The Quantum Spin Hall Effect

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with Andrei Bernevig, Taylor Hughes

Science*, 314,*1757 (2006)

Molenkamp et al,
Science*, 318, *766 (2007)

XL Qi, T. Hughes, SCZ
Nature Physics*, 4, *273 (2008)

The search for new states of matter

The search for new elements led to a golden age of chemistry.

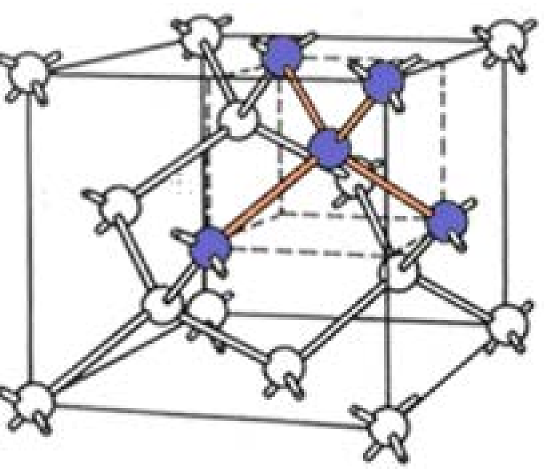
The search for new particles led to the golden age of particle physics.

In condensed matter physics, we ask what are the fundamental states of matter?

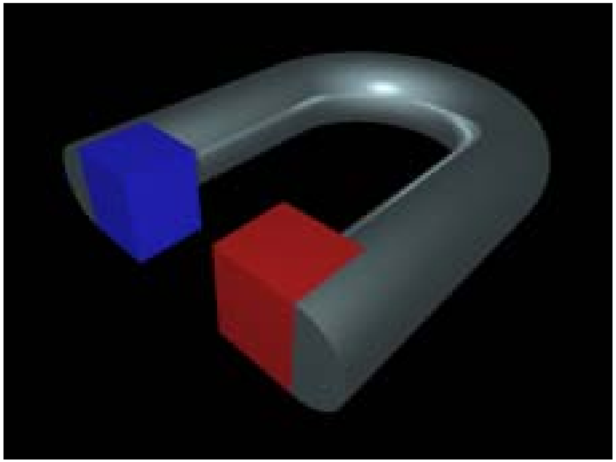
In the classical world we have solid, liquid and gas. The same H_2O molecules can condense into ice, water or vapor.

In the quantum world we have metals, insulators, superconductors, magnets etc.

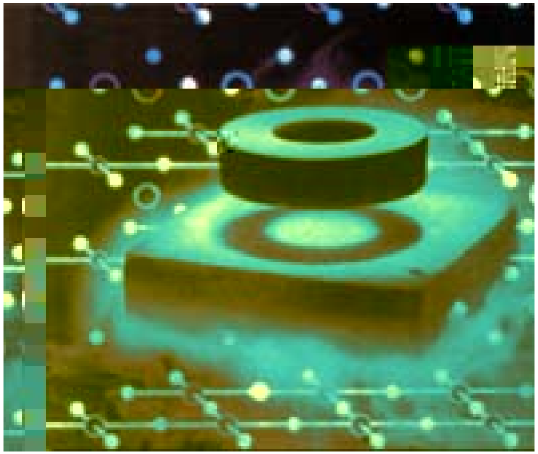
Most of these states are differentiated by the broken symmetry.



