

Outline

- Introduction
- Information refrigerator
- Clausius's inequality
- Summary

Introduction

Is there any relation between information and heat?

Or is there any relation between the **computer** and the **refrigerator**?



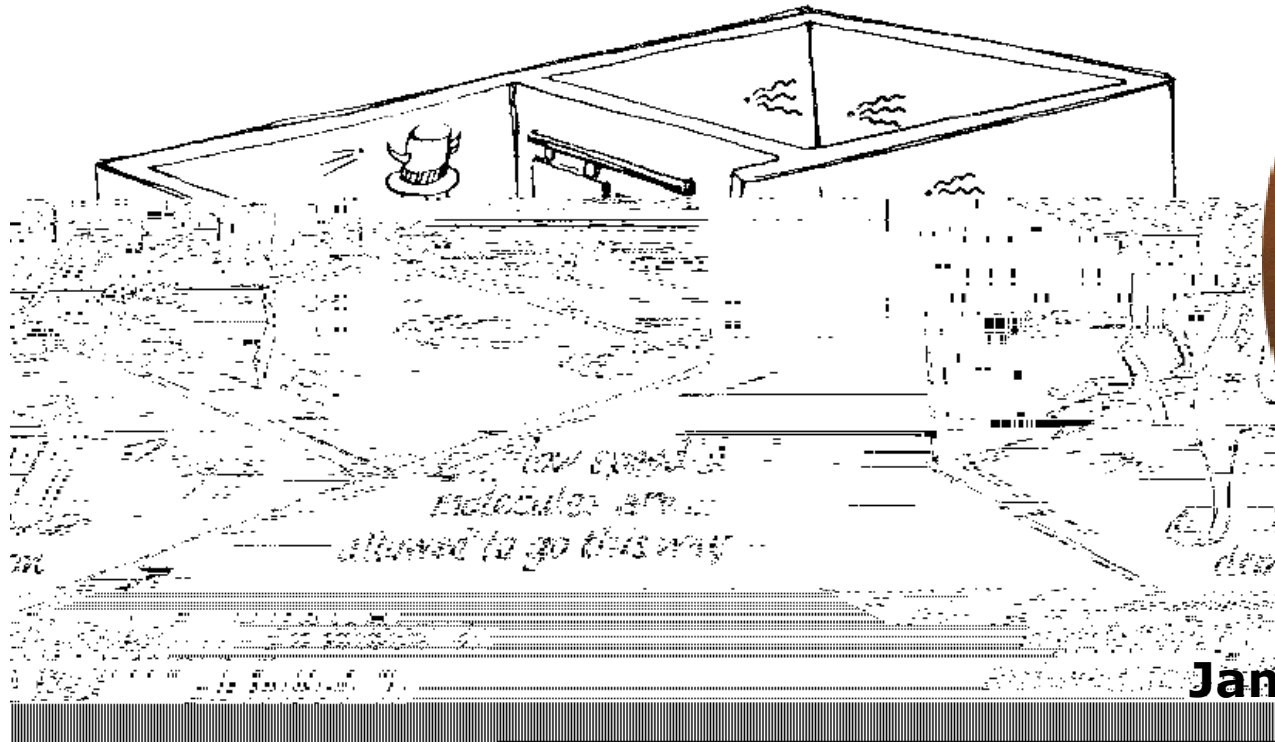
What is information?

What is one bit of information?

- Information is related to probability, oncto

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Maxwell's demon thought experiment



James Clerk Maxwell

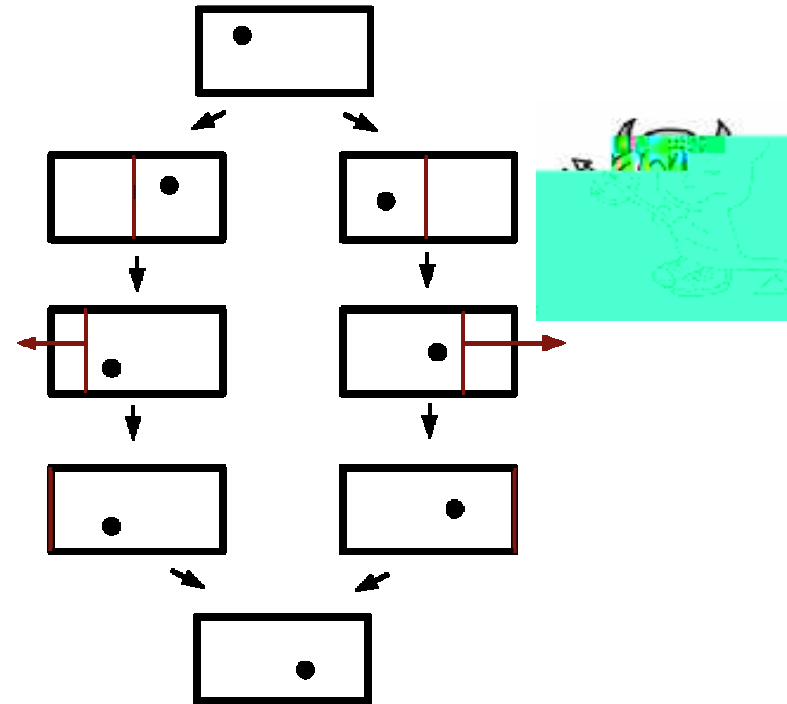
Theory of Heat (Longmans, London, 1871)

