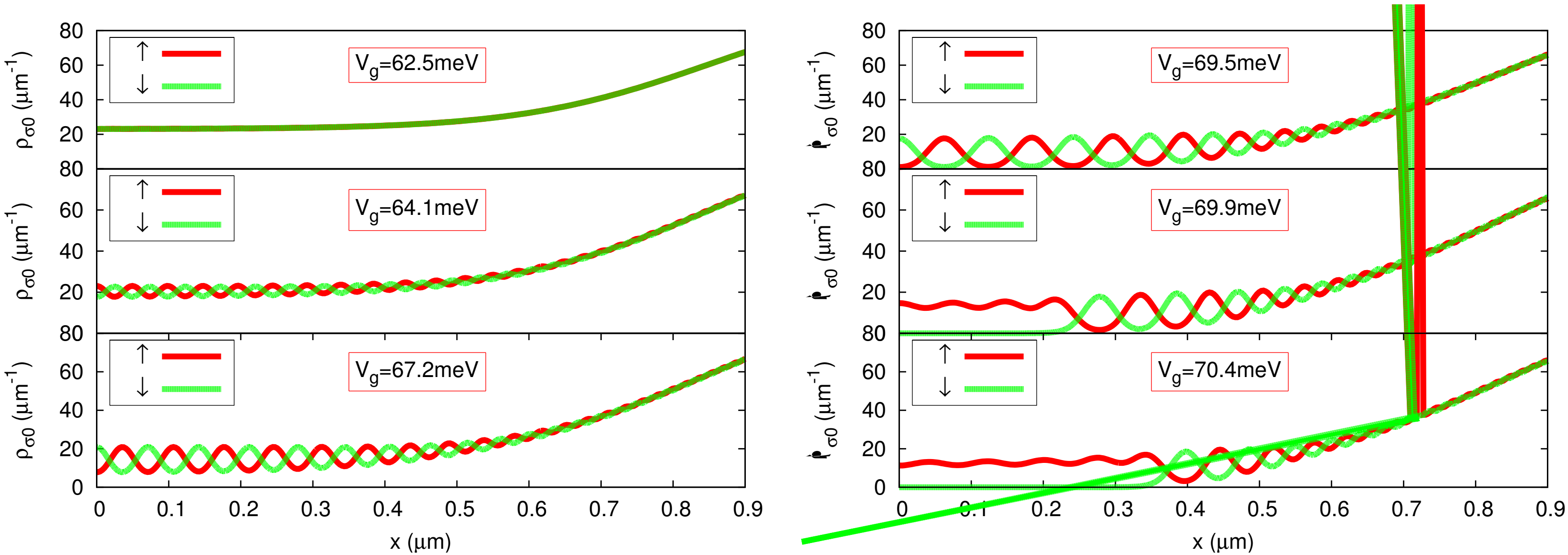
$$
\begin{aligned}
H\psi_{\sigma\beta}(x) = {} & -\frac{\hbar^2}{2m^*}\frac{\partial^2\psi_{\sigma\beta}(x)}{\partial x^2} + (V_G(x)+\Delta_\beta)\psi_{\sigma\beta}(x) + \mu_B B S_z \psi_{\sigma\beta}(x) \\
& + V_H(x)\psi_{\sigma\beta}(x) - \int dx' V_F^{\sigma\beta}(x,x')\psi_{\sigma\beta}(x')
\end{aligned}
$$

$$
V_H(x) = \int dx' \Big(\sum_{i,\sigma',\beta'} |\psi_{i\sigma'\beta'}(x')|^2\Big) V_{\text{eff}}(x-x')
$$

$$
V_F^{\sigma\beta} = \sum_i \psi_{i\sigma\beta}(x)\psi^*_{i\sigma\beta}(x') V_{\text{eff}}(x-x').
$$

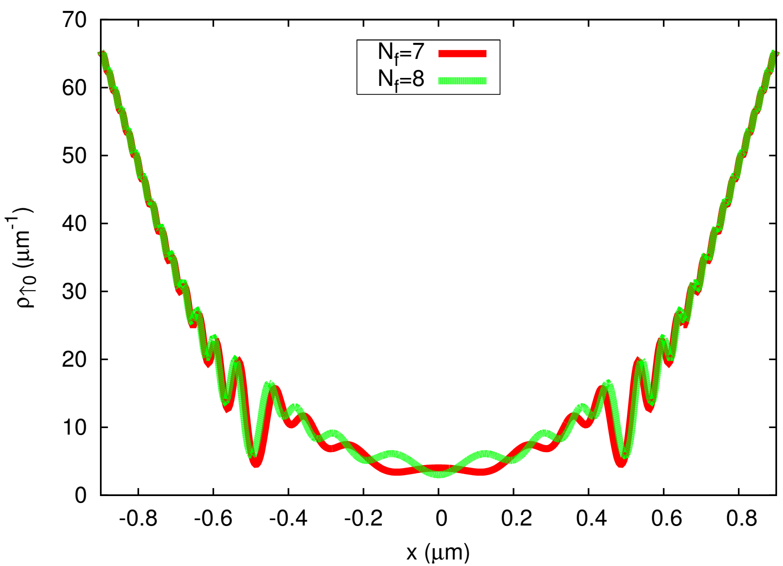
Magnetic Phases at the Low Density Region