Crystal: Broken translational symmetry



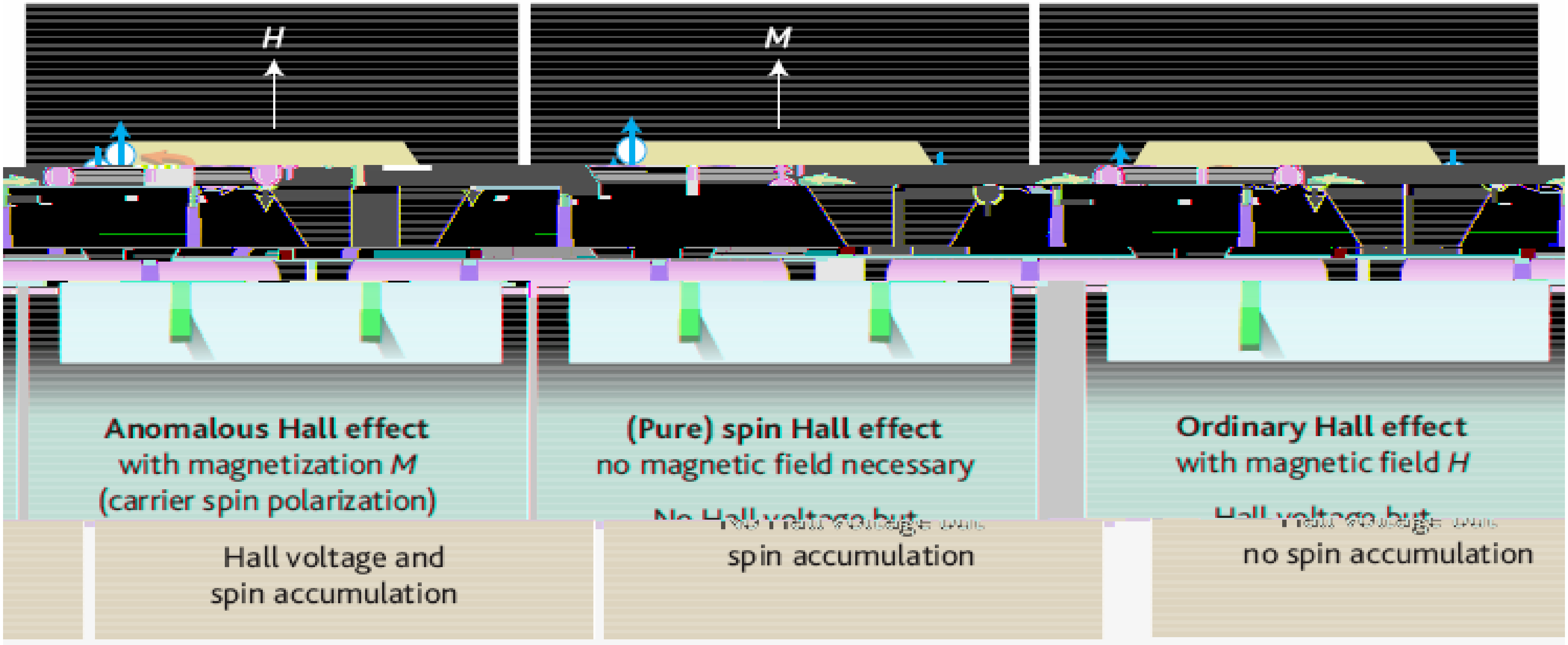
Magnet: Broken rotational symmetry



Superconductor: Broken gauge symmetry

The quantum Hall state, a topologically non-trivial state of matter

The Generalizations of the Hall Effect



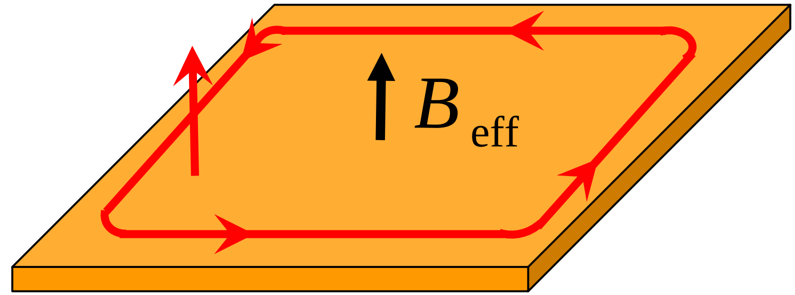
- Theoretical predictions of the intrinsic spin Hall effect
- The spin Hall effect has now been experimentally observed.