1831 - 1879

Szilard's engine



Leo Szilard
1898-1964



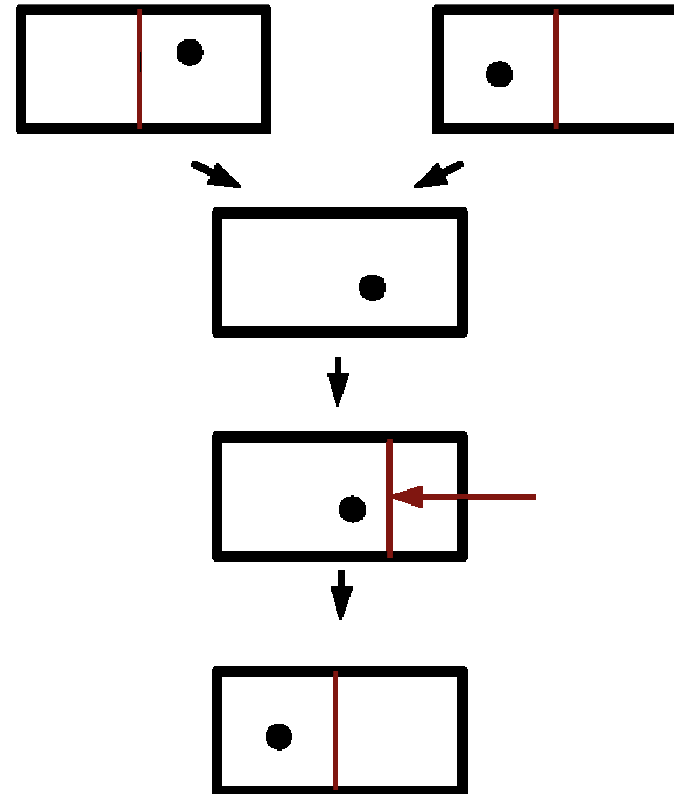
Szilard's Single Molecule
Engine (1929)

Landauer's principle (1961):

Each bit of lost information will lead to an release of amount of $KT \ln 2$ heat



Rolf Landauer
1927-1999

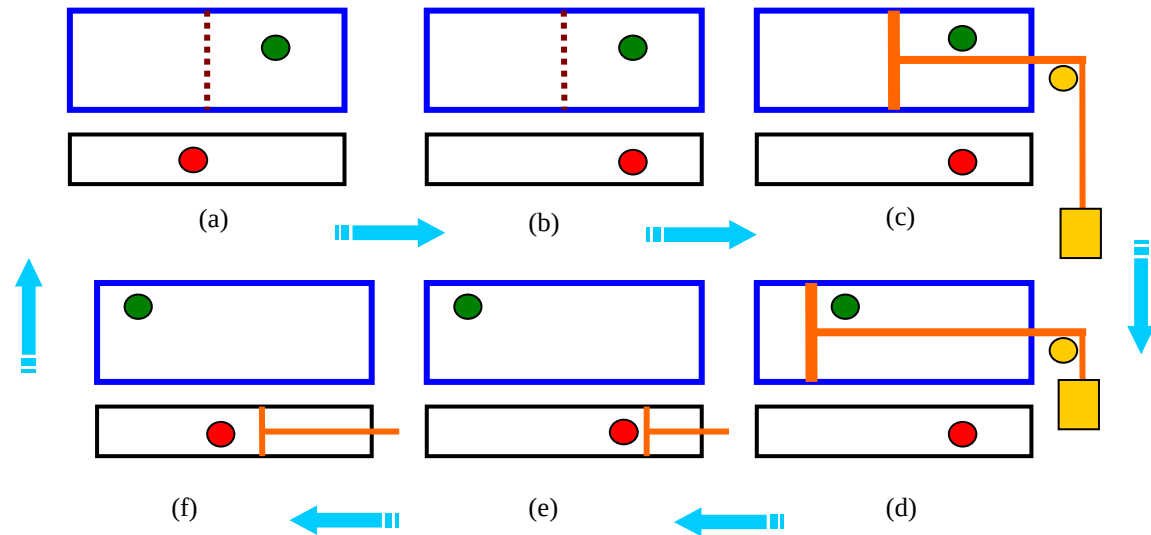


$$W = \int_V^{V/2} P dV = \int_V^{V/2} \frac{KT}{V} dV = -KT \ln 2$$

There is a lower bound of heat a computer must dissipate to process a given amount of information.



Charles H. Bennett
1943-



“The erasure of the memory of the demon compensates the entropy decreases and thus save the second law.”