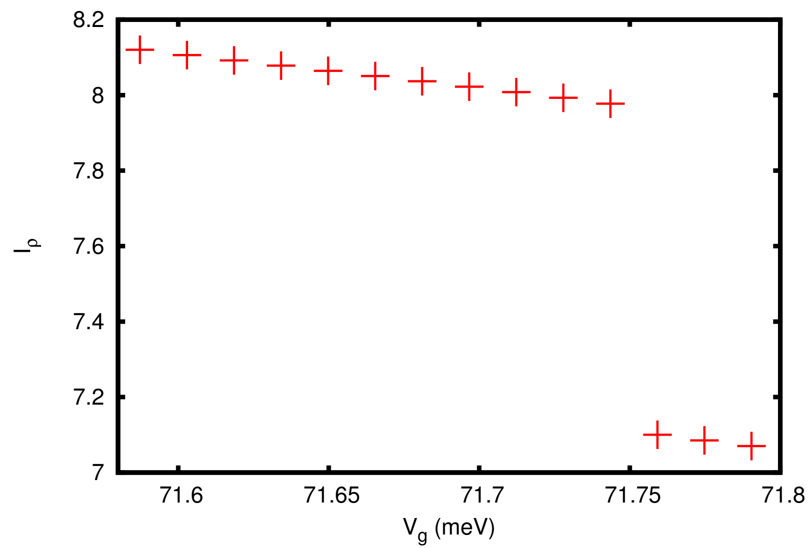
- The emergence of an antiferromagnetic order at the low density region (left)
- The emergence of spin-aligned region at the center of the wire (right)



Discrete Density Changes in the Spin-aligned Phase

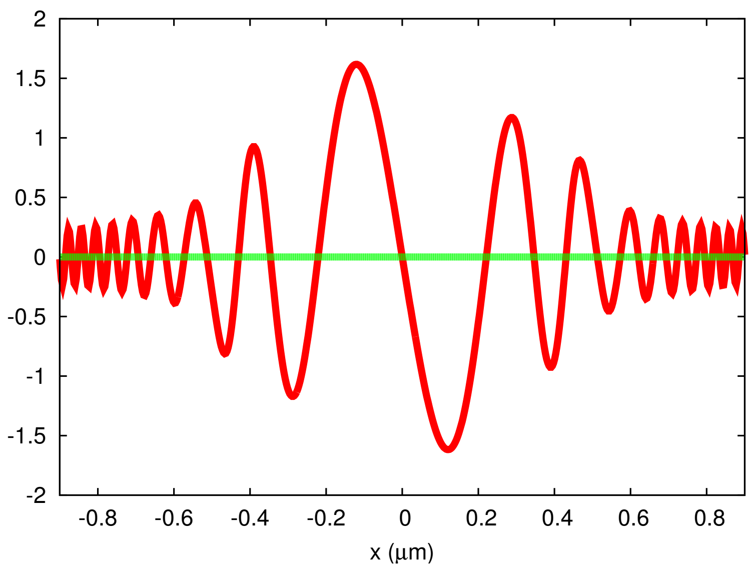
Abrupt density rearrangements occur due to the successive expulsion of a single electron from the spin-aligned region

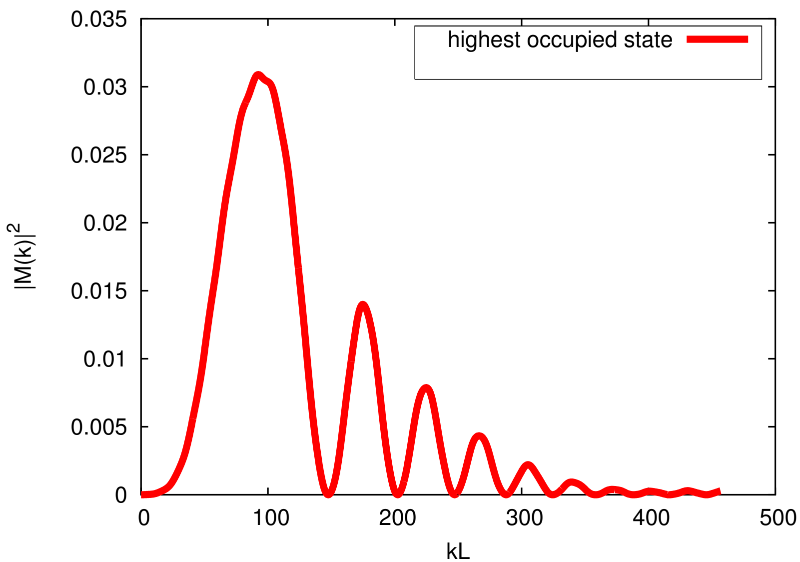




Nature of Electronic States in the Spin-aligned Region

- The wavefunctions near the Fermi level have large weights near the center
- No sign of self-consistent barrier at the ends of spin-aligned region
- Little spectral weight near $k = 0$ in the momentum-dependent tunneling matrix element