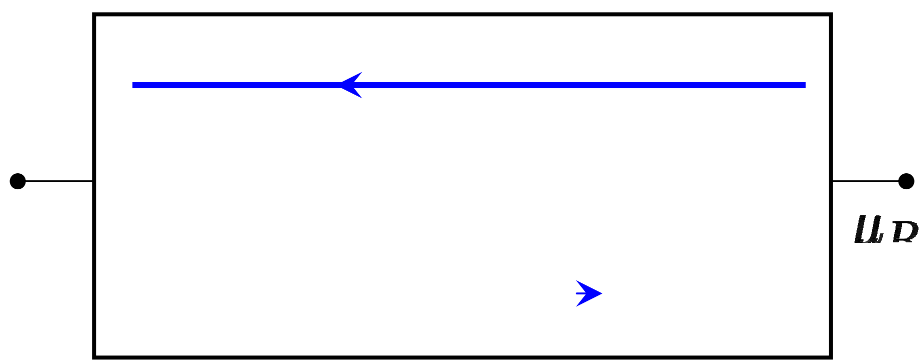
What about the quantum spin Hall effect?

Quantum Spin Hall Effect

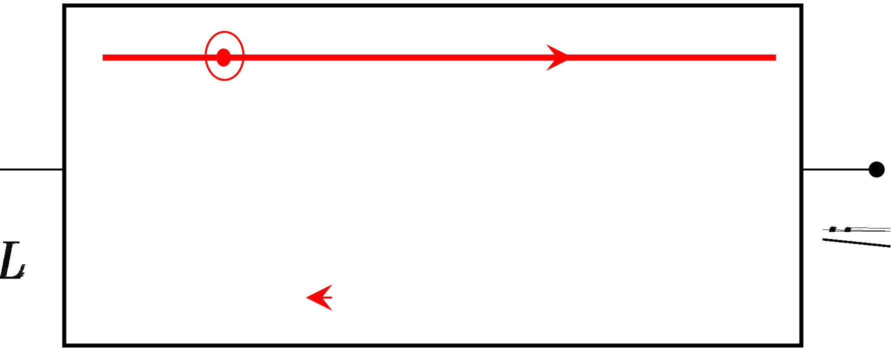
• The QSH state can be thought of as two copies of QH states, one for each spin component, each seeing the opposite magnetic field.

• The QSH state does not break the time reversal symmetry, and can exist without any external magnetic field.





Chiral (QHE) and helical (QSHE) liquids in D=1



Taking the square root in math and physics

Time reversal symmetry in quantum mechanics

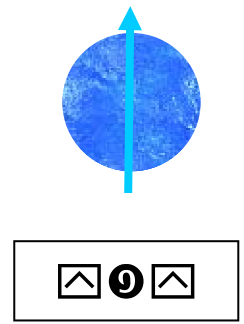
• Wave function of a particle with integer spin changes by 1 under 2π rotation.

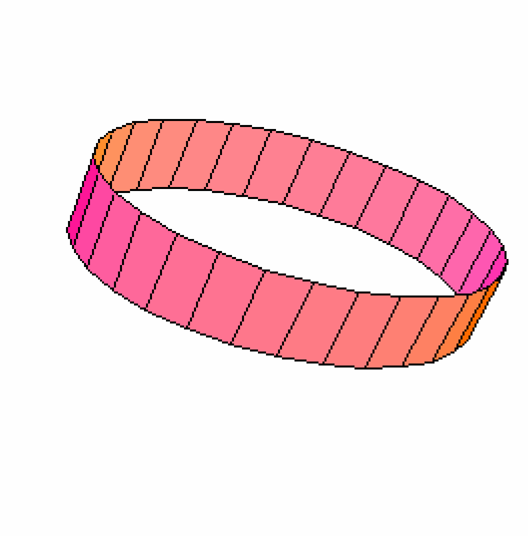
• Wave function of a half-integer spin changes by -1 under 2π rotation.

• Kramers theorem, in a time reversal invariant system with half-integer spins, $T^2=-1$, all states for degenerate doublets.

