

Pro|er Scaling of Anomalous Hall Effect

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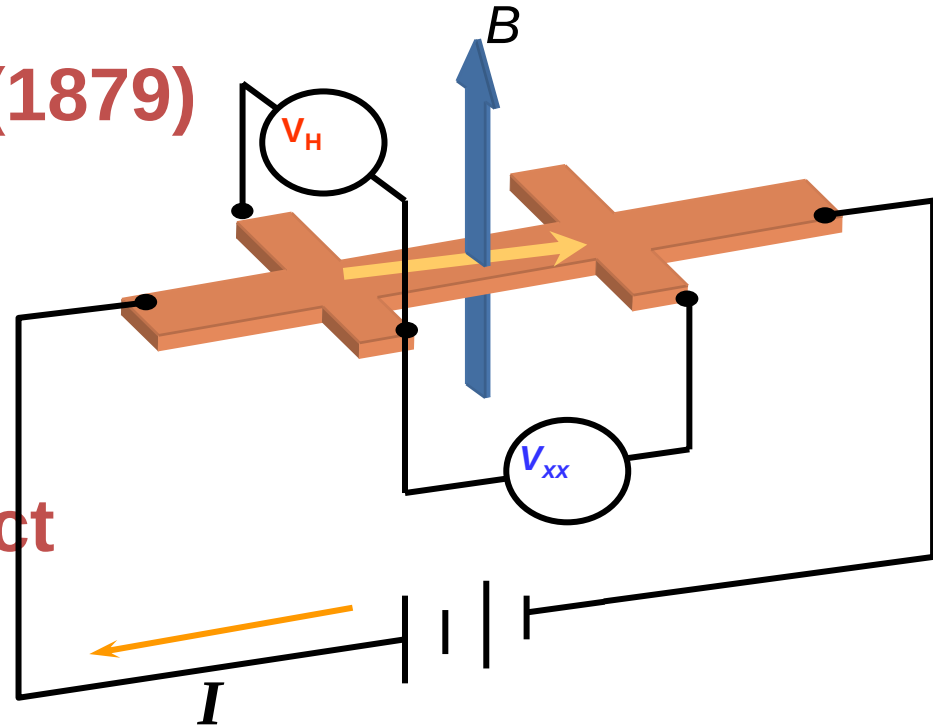
Hall effect

- Ordinary Hall effect (1879)

$$\rho_{xy} = R_0 B$$

- Anomalous Hall effect (1880&1881)

$$\rho_{xy} = R_0 B + R_{ah} M_z$$



$$\rho_{xy} = R_0 B + \rho_{ah} M_z$$

$$\rho_{ah} = f(\rho_{xx})$$

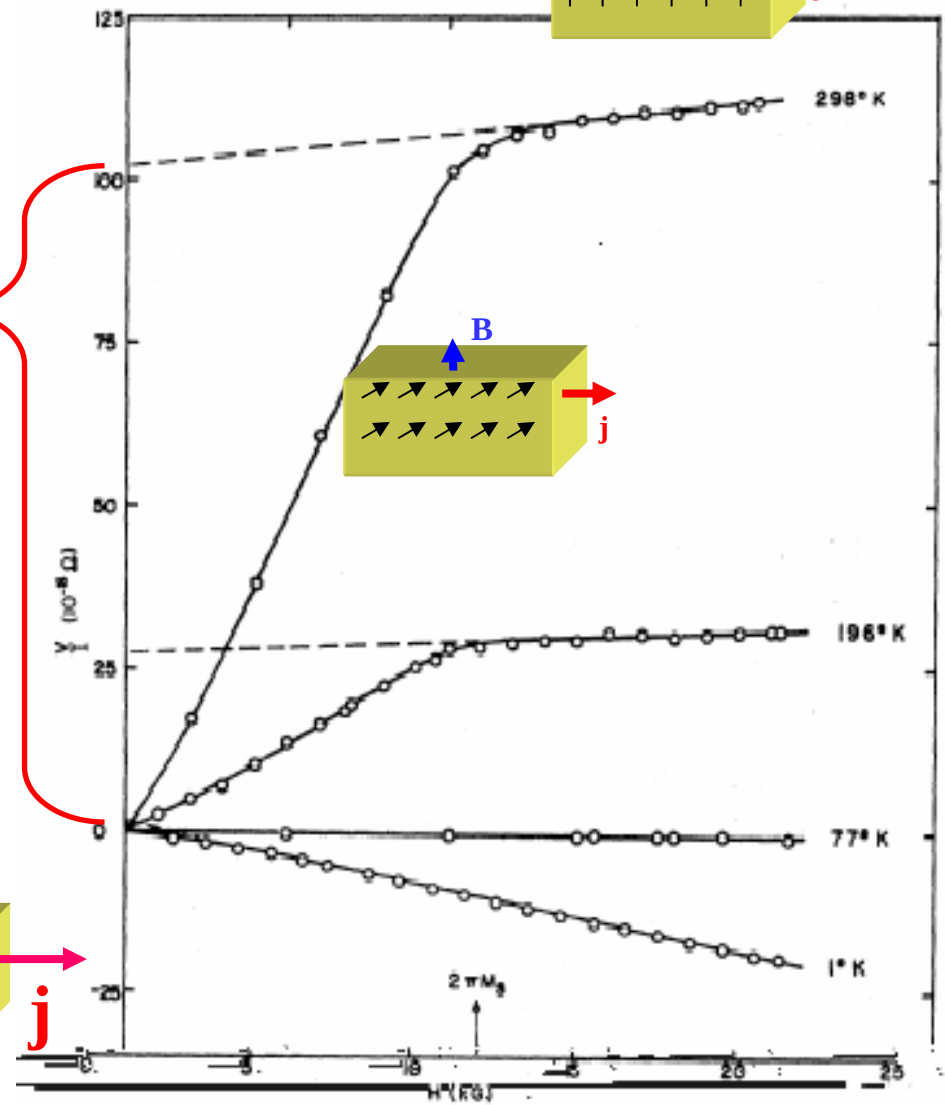
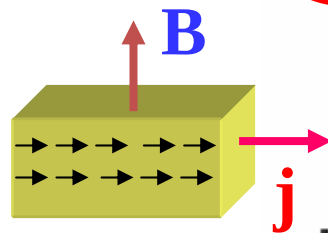
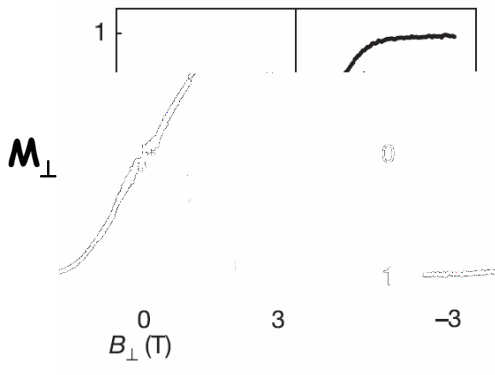
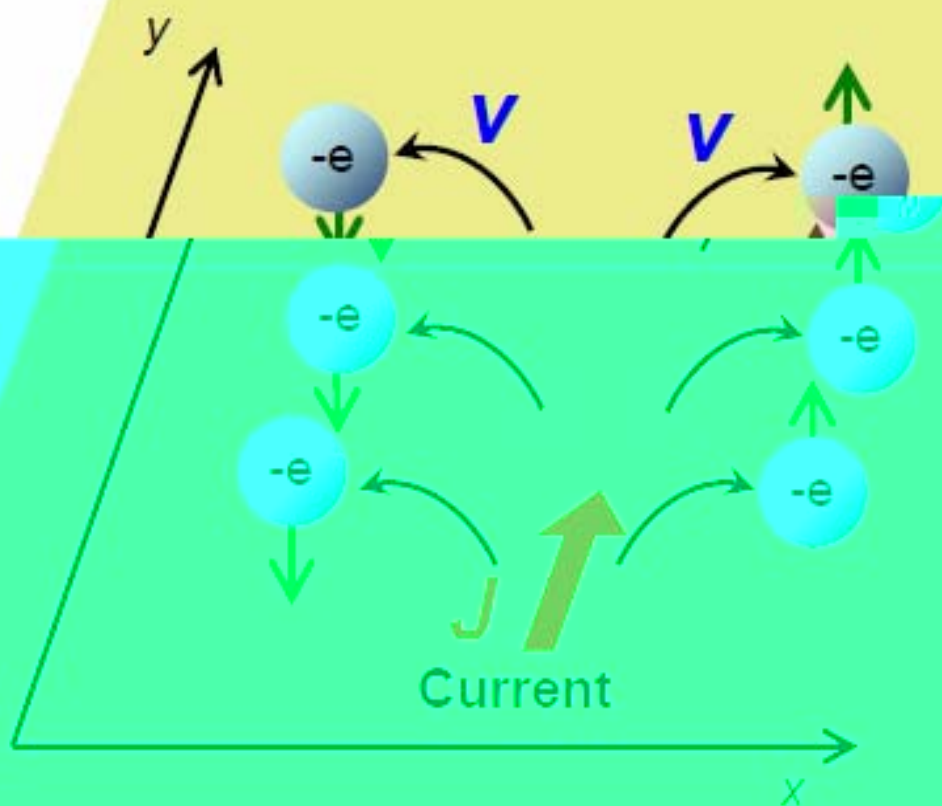


FIG. 1. Variation of the Hall resistance with applied magnetic field for whisker Fe

Spin Hall Effect



Theories

- Karplus & Luttinger (1954)**

Intrinsic: s-p in-orbit interaction with nonzero interband matrix element of the current.

Anomalous Velocity

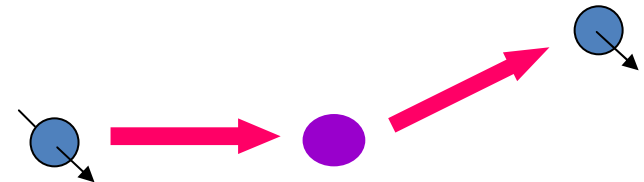


$$\rho_{in} \propto \rho_{xx}^2$$

- J. Smit (1955)**

Extrinsic: s-p in-orbit interaction with asymmetry scatterings at impurities.

Skew Scattering



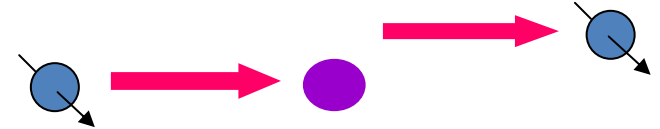
$$\rho_{sk} \propto \rho_{xx}$$

Theories

- **L. Berger (1970)**

Side Jump

Extrinsic spin-orbit interaction with side-jump scatterings at impurities.



$$\rho_{sj} \propto \rho_{xx}^2$$

$$\rho_{ah} = a\rho_{xx} + b\rho_{xx}^2$$

G. Sundaram and Q. Niu, Phys. Rev. B 59 (1999) 14915.

$$\frac{d \vec{r}(t)}{dt} = \frac{\partial \mathcal{E}_n(\vec{k})}{\partial \vec{k}} - \vec{B}_n(\vec{k}) \times \frac{d \vec{k}(t)}{dt}$$

k- space curvature

$$\frac{d \vec{k}(t)}{dt} = \frac{\partial V(\vec{r})}{\partial \vec{r}} - \vec{B}(\vec{r}) \times \frac{d \vec{r}(t)}{dt}$$

r- space curvature

Theories

Karplus-Luttinger Term — Berry Curvature

T. Jungwirth, Q. Niu and A. H. MacDonald: Phys. Rev. Lett. 88 (2002) 207208.
M. Onoda and N. Nagaosa: J. Phys. Soc. Jpn. 71 (2002) 19.

Berry curvature

$$\mathbf{j}_y = -e^2 \mathbf{E} \times \int d^3k f(\mathbf{k}) \boxed{\Omega}$$

$\propto \sigma_{int}$

$$\sigma_{int} = \text{constant}$$



$$\rho_{int} = \sigma_{int} \rho_{xx}^2$$

Extrinsic or Intrinsic ???