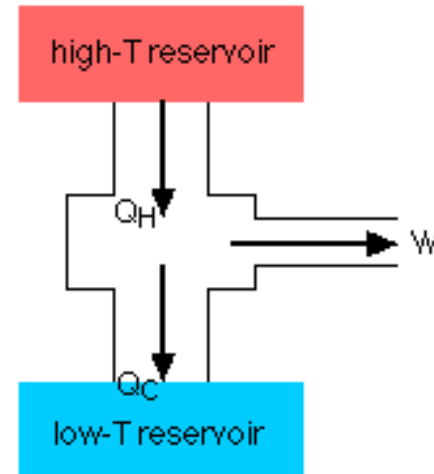
Is it possible to build an autonomous Maxwell's refrigerator without a demon?

- This refrigerator needs to rectify thermal fluctuations
- No intelligent creature (demon) is involved
- Achieving heat flow against temperature gradient by utilizing information
- The erasure of information compensates the entropy decrease

Information refrigerator

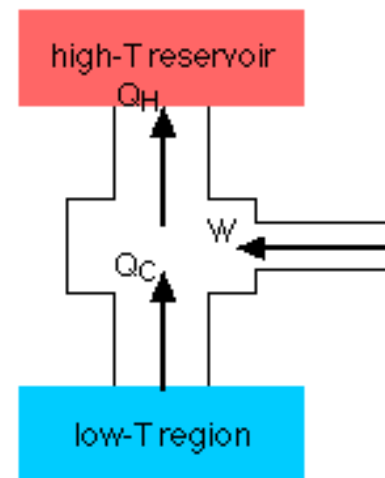
Heat engine and Refrigerator

Heat engine

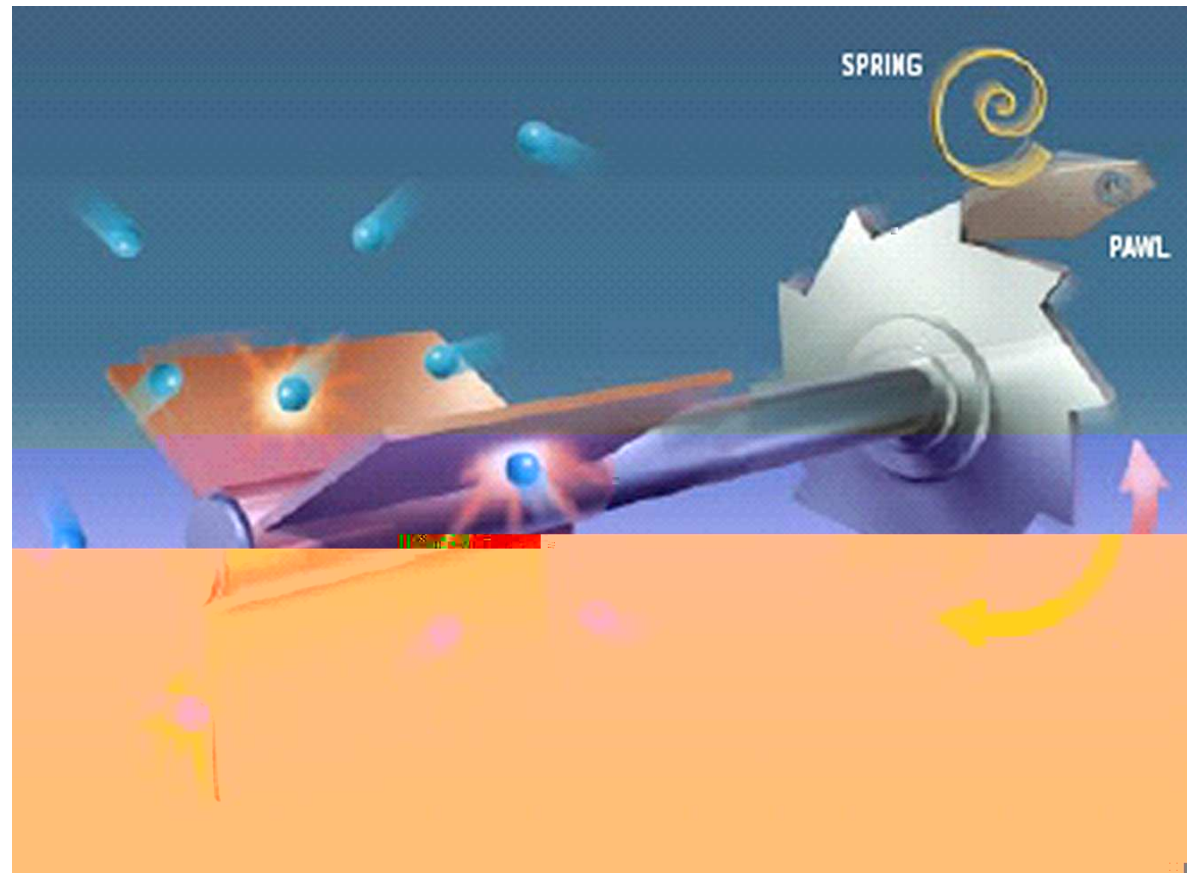


No thermal
Fluctuation!