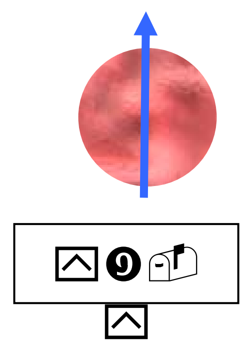
• Application in condensed matter physics: Anderson's theorem. BCS pair=(k,✸)+(-k,✵). General pairing between Kramers doublets.

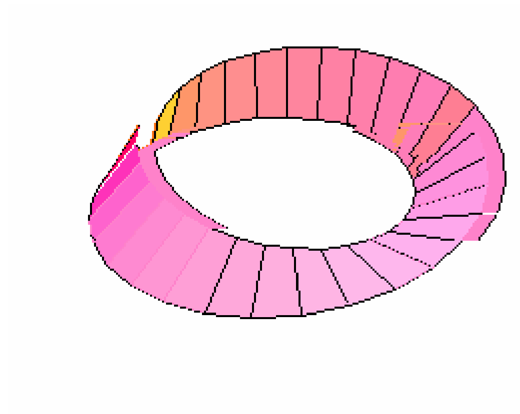
Spin=1





Spin=1/2





The topological distinction between a conventional insulator and a QSH insulator

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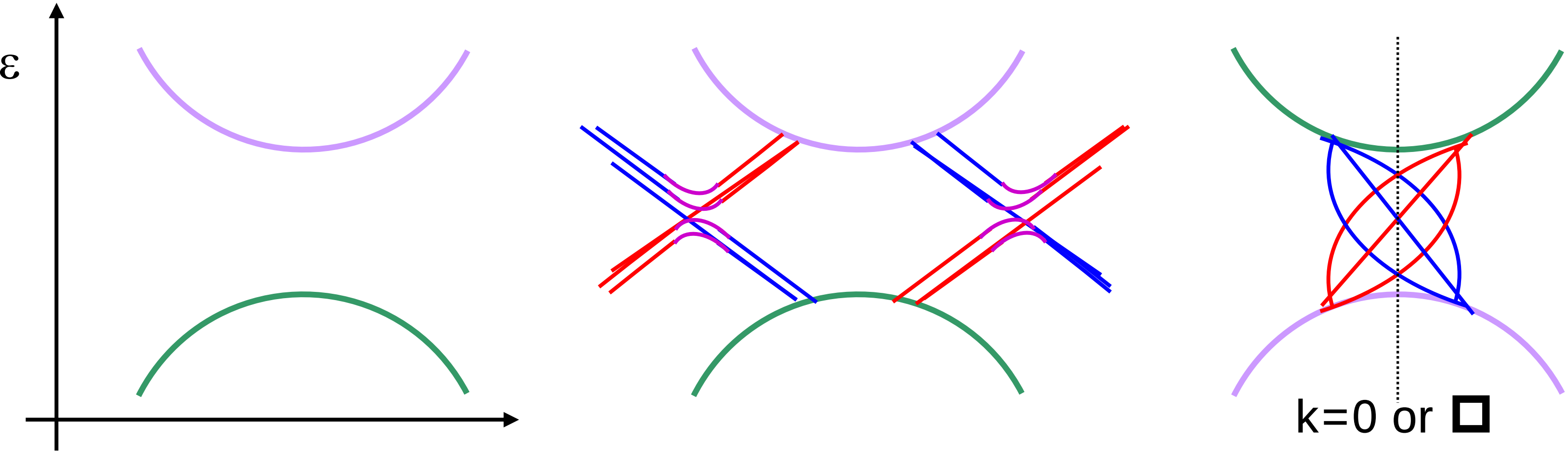