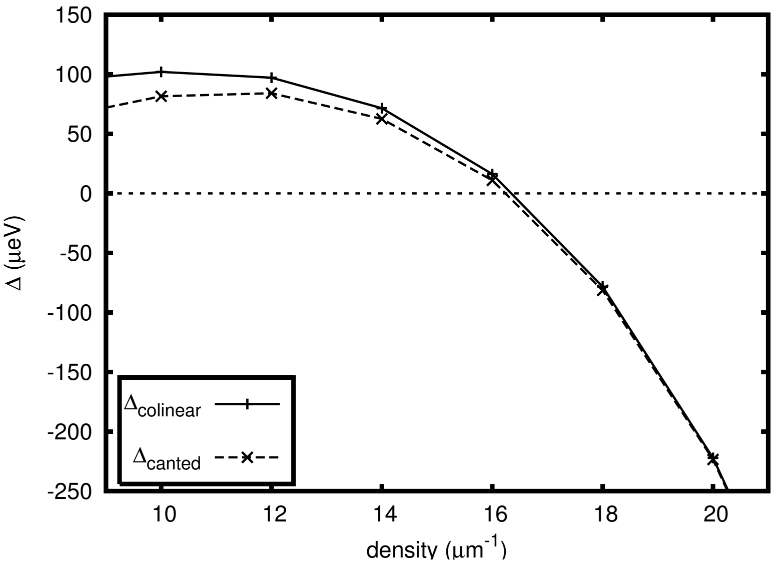


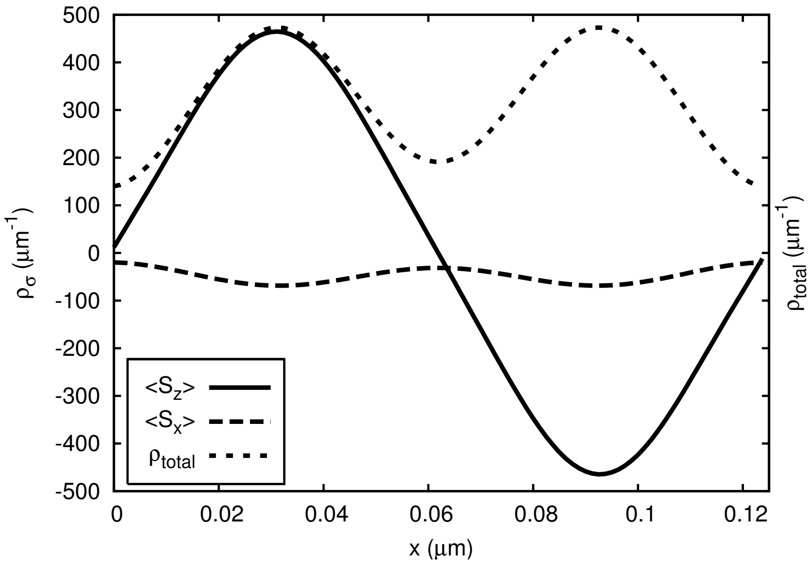


Left wavefunction of $N = 7$ solution at E_f Right transition from $N = 7$ to $N = 8$ solutions

Unrestricted Hartree-Fock in a Homogeneous system: E ects of Canting in the Antiferromagnetic Phase

- The correction to the energy per unit cell due to canting is small in the range where the ground state is antiferromagnetic
- The S_x magnetization is small for canted solution at $r = 16mm^{-1}$, where the system make a transition to a ferromagnetic ground state





Conclusions for Part II

We investigated the density and spin con guration an inhomogeneous quantum wire using the restricted Hartree-Fock method. We found:

- When lowering its density, the depleted region goes from a non-magnetic state to an antiferromagnetic state, and nally to a spin-aligned state sandwiched by antiferromagnetic states
- In the spin-aligned phase, the spin-aligned region undergoes abrupt density changes by successively losing a single electron
- The wavefunctions near the Fermi surface are relatively localized near the center, but they are not Coulomb-blockade states con ned by barrier potentials
- Additional mechanisms, such as impurity potentials or multiple spin state contributions, are needed to explain the observed large spectral weigh near $k = 0$ in the momentum dependent tunneling
- In our model, the e ects due to the canting of the spins in the unrestricted Hartree-Fock model are small