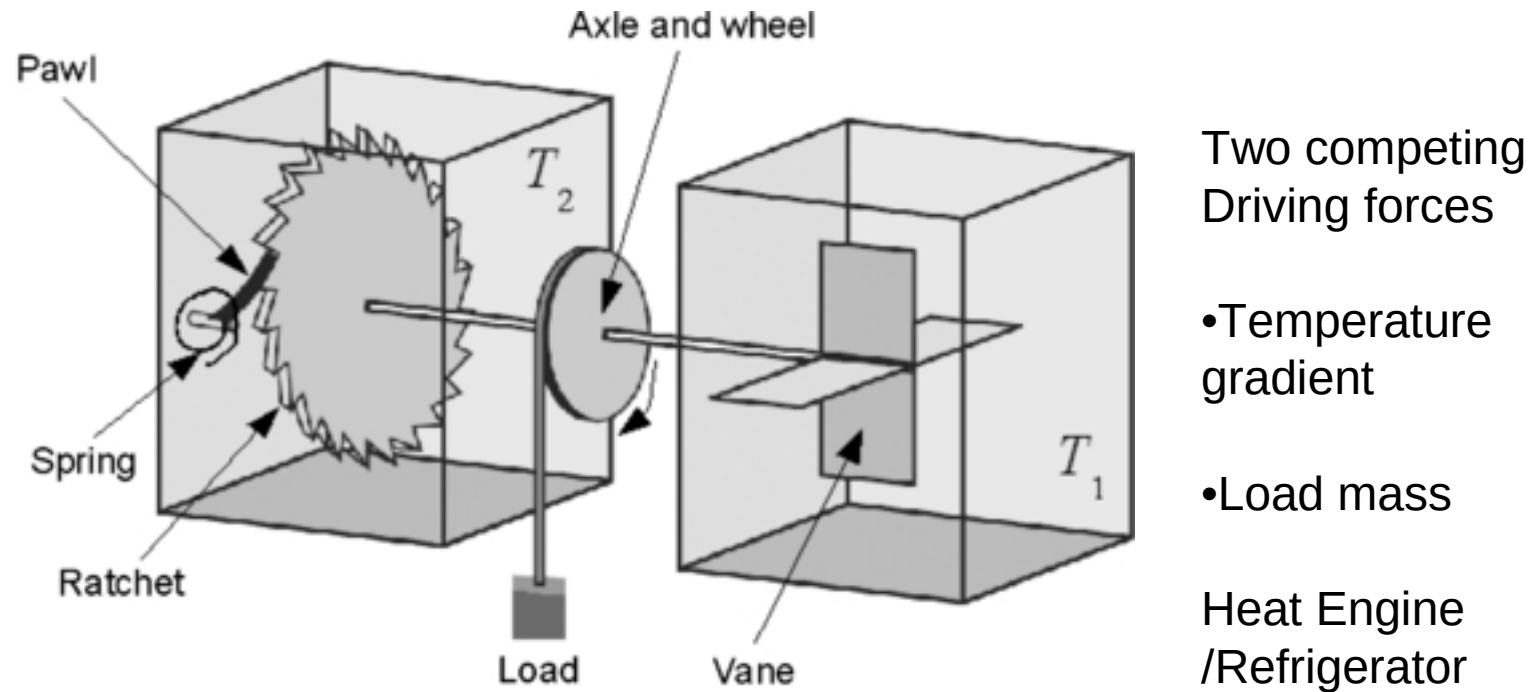
Refrigerator
(Heat pump)



Fluctuations in small systems



Feynman's Ratchet and Pawl (1963)



C. Jarzynski, et al PRE, 59, 6448 (1999); Z. C. Tu, J. Phys. A, 41, 312003 (2008)

- Rectifying thermal fluctuations, either a heat engine or a refrigerator
- No information is involved

In order to mimicking Maxwell's thought experiment, we need to input information instead of mechanical work