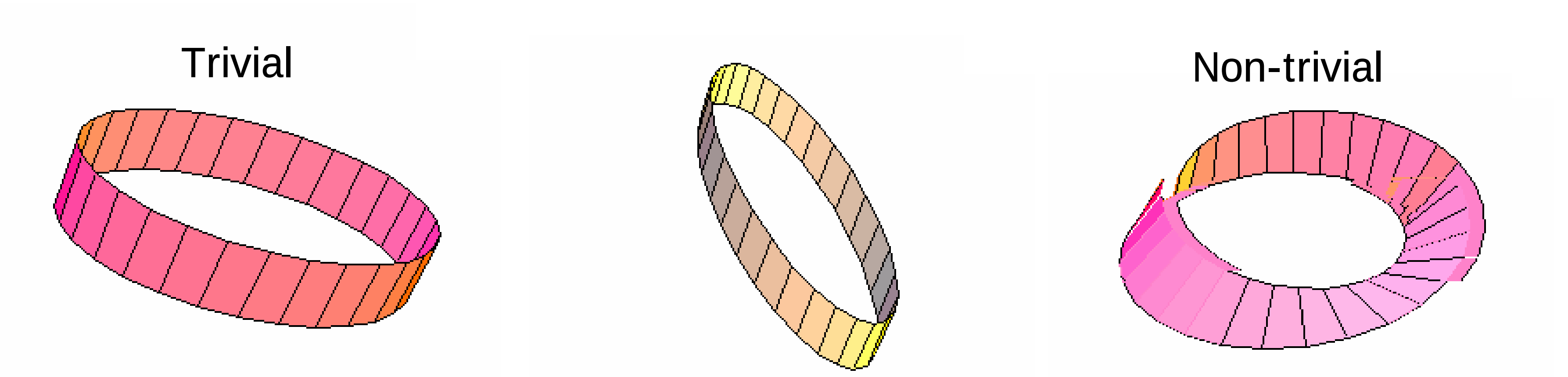
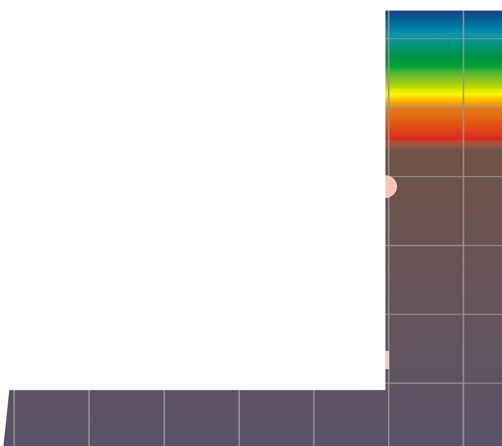
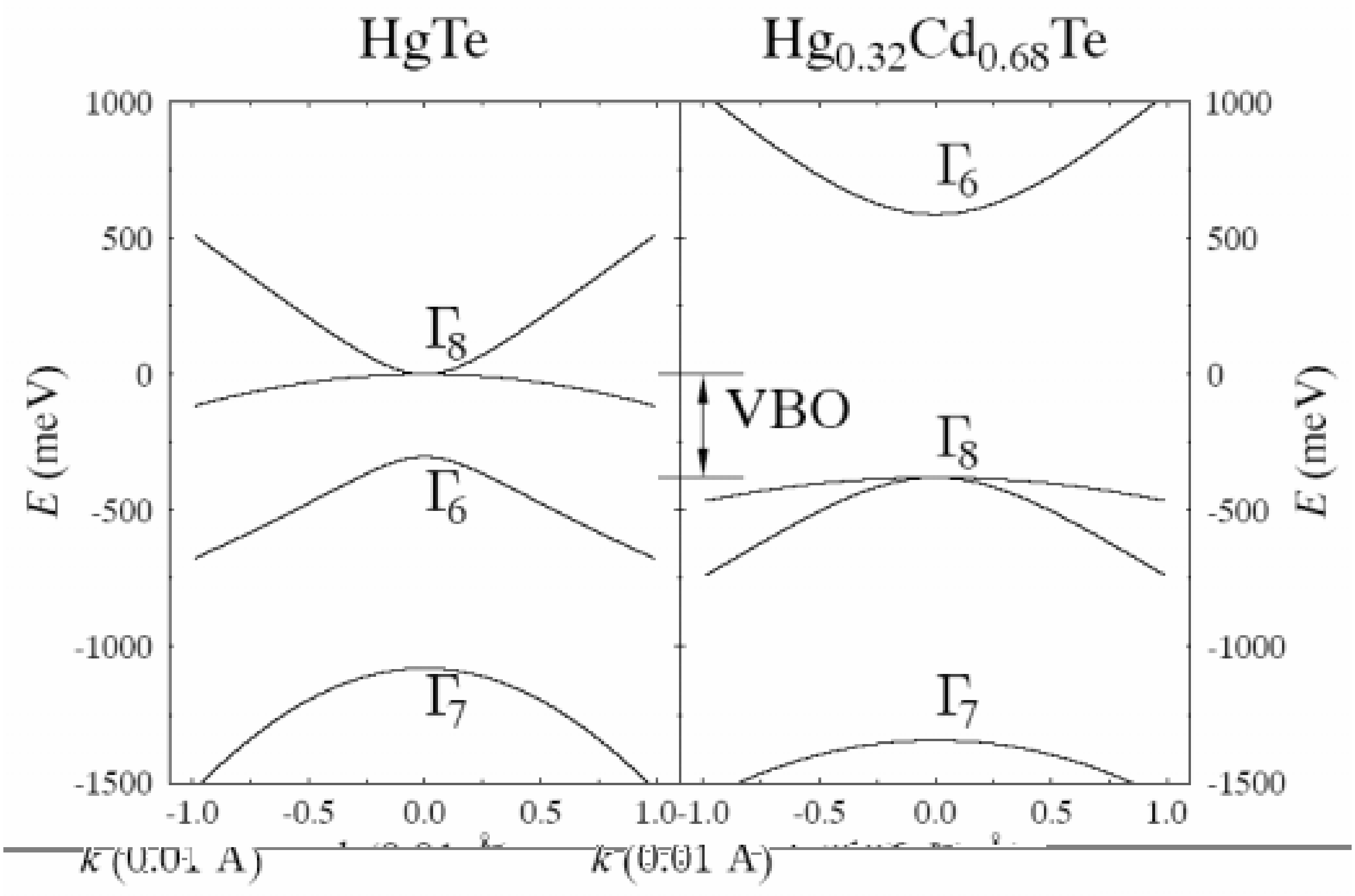
• Band diagram of a conventional insulator, a conventional insulator with accidental surface states (with animation), a QSH insulator (with animation). Blue and red color code for up and down spins.