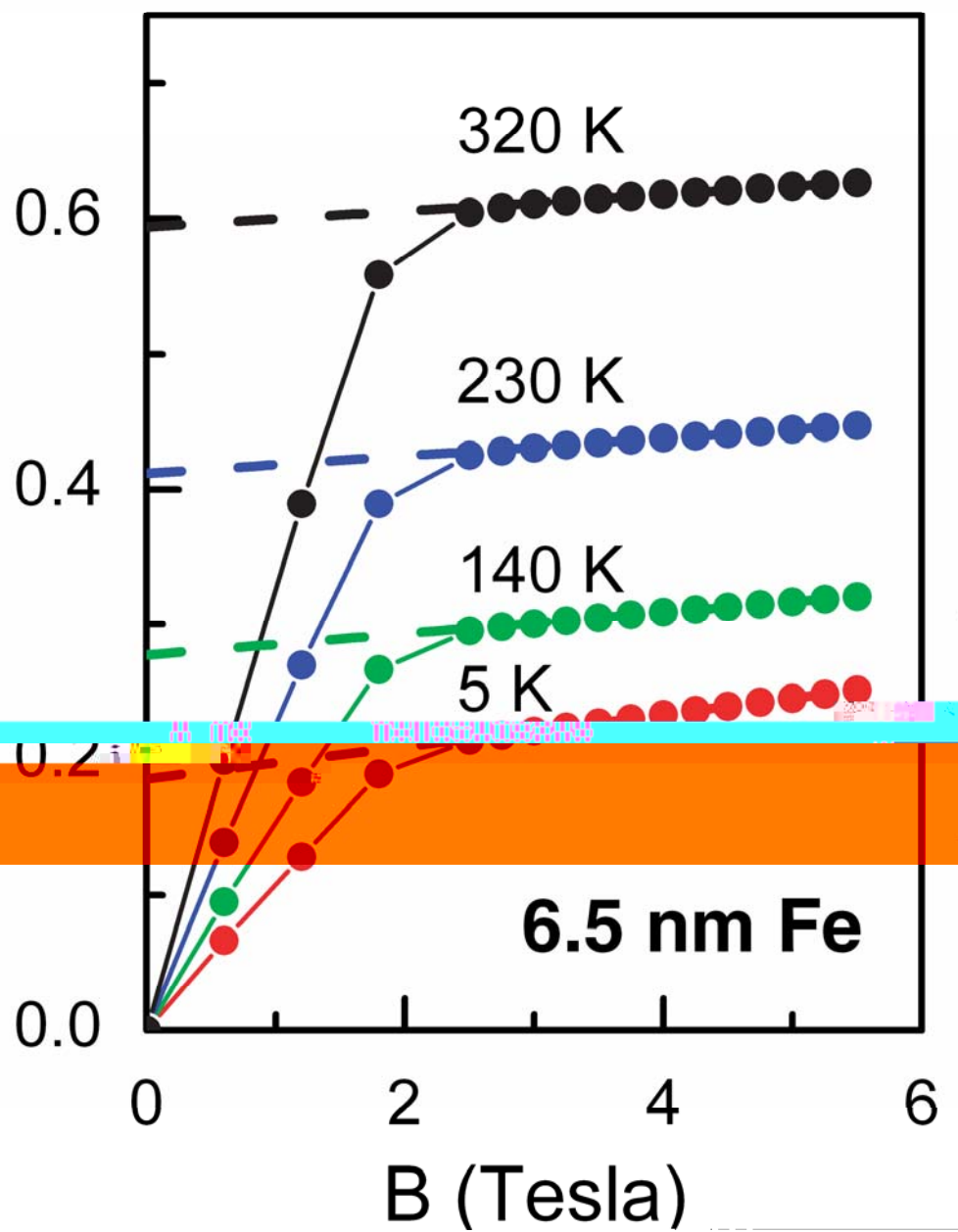
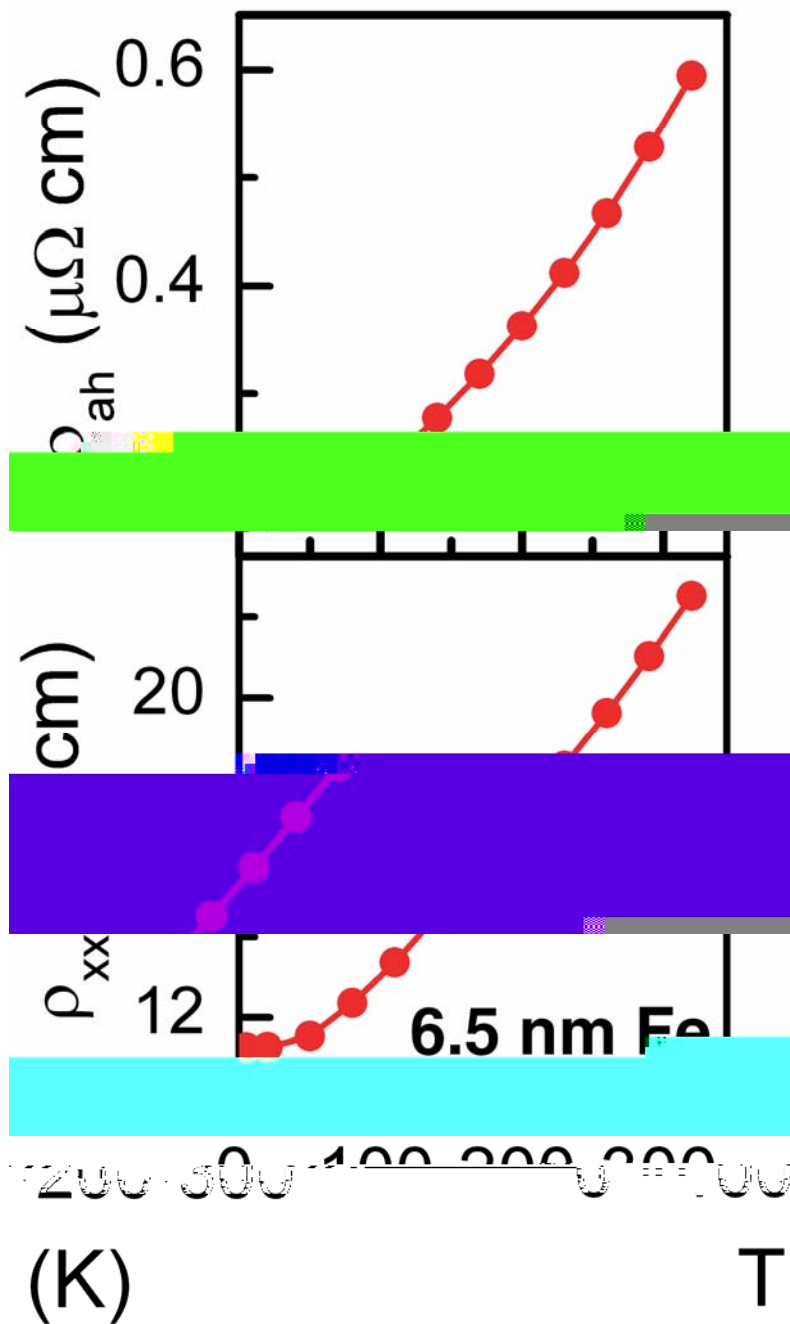


$$\rho_{ah} = a\rho_{xx} + b\rho_{xx}^2$$

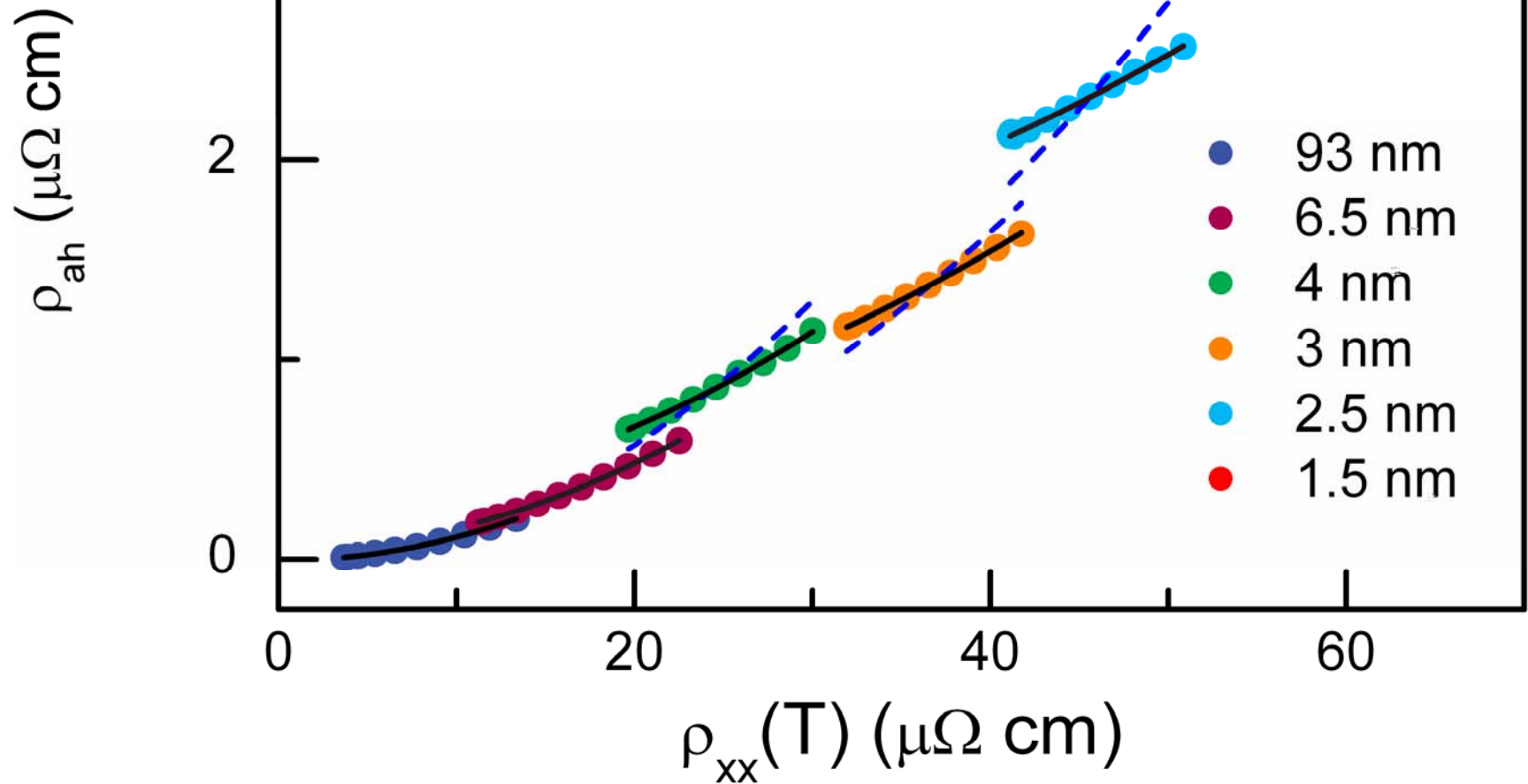
Question 1:
Proper Scaling of the AHE ?

$$\rho_{ah} = f(\rho_{xx})$$

Question 2:
Intrinsic and Extrinsic in the AHE ?



$$\rho_{ah} = f(\rho_{xx})$$



Dashed lines are fittings with: $\rho_{ah} = b\rho_{xx}^2$

Can we use $\rho_{ah} = a\rho_{xx} + b\rho_{xx}^2$?