Feynman refrigerator and information refrigerator

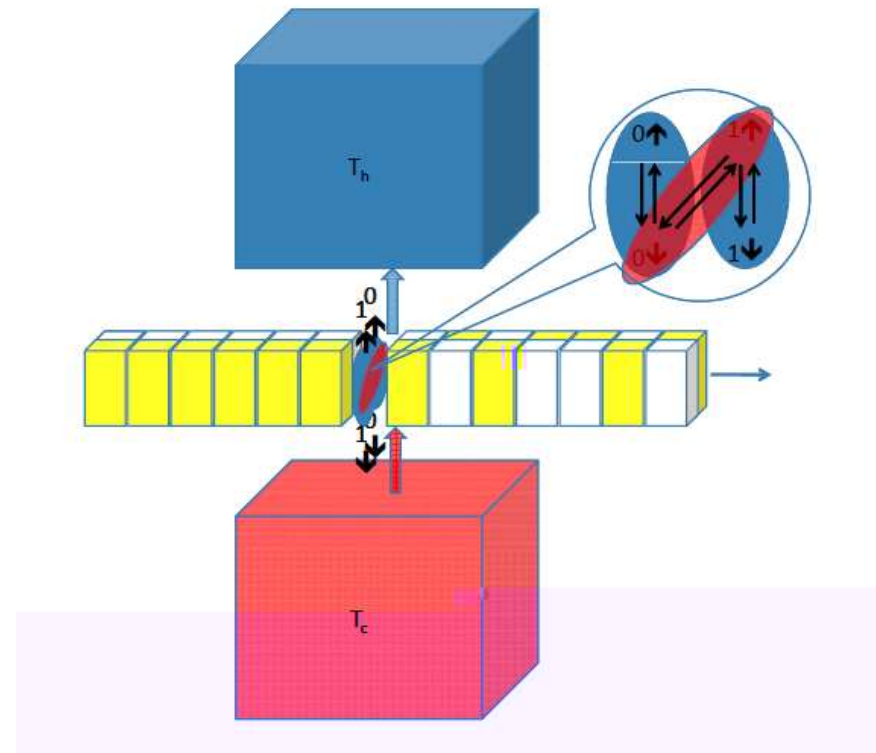
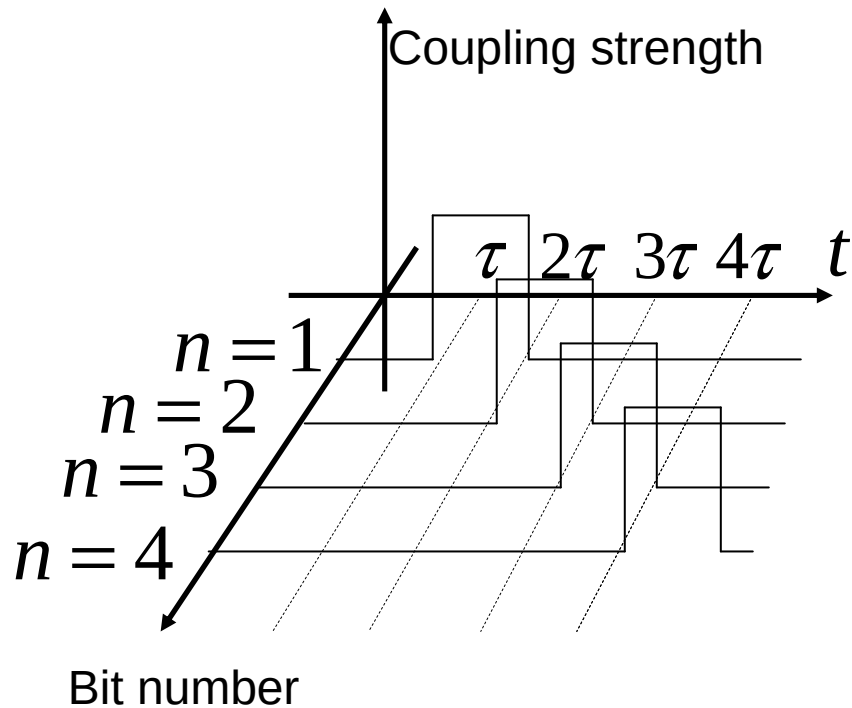
The Feynman refrigerator

- This refrigerator needs to rectify thermal fluctuations
- Input mechanical work
- The conjugate cycle is a heat engine
- No information content is involved

The information refrigerator

- This refrigerator needs to rectify thermal fluctuations
- Input low entropy memory unit
- The conjugate is an information eraser
- No mechanical work is involved

Schematic figure of the information refrigerator



What is fixed?

Temperatures of the two reservoirs

T_h

T_c

The initial probability distribution of the bits

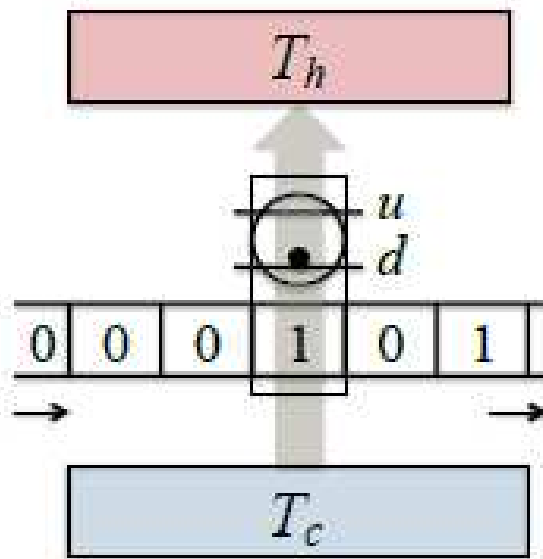
P_0^B

P_1^B

The period of interaction between the two-level system and every bit

τ

Information refrigerator



Two-level system

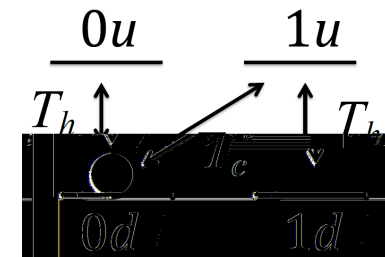
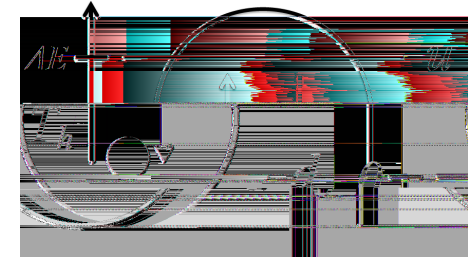
- Different energy
- Detailed balance