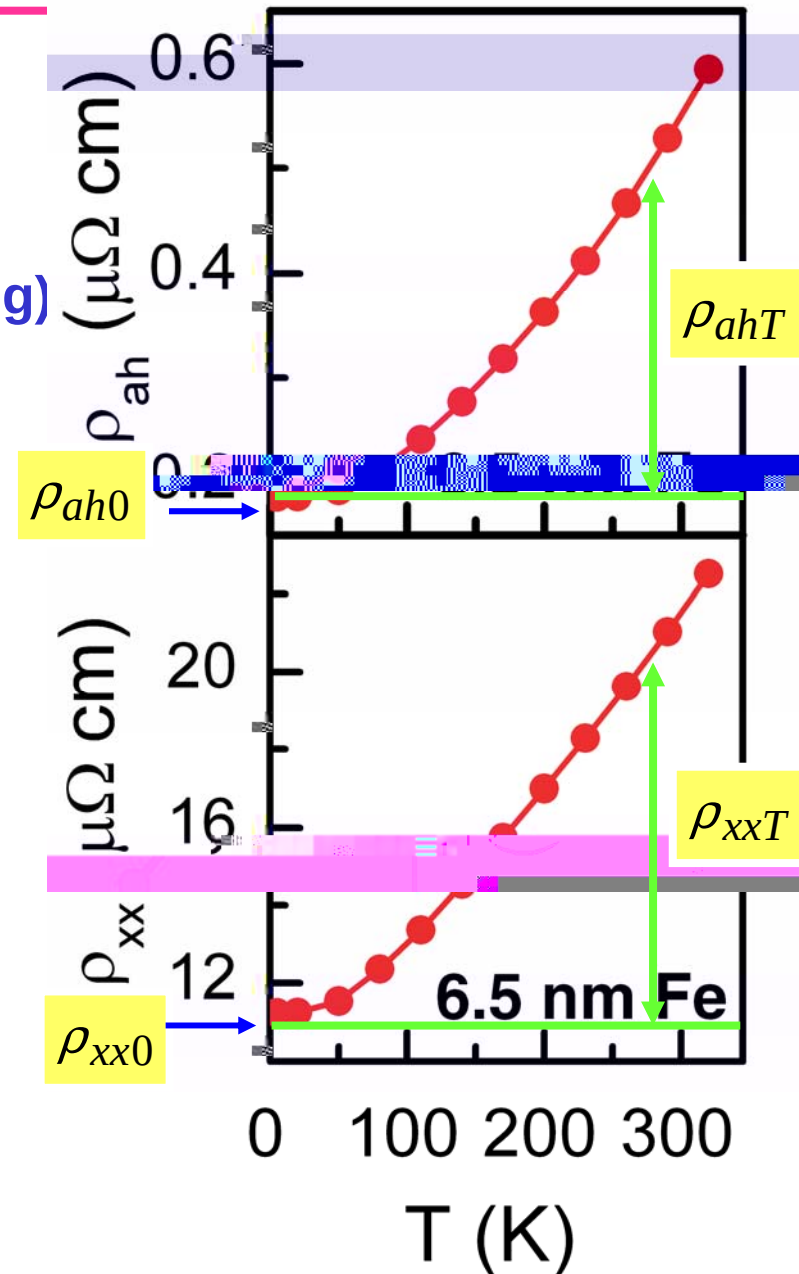
Matthiessen's rule :

$\rho_{xx0} \sim$ Impurity (Skew scattering)

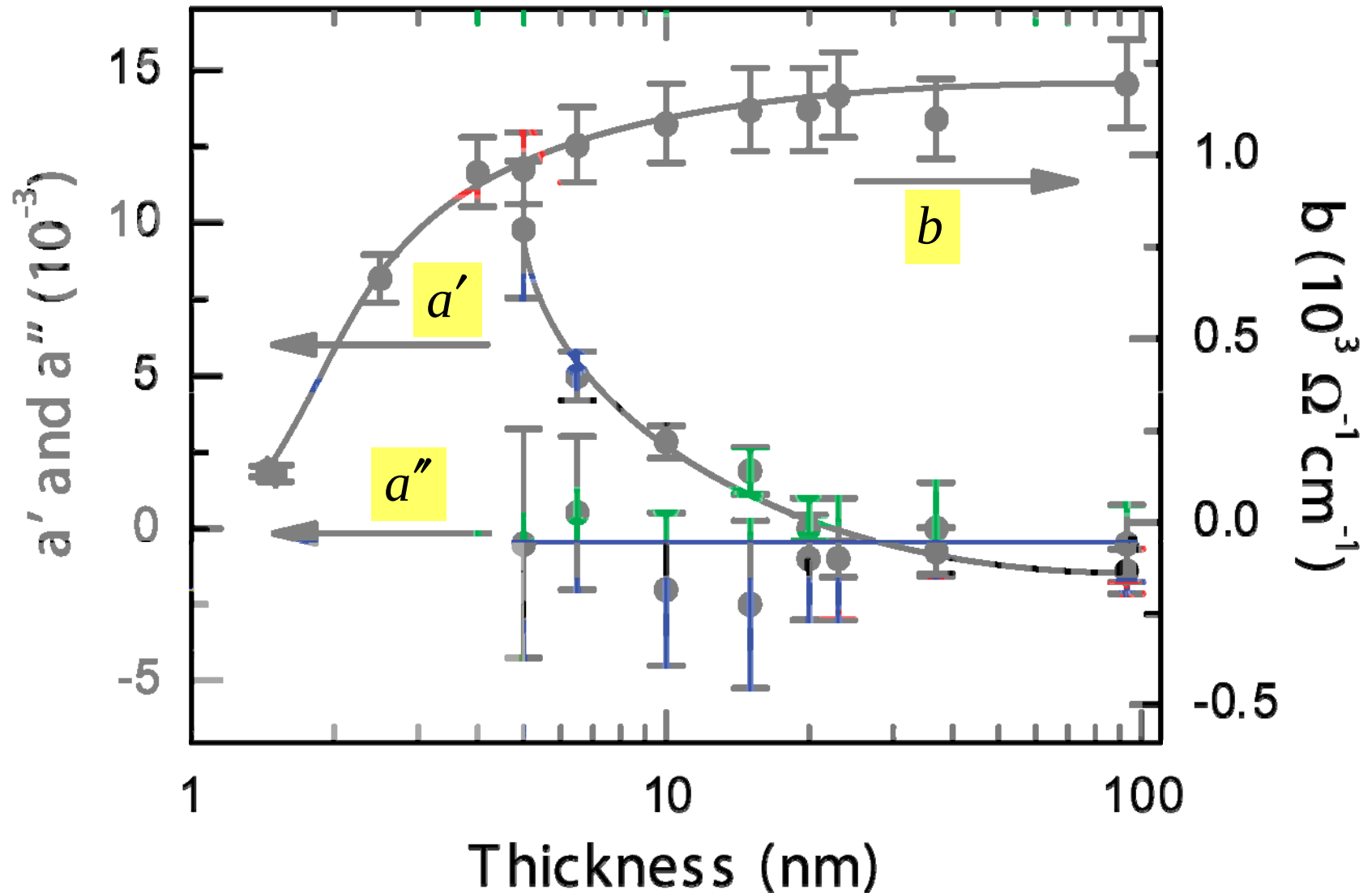
$\rho_{xx} = \rho_{xx0} + \rho_T$
 $\rho_T \sim$ Phonon (Unknown)

$$a\rho_{xx} = a\rho_{xx0} + a\rho_T$$

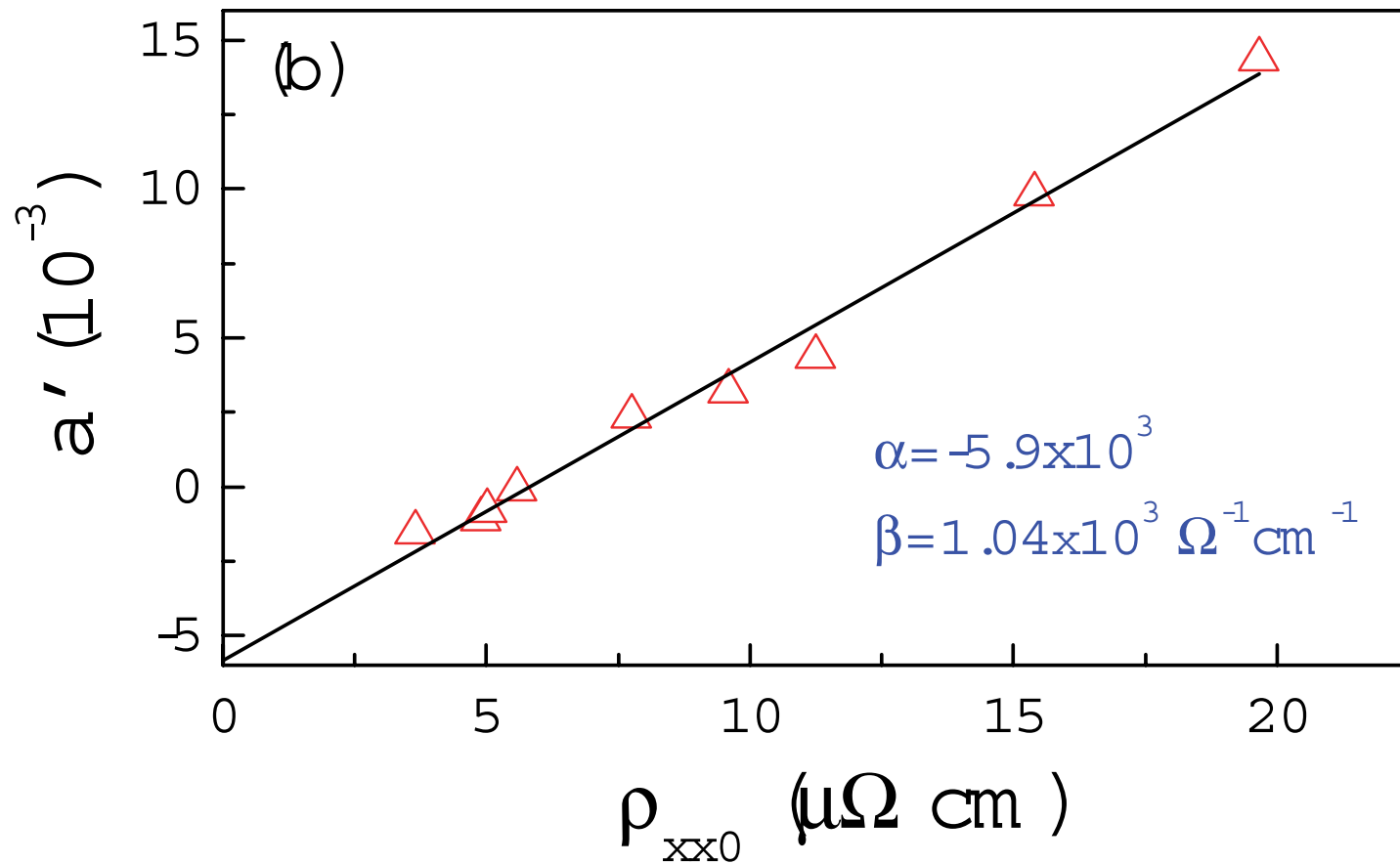
$$\rightarrow a'\rho_{xx0} + a''\rho_T$$



$$\rho_{ah} = a' \rho_{xx0} + a'' \rho_{xxT} + b(\rho_{xx0} + \rho_{xxT})^2$$



$$a' = \alpha + \beta \rho_{xx0}$$



Old scaling:

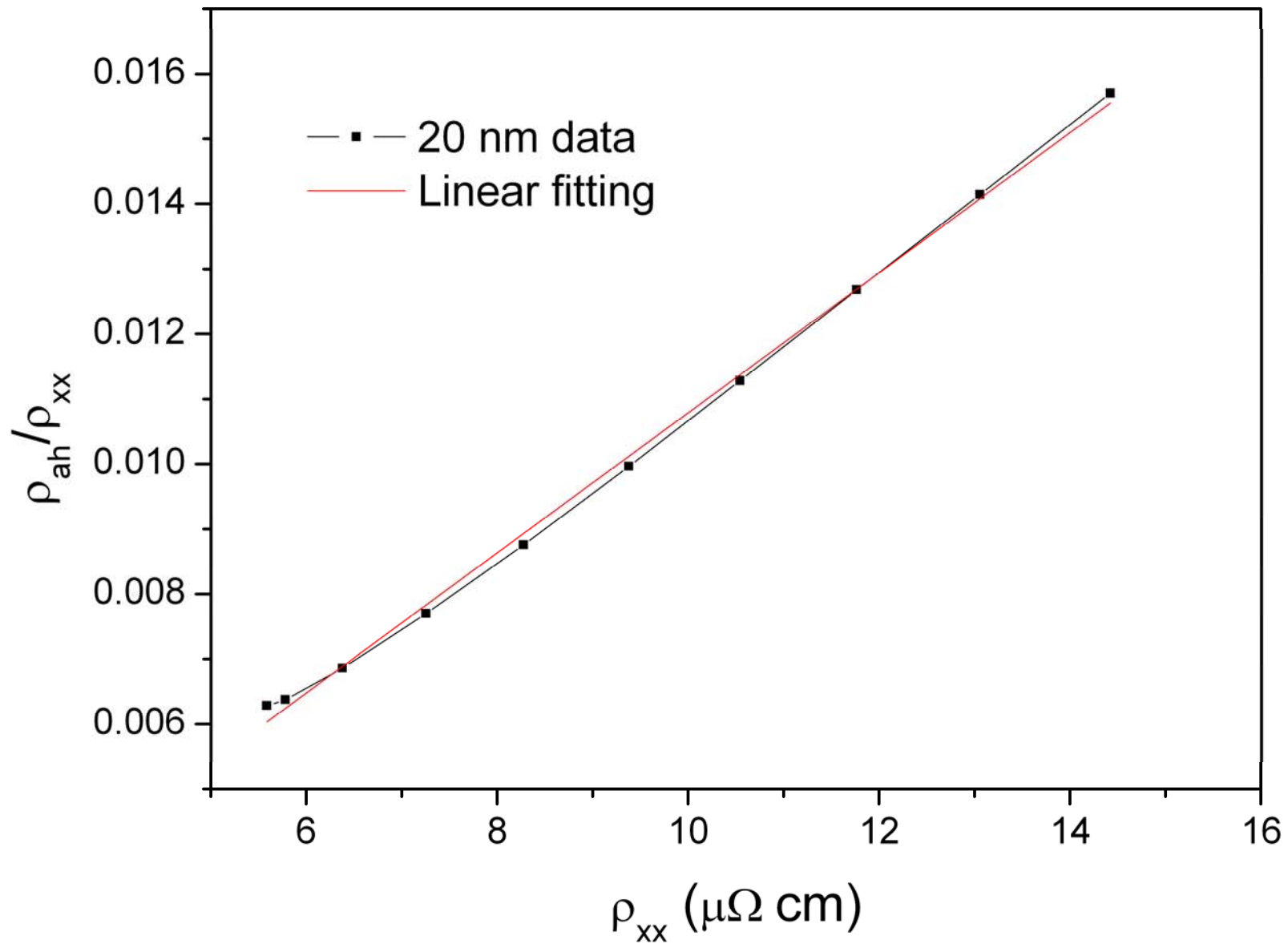
$$\rho_{ah} = a\rho_{xx} + b\rho_{xx}^2$$

$$\frac{\rho_{ah}}{\rho_{xx}} = a + b\rho_{xx}$$

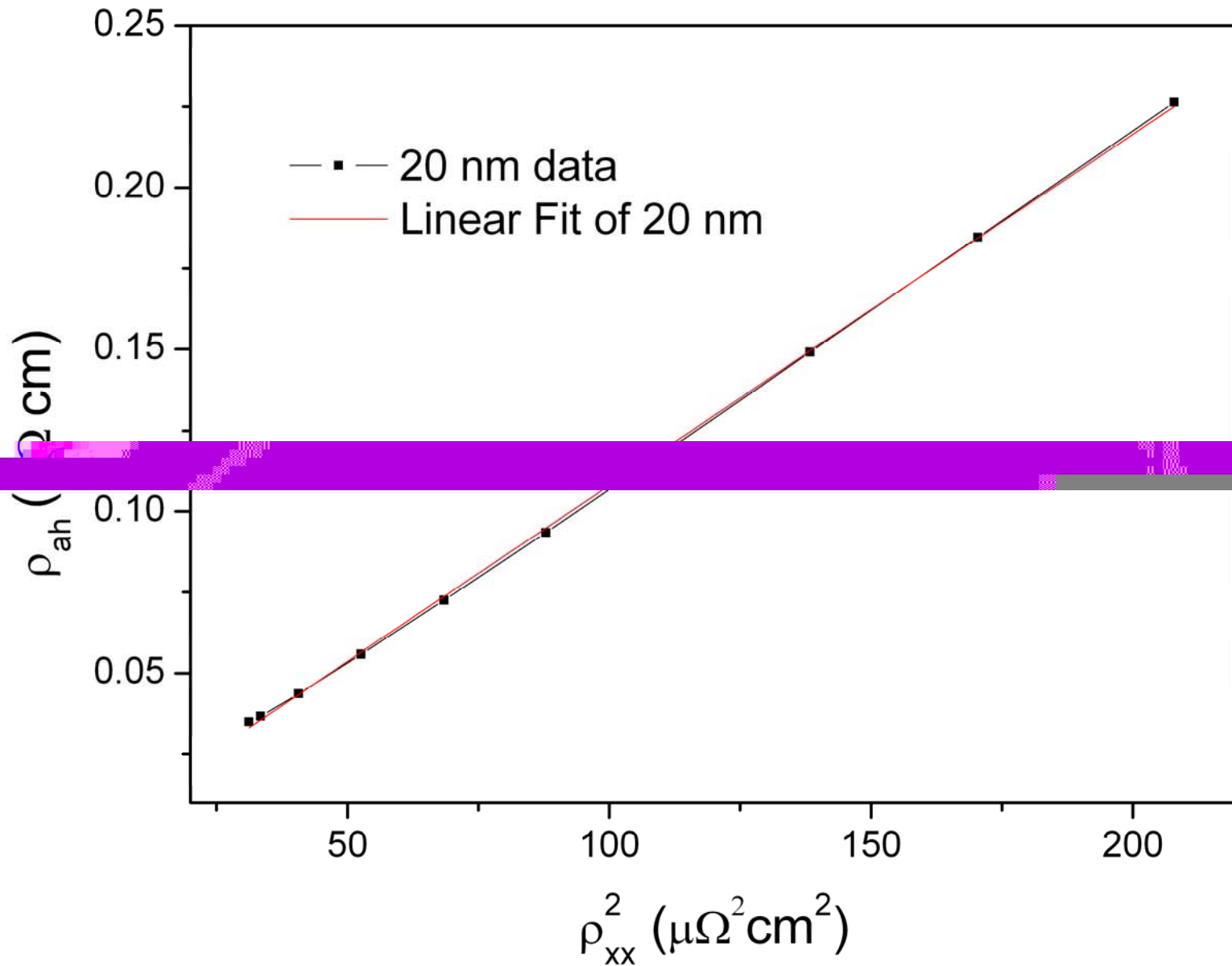
New scaling:

$$\rho_{ah} = \alpha\rho_{xx0} + \beta\rho_{xx0}^2 + b\rho_{xx}^2$$

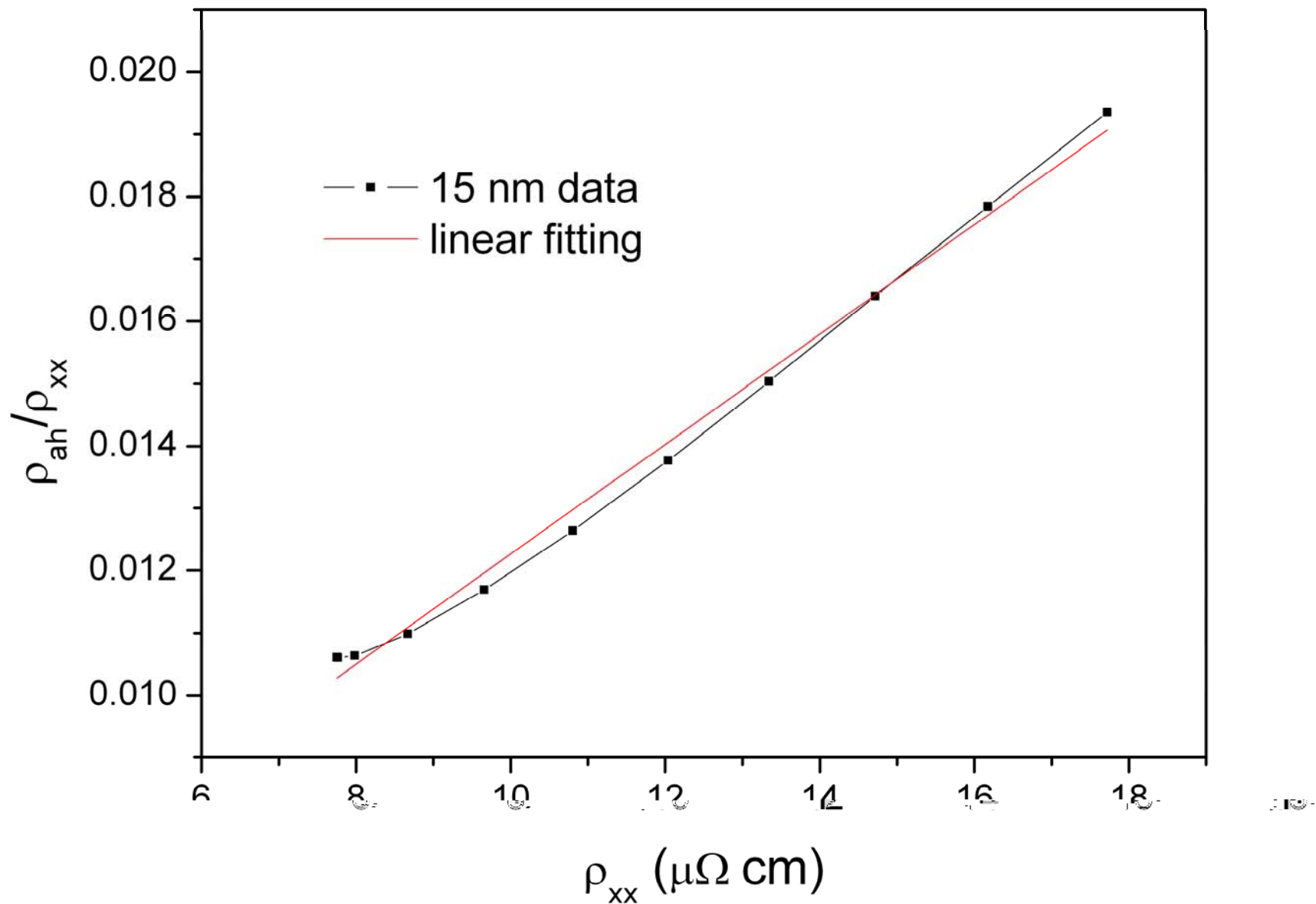
Old scaling:



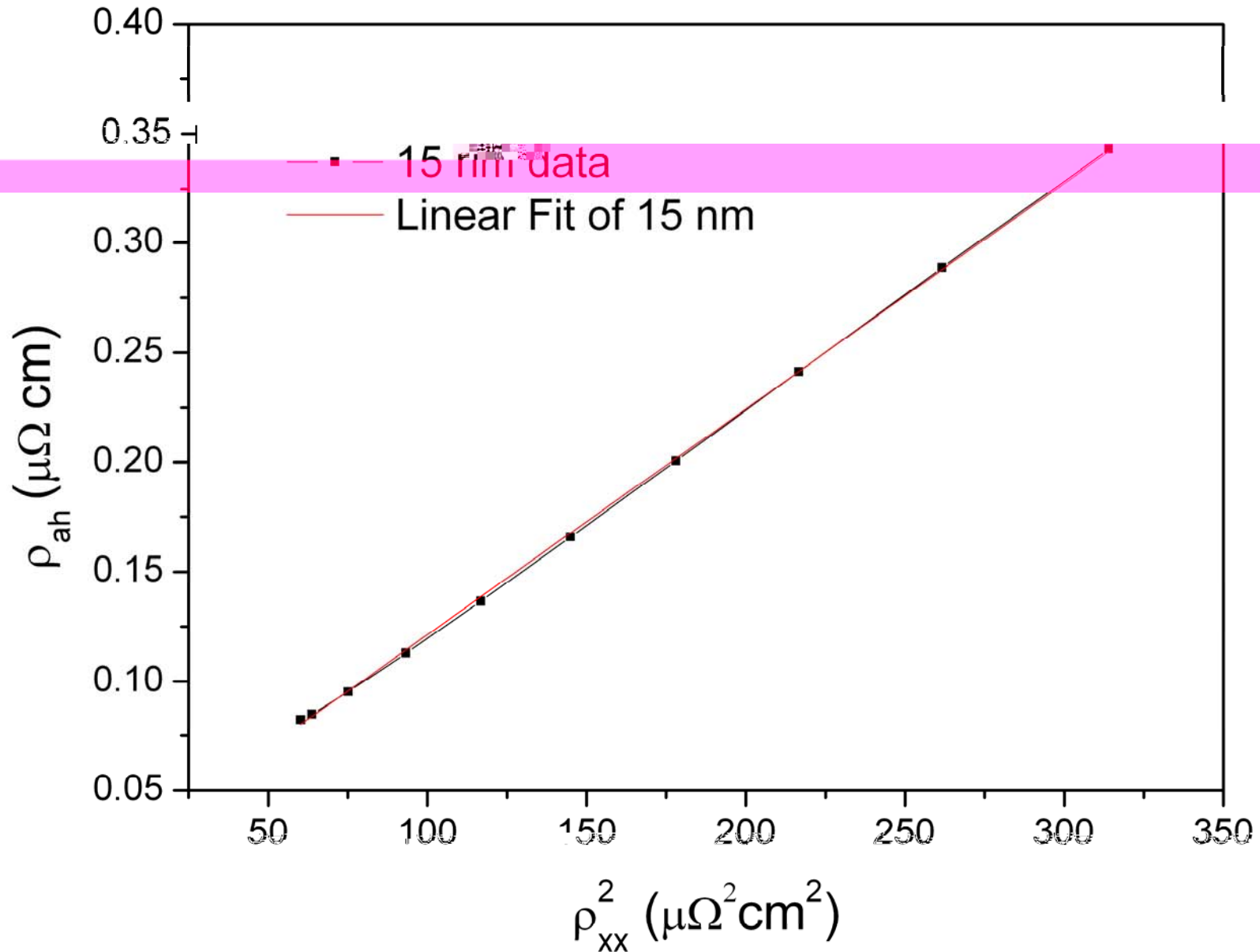
New scaling:



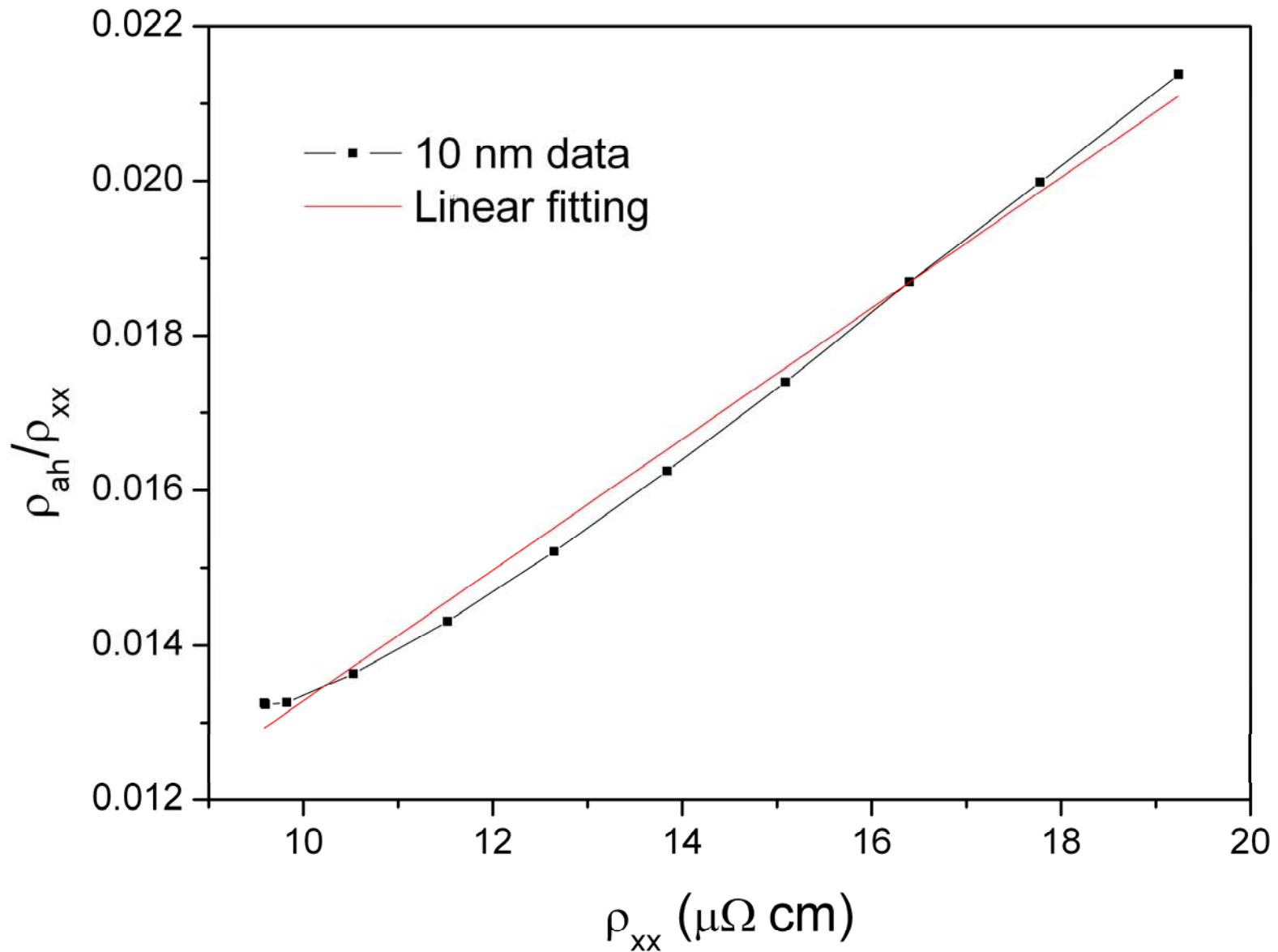
Old scaling:



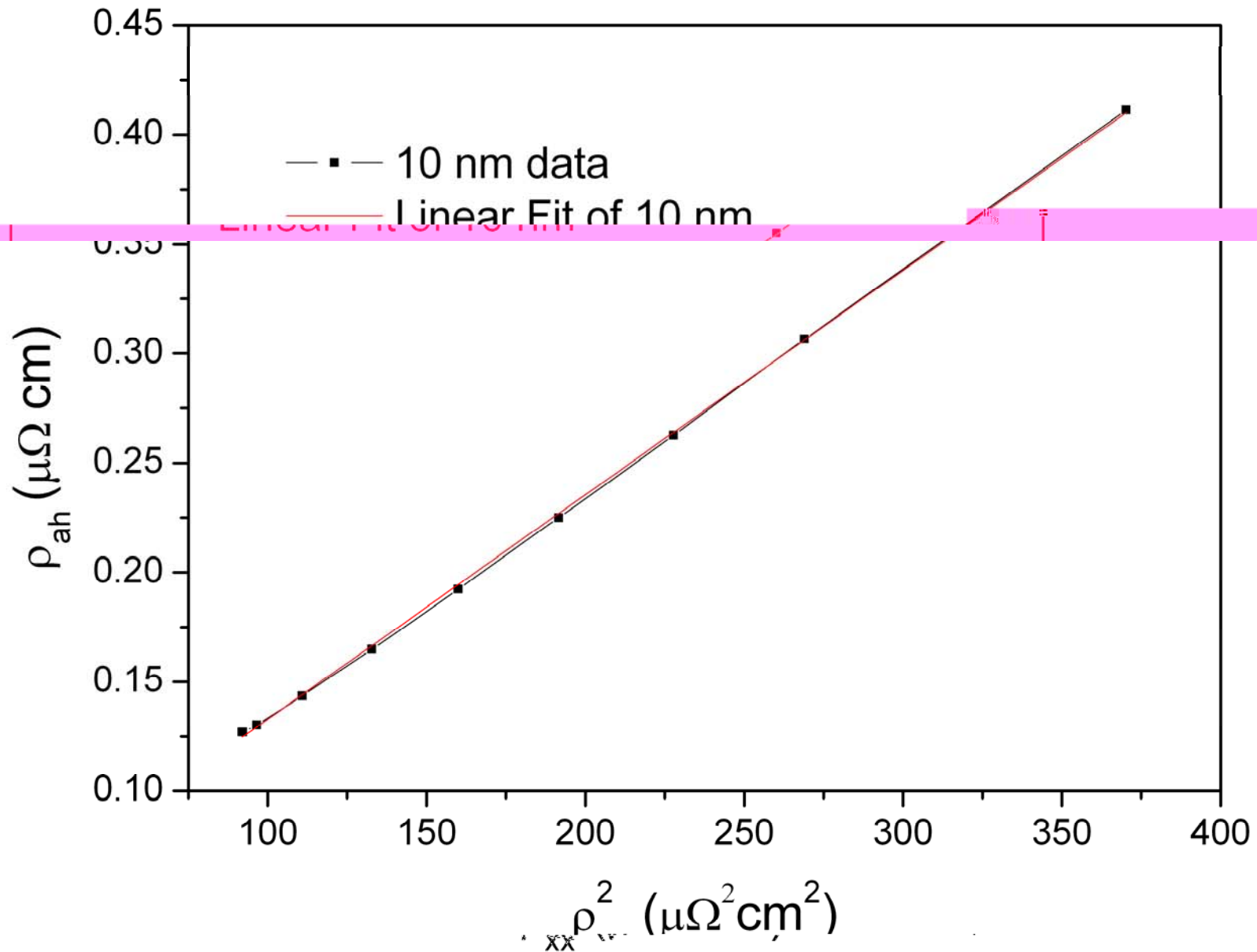
New scaling:



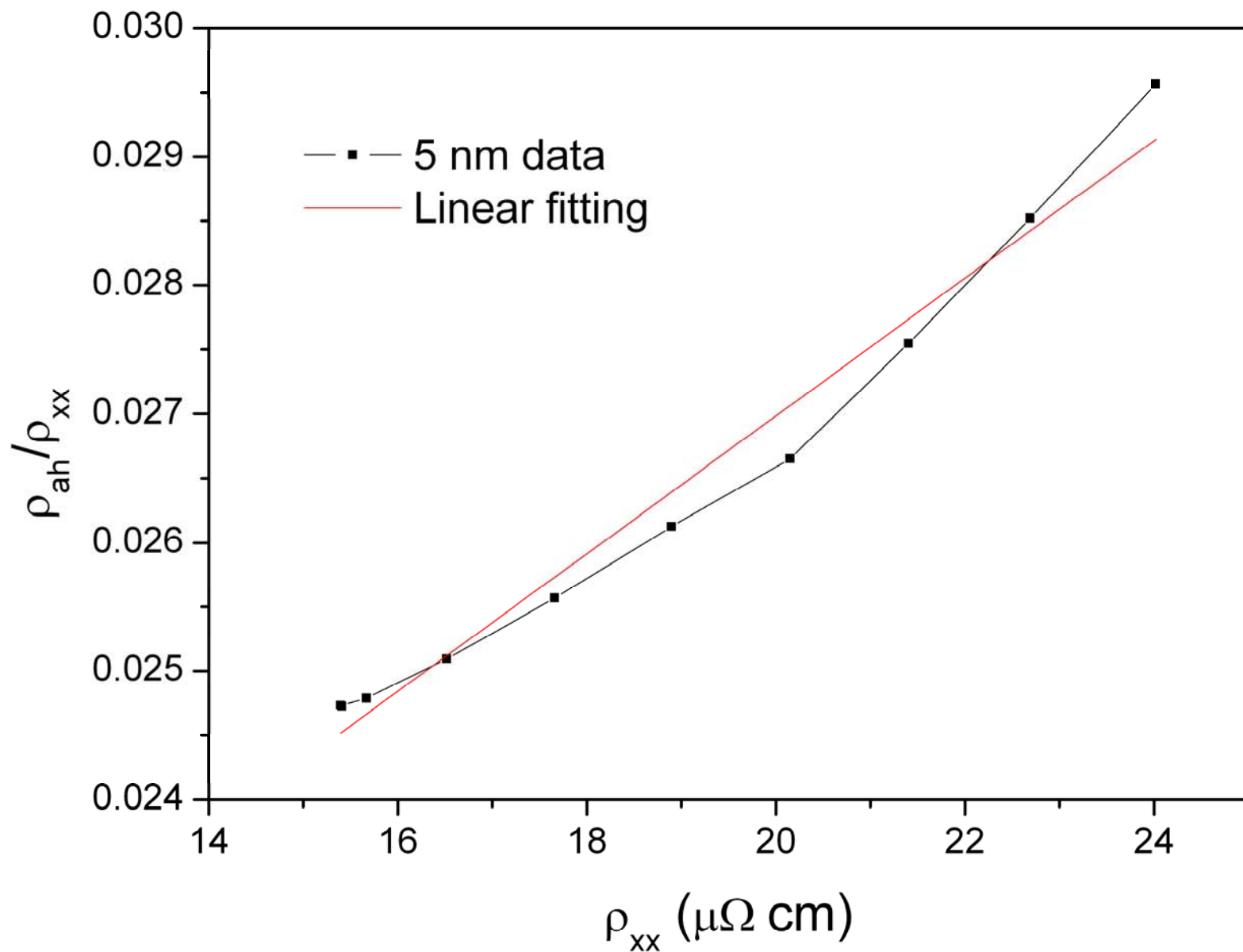
Old scaling:



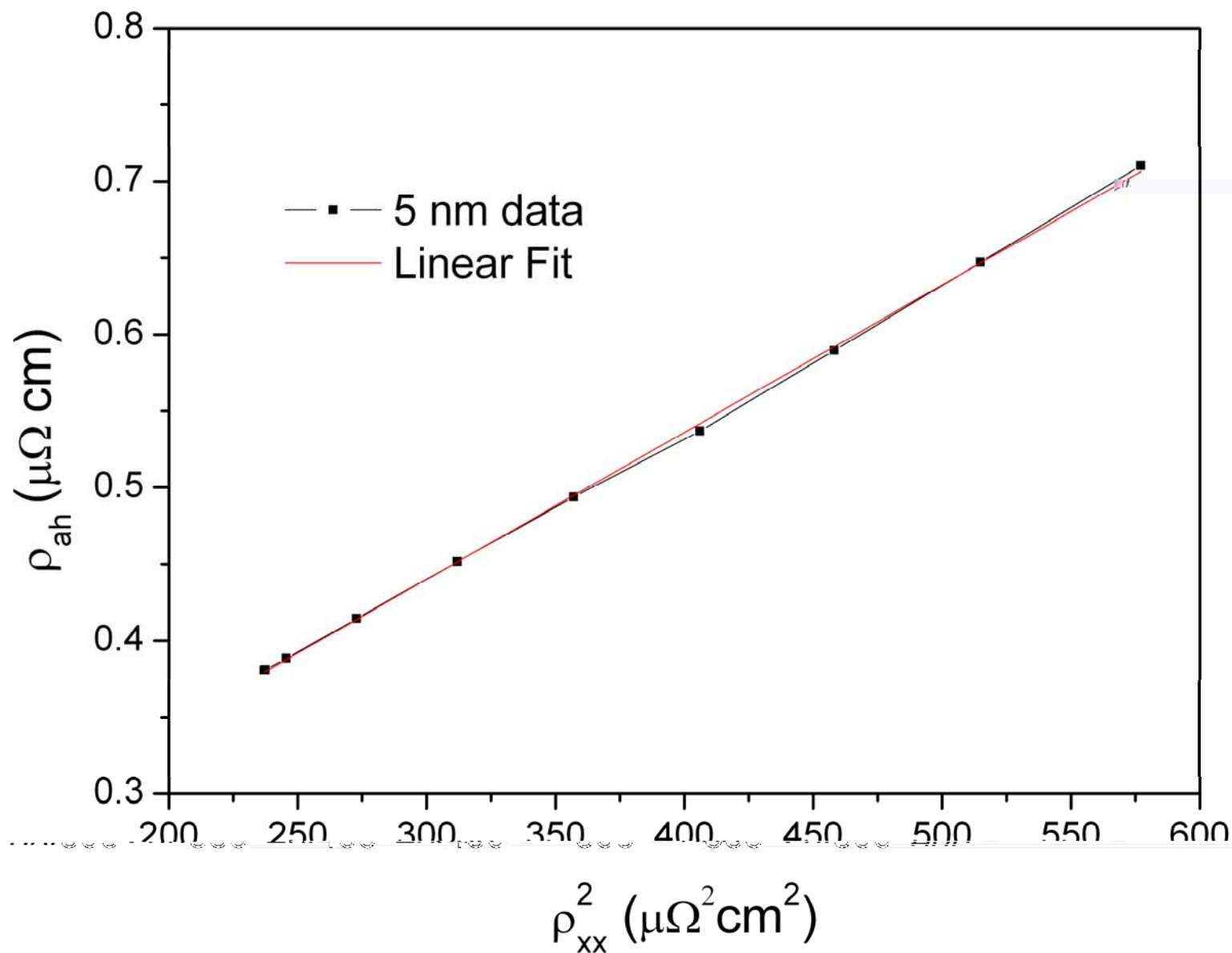
New scaling:



Old scaling:



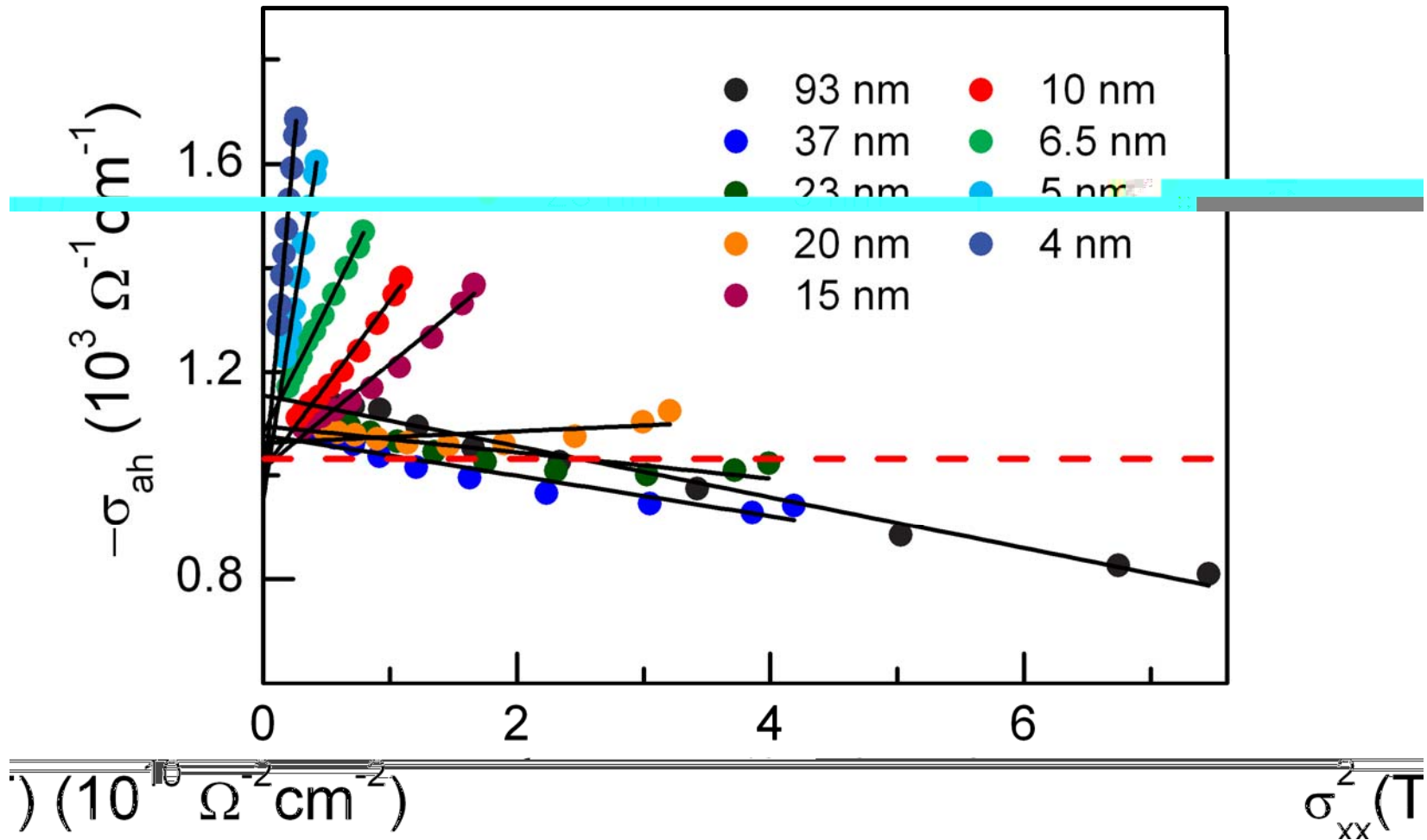
New scaling:



Proper Scaling of the AHE

$$\begin{pmatrix} 0 & 0 \end{pmatrix} \quad 0 \quad 2 \quad 0 \quad 2$$

$$\sigma_{ah} = -(\alpha\sigma_{xx0}^{-1} + \beta\sigma_{xx0}^{-2})\sigma_{xx}^2 - b$$