Input bits

- Equal energy
- No transition

Cooperative transition

- Heat exchange



Heuristic analysis: All bits prepared in “0”

Microscopic equations of motion

Classical master equation $\frac{d\vec{P}(t)}{dt} = \mathcal{R}\vec{P}(t)$

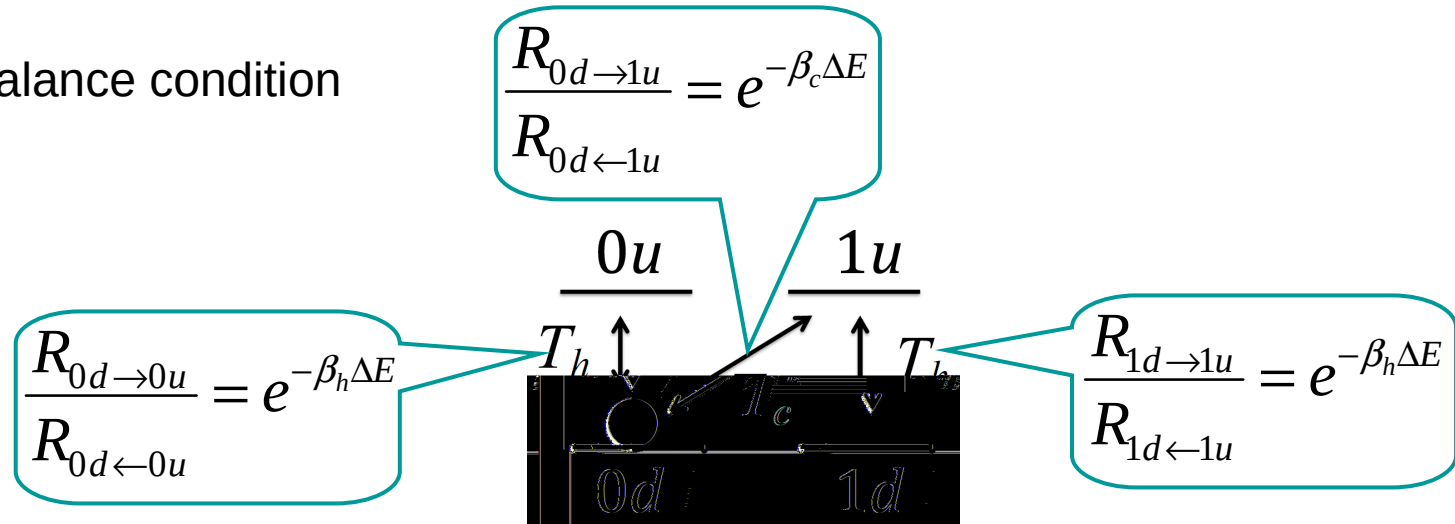
Transition matrix

$$\vec{P}(t) = \begin{pmatrix} P_{0u}(t) \\ P_{0d}(t) \\ P_{1u}(t) \\ P_{1d}(t) \end{pmatrix}$$

$$\mathcal{R} = \begin{pmatrix} \bullet & \gamma(1-\sigma) & 0 & 0 \\ \gamma(1+\sigma) & \bullet & 1+\omega & 0 \\ 0 & 1-\omega & \bullet & \gamma(1-\sigma) \\ 0 & 0 & \gamma(1+\sigma) & \bullet \end{pmatrix}$$

Microscopic equations of motion

Detailed balance condition



$$R_{0d \leftarrow 1u} = 1 + \omega$$

$$R_{0d \rightarrow 1u} = 1 - \omega$$

$$R_{d \rightarrow u} = \gamma(1 - \sigma)$$

$$R_{d \leftarrow u} = \gamma(1 + \sigma)$$

Strategy to solve the dynamics

Initial state of the two-level system and the bit

$$\begin{pmatrix} P_{0u}(0) \\ P_{0d}(0) \\ P_{1u}(0) \\ P_{1d}(0) \end{pmatrix} = \begin{pmatrix} P_0^B \times P_u^D \\ P_0^B \times P_d^D \\ P_1^B \times P_u^D \\ P_1^B \times P_d^D \end{pmatrix} \Rightarrow \begin{pmatrix} P_{0u}(\tau) \\ P_{0d}(\tau) \\ P_{1u}(\tau) \\ P_{1d}(\tau) \end{pmatrix} = e^{\Re \tau} \begin{pmatrix} P_0^B \times P_u^D \\ P_0^B \times P_d^D \\ P_1^B \times P_u^D \\ P_1^B \times P_d^D \end{pmatrix}$$

The marginal distribution of the two-level system

$$\begin{pmatrix} P_u^D(\tau) \\ P_d^D(\tau) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_{0u}(\tau) \\ P_{0d}(\tau) \\ P_{1u}(\tau) \\ P_{1d}(\tau) \end{pmatrix}$$