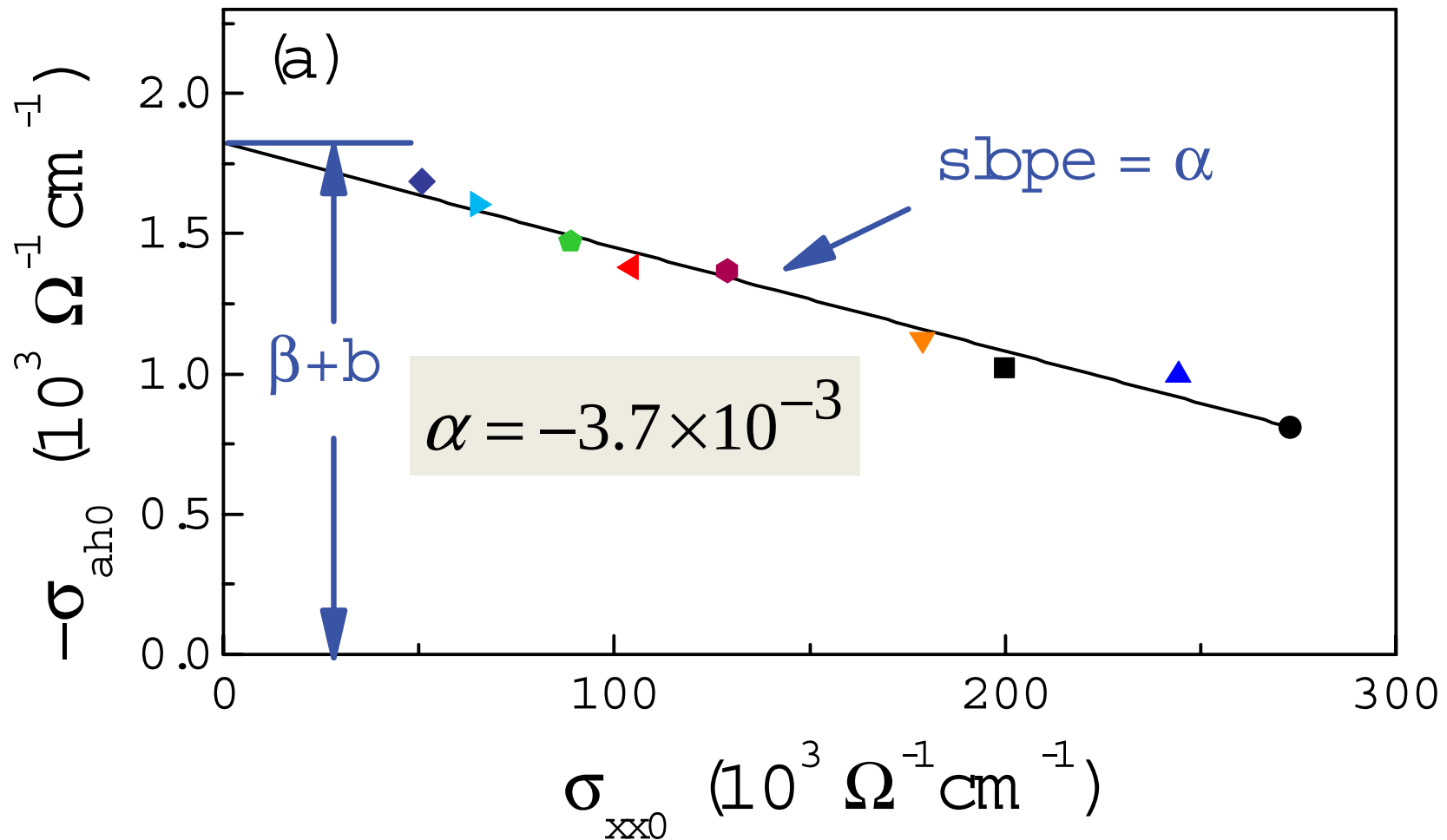


$$b \approx 1.1 \times 10^3 \Omega^{-1} cm^{-1} = -\sigma_{int}$$

$$\sigma_{ah} = -(\alpha\sigma_{xx0}^{-1} + \beta\sigma_{xx0}^{-2})\sigma_{xx}^2 - b$$

at 5K

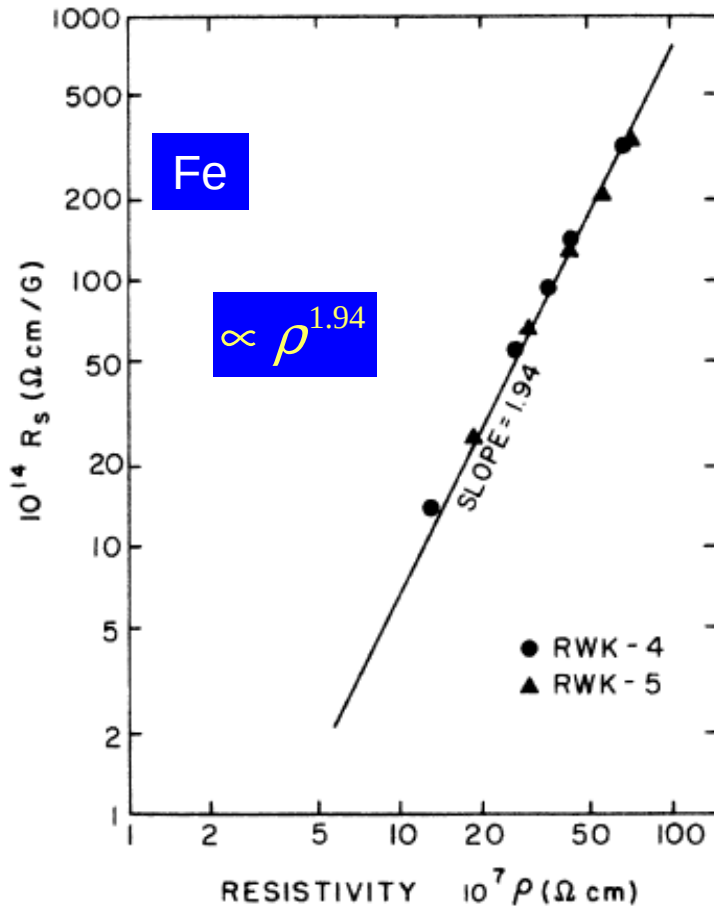
$$-\sigma_{ah0} = \alpha\sigma_{xx0} + (\beta + b)$$



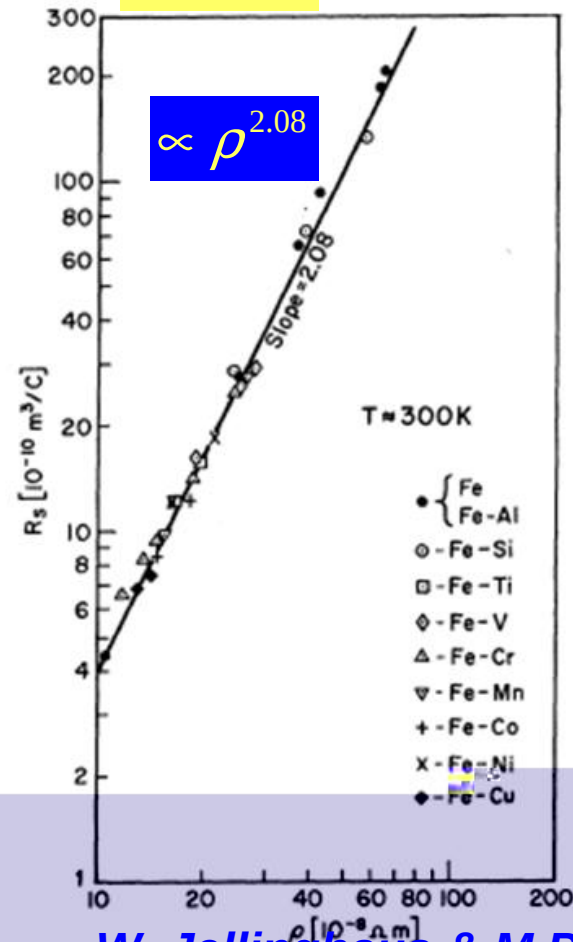
$$\rho_{ah} = \alpha \rho_{xx0} + \beta \rho_{xx0}^2 + b \rho_{xx}^2$$

Category 1:

$$b \rho_{xx}^2$$



*R. W. Klaffy & R.V. Coleman,
Phy. Rev B. 10, 2915 (1974).*

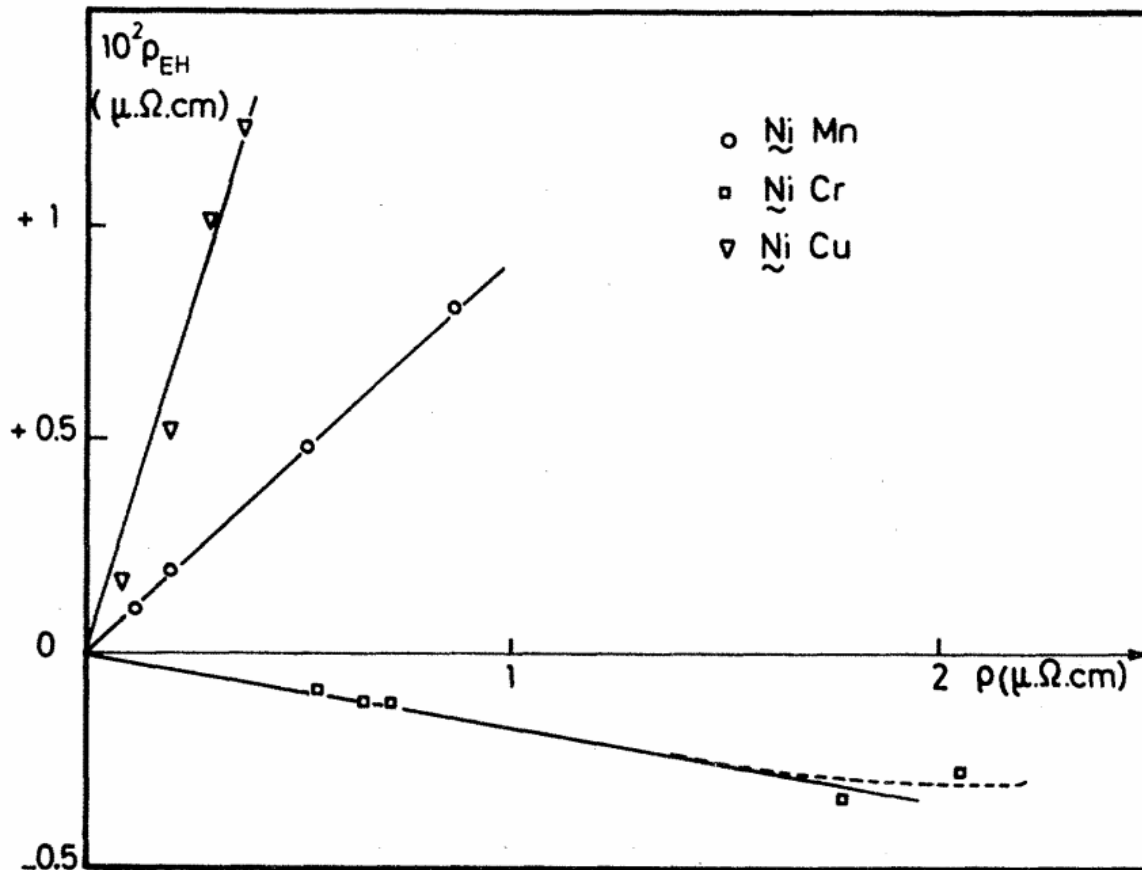


*W. Jellinghaus & M.P. DeAndres,
Ann. Physik, 462, 189 (1961)*

$$\rho_{ah} = \alpha\rho_{xx0} + \beta\rho_{xx0}^2 + b\rho_{xx}^2$$

Category 2:

$$a\rho_{xx}$$

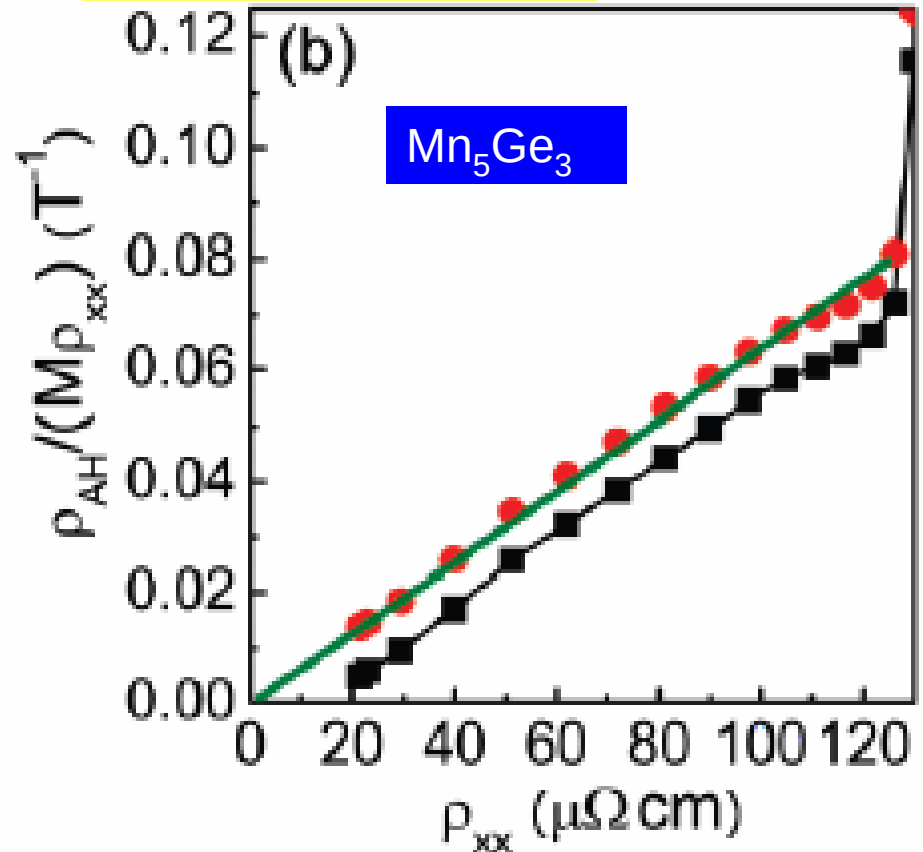
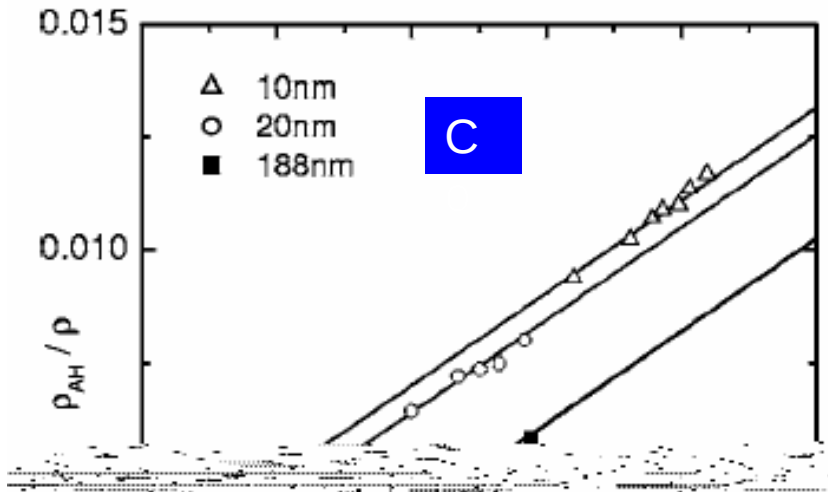


A. Fert and O. Jaoul, PRL. 28, 303 (1972)

$$\rho_{ah} = \alpha\rho_{xx0} + \beta\rho_{xx0}^2 + b\rho_{xx}^2$$

Category 3:

$$a\rho_{xx} + b\rho_{xx}^2$$

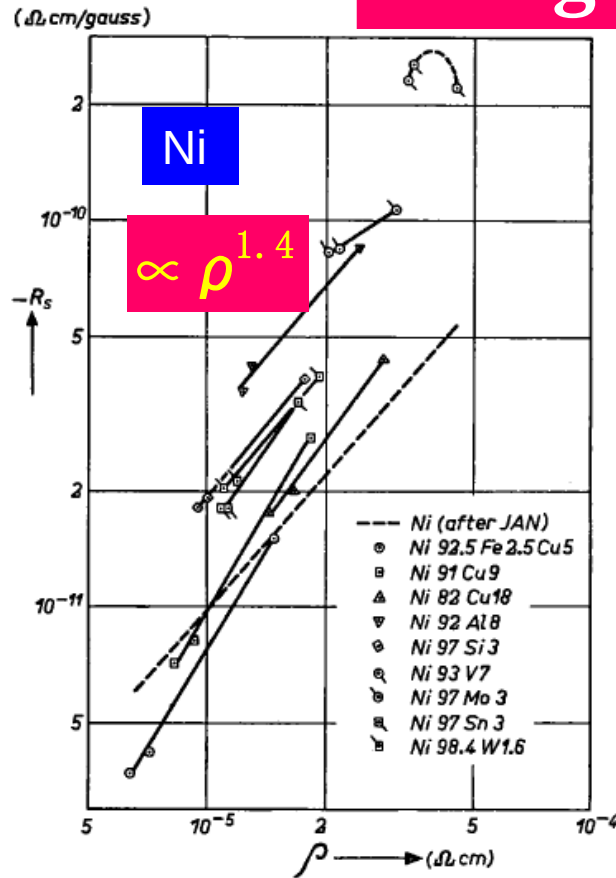


*J. Kotzler and W. Gil,
Phys. Rev. B 72, 060412 (2005)*

C. G. Zeng, et, al, PRL. 96, 037204 (2006)

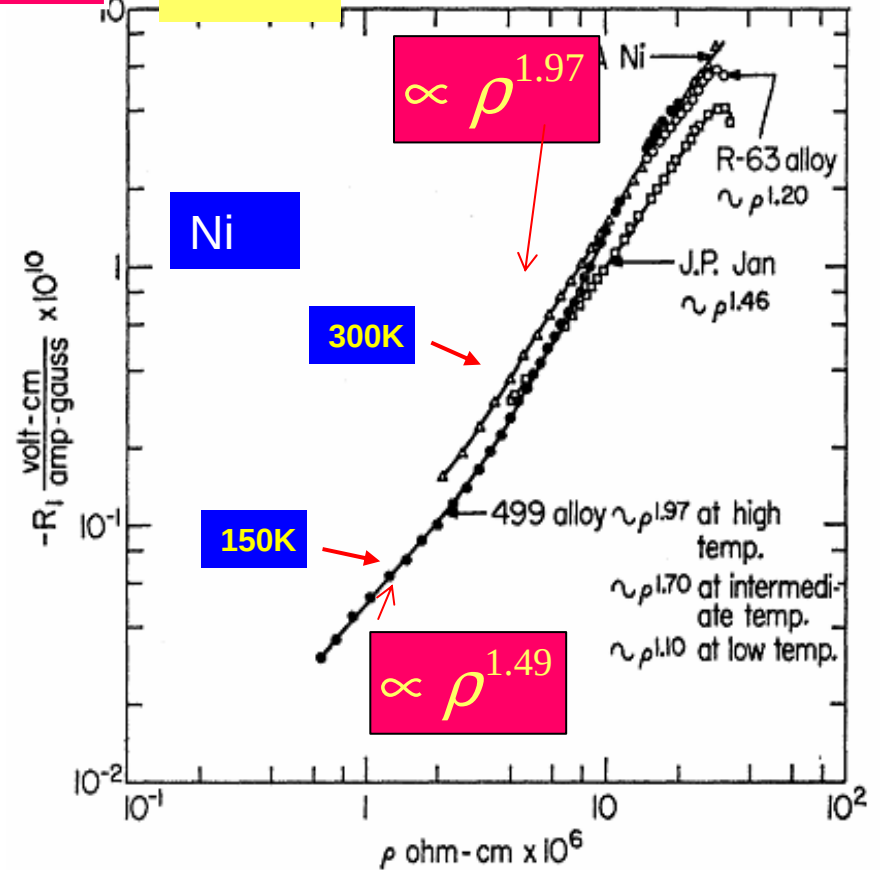
$$\rho_{ah} = \alpha \rho_{xx0} + \beta \rho_{xx0}^2 + b \rho_{xx}^2$$

Category 4:



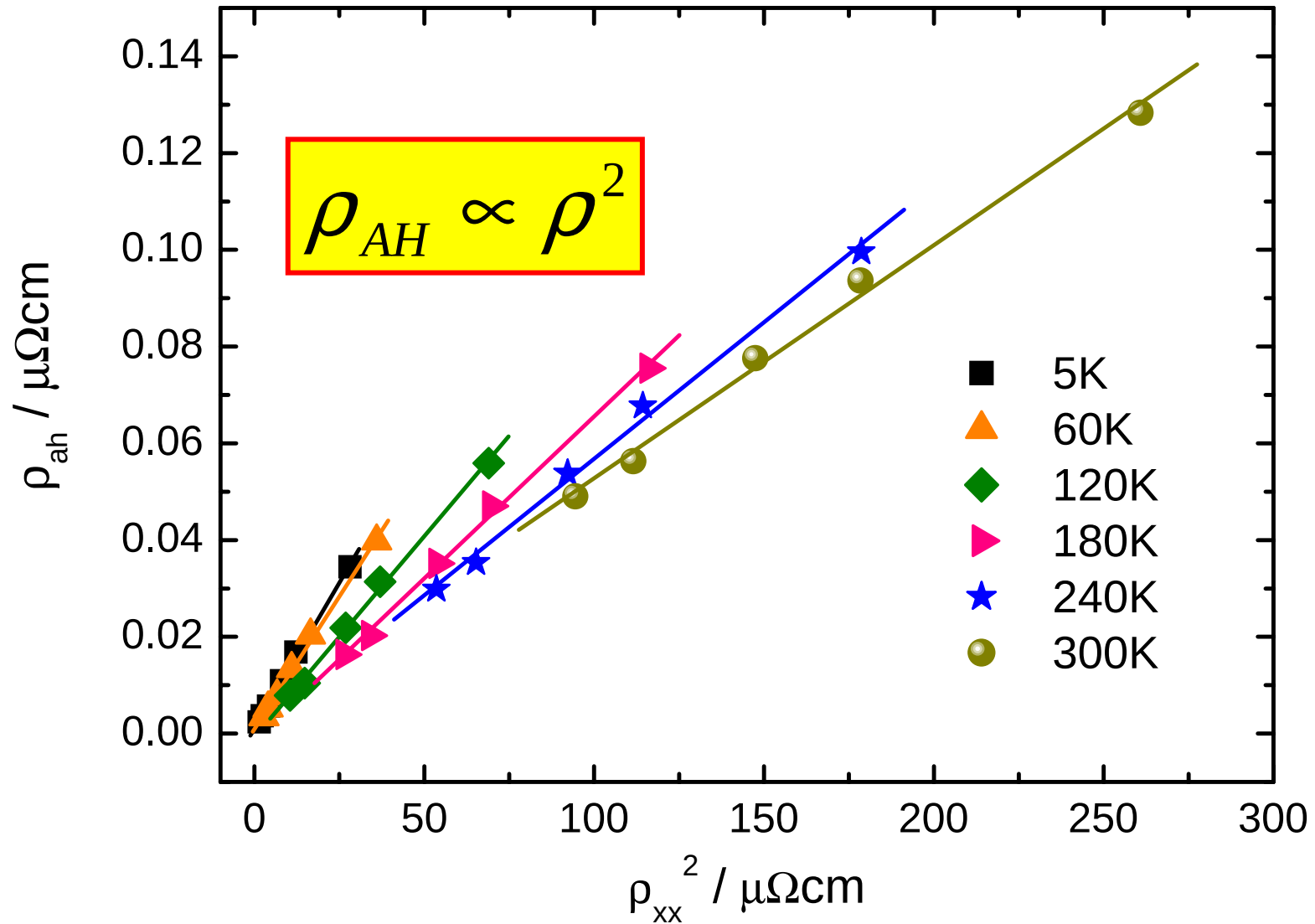
J. Smit, Physica 24, 39 (1954)

$$b \rho_{xx}^\alpha$$



J. Mavine, Phys. Rev. 123, 1273 (1961)

Anomalous Hall Resistivity **vs** Film Thickness



Conclusion 1:

Proper Scaling of the AHE ?

$$\rho_{ah} = f(\rho_{xx0}, \rho_{xx})$$

Conclusion 2:

Intrinsic and Extrinsic in the AHE ?

$$\rho_{ah} = \alpha \rho_{xx0} + \beta \rho_{xx0}^2 + b \rho_{xx}^2$$



THANK YOU