Periodic steady state of the two-level system

$$\begin{pmatrix} P_u^{D,ps}(\tau) \\ P_d^{D,ps}(\tau) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_{0u}(\tau) \\ P_{0d}(\tau) \\ P_{1u}(\tau) \\ P_{1d}(\tau) \end{pmatrix} = \begin{pmatrix} P_u^{D,ps}(0) \\ P_d^{D,ps}(0) \end{pmatrix}$$

The probability distribution of the outgoing bits

$$\begin{pmatrix} P_0^B(\tau) \\ P_1^B(\tau) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} e^{\Re \tau} \begin{pmatrix} P_0^B(0) & 0 \\ 0 & P_0^B(0) \\ P_1^B(0) & 0 \\ 0 & P_1^B(0) \end{pmatrix} \begin{pmatrix} P_u^{D,ps}(0) \\ P_d^{D,ps}(0) \end{pmatrix}$$

The periodic steady state depends on the period τ

Two competing driving forces

Temperature gradient

$$\varepsilon = \frac{\omega - \sigma}{1 - \omega\sigma} = \tanh \frac{(\beta_c - \beta_h)\Delta E}{2}$$

Low entropy of the input bits

$$\delta = P_0^B(0) - P_1^B(0)$$

If the former wins $\varepsilon > \delta$ It is an information erasure

If the later wins $\varepsilon < \delta$ It is an information refrigerator

Heat flux from the master equation

$$Q_{c \rightarrow h} = [P_1^B(\tau) - P_1^B(0)] \Delta E$$

Where

$$P_1^B(\tau) - P_1^B(0) = \frac{\delta - \varepsilon}{2} \eta(\Lambda)$$

$$\eta(\Lambda) = \frac{\nu_2 P + \nu_3 Q}{P + Q},$$

$$P = \mu_2 (\mu_4 \nu_3 + \mu_1 \nu_1) \quad , \quad Q = \mu_3 (\mu_4 \nu_2 + \mu_1 \nu_1),$$

$$\nu_1 = 1 - e^{-2\gamma \tau} \quad , \quad \mu_1 = (\delta + \sigma) \omega,$$

$$\nu_2 = 1 - e^{-(1+\gamma-\alpha) \tau} \quad , \quad \mu_2 = \alpha + \gamma + \sigma \omega,$$

$$\nu_3 = 1 - e^{-(1+\gamma+\alpha) \tau} \quad , \quad \mu_3 = \alpha - \gamma - \sigma \omega,$$

$$\alpha = \sqrt{1 + \gamma^2 + 2\gamma\sigma\omega} \quad , \quad \mu_4 = 1 - \delta \omega.$$

Heat flow from low temperature to high temperature

$$Q_{c \rightarrow h} = \Delta E \frac{\delta - \varepsilon}{2} \eta(\Lambda)$$

Shannon entropy change of every bit

$$\begin{aligned} \Delta S_B = & -k_B \left[P_0^B(\tau) \ln P_0^B(\tau) + P_1^B(\tau) \ln P_1^B(\tau) \right] \\ & + k_B \left[P_0^B(0) \ln P_0^B(0) + P_1^B(0) \ln P_1^B(0) \right] \end{aligned}$$

“Phase diagram” of the device

Information
Refrigerator

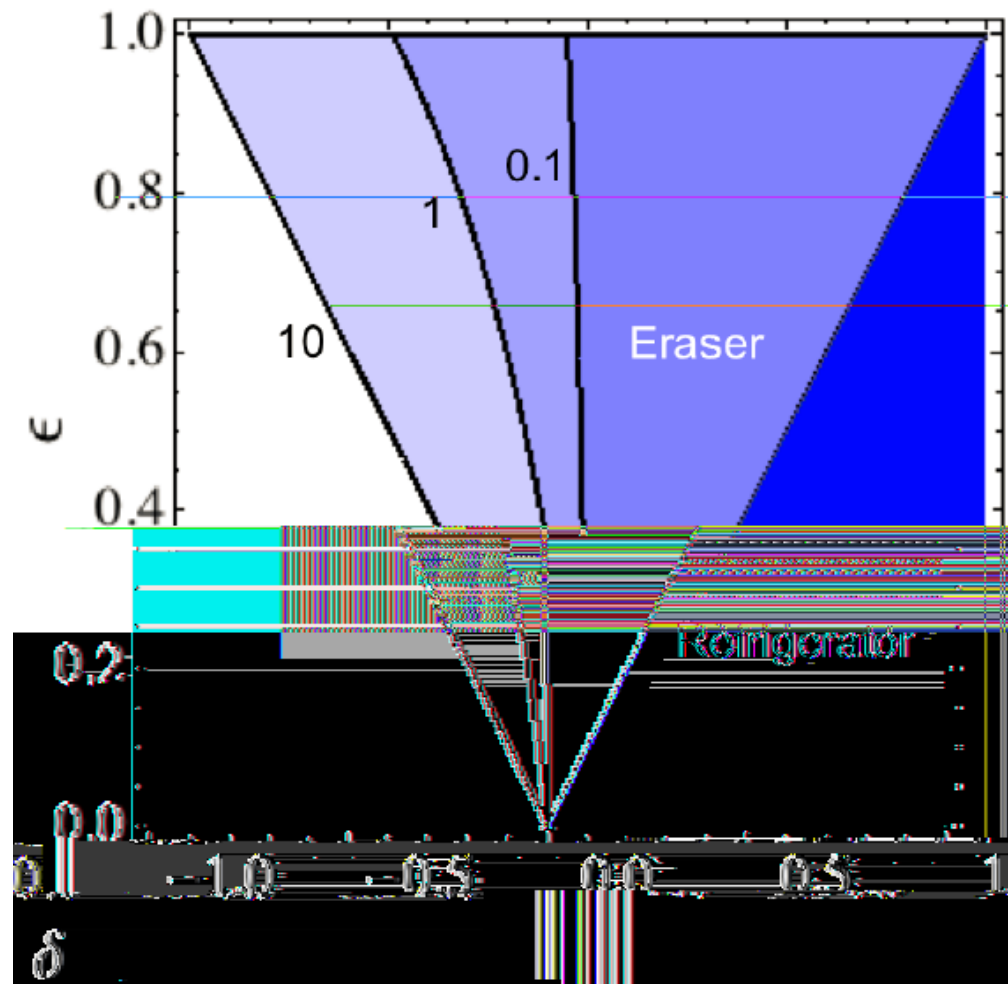
$$Q_{c \rightarrow h} > 0$$

$$\Delta S_B > 0$$

Information
Eraser

$$Q_{c \rightarrow h} < 0$$

$$\Delta S_B < 0$$



Clausius inequality

$$Q_{c \rightarrow h}(\beta_h - \beta_c) + \Delta S_B \geq 0$$

Information entropy increase in every bit

$$\begin{aligned} \Delta S_B = & -k_B \left[P_0^B(\tau) \ln P_0^B(\tau) + P_1^B(\tau) \ln P_1^B(\tau) \right] \\ & + k_B \left[P_0^B(0) \ln P_0^B(0) + P_1^B(0) \ln P_1^B(0) \right] \end{aligned}$$

The second law is not violated if we identify the information entropy with the thermodynamic entropy

Lower bound on heat dissipation in information processing



$10^{-21} J$ Heat dissipated when one bit is erased

In modern silicon device it is 1000 times higher than that limit

Antoine Berut et al., Nature, 483, 187 (2012)

Experimental relevance

Colloid particle

- G. M. Wang et al, Phys. Rev. Lett., 89, 050601 (2002)
- V. Blickle et al, Phys. Rev. Lett., 96, 070603 (2006)
- Tongcang Li et al, Science, 328, 1673 (2010)

Biosystems

Jan liphardt et al Science, 296, 1832 (2002)

Superconducting qubit

H. T. Quan et al Rev. Lett. 97, 180402 (2006)

Trapped ion system

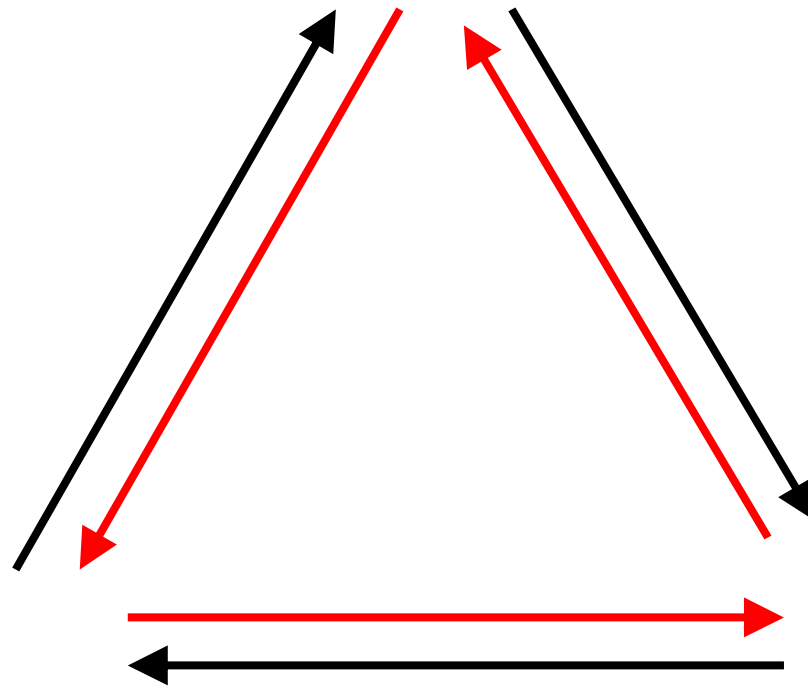
G. Huber et al Phys. Rev. Lett. 101, 070403 (2008)

Reminding

“Heat can never pass from a colder to a warmer body **without some other change** connected therewith, occurring at the same time.” --- R. Clausius, (1854)

~~循环运转装置，在不产生使热由冷体传向热体之效，---，（2013）~~

Summary



Summary

- We introduce an autonomous model to generate heat flow against the thermal gradient mimicking the Maxwell's original idea
- Heat can be pumped from a low temperature to a high temperature reservoir if there is a memory register to which we can write information
- The increases of the information entropy of the memory compensates the decreases in the thermodynamic entropy.