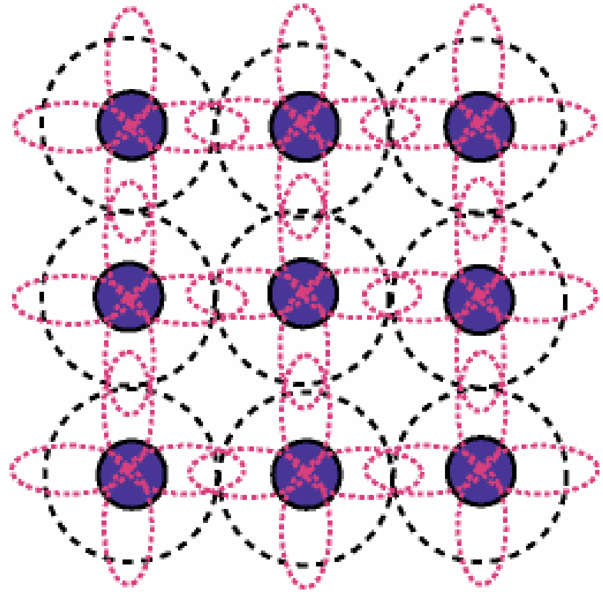
Band Structure of HgTe



Effective tight-binding model

Square lattice with 4-orbitals per site:

$$|s,\uparrow\rangle, |s,\downarrow\rangle, |(p_x+ip_y,\uparrow\rangle, |-(p_x-ip_y),\downarrow\rangle$$



Nearest neighbor hopping integrals. Mixing matrix elements between the s and the p states must be odd in k.

$$H_{eff}(k_x,k_y)=\begin{pmatrix} h(k) & 0 \\ 0 & h^*(-k) \end{pmatrix}$$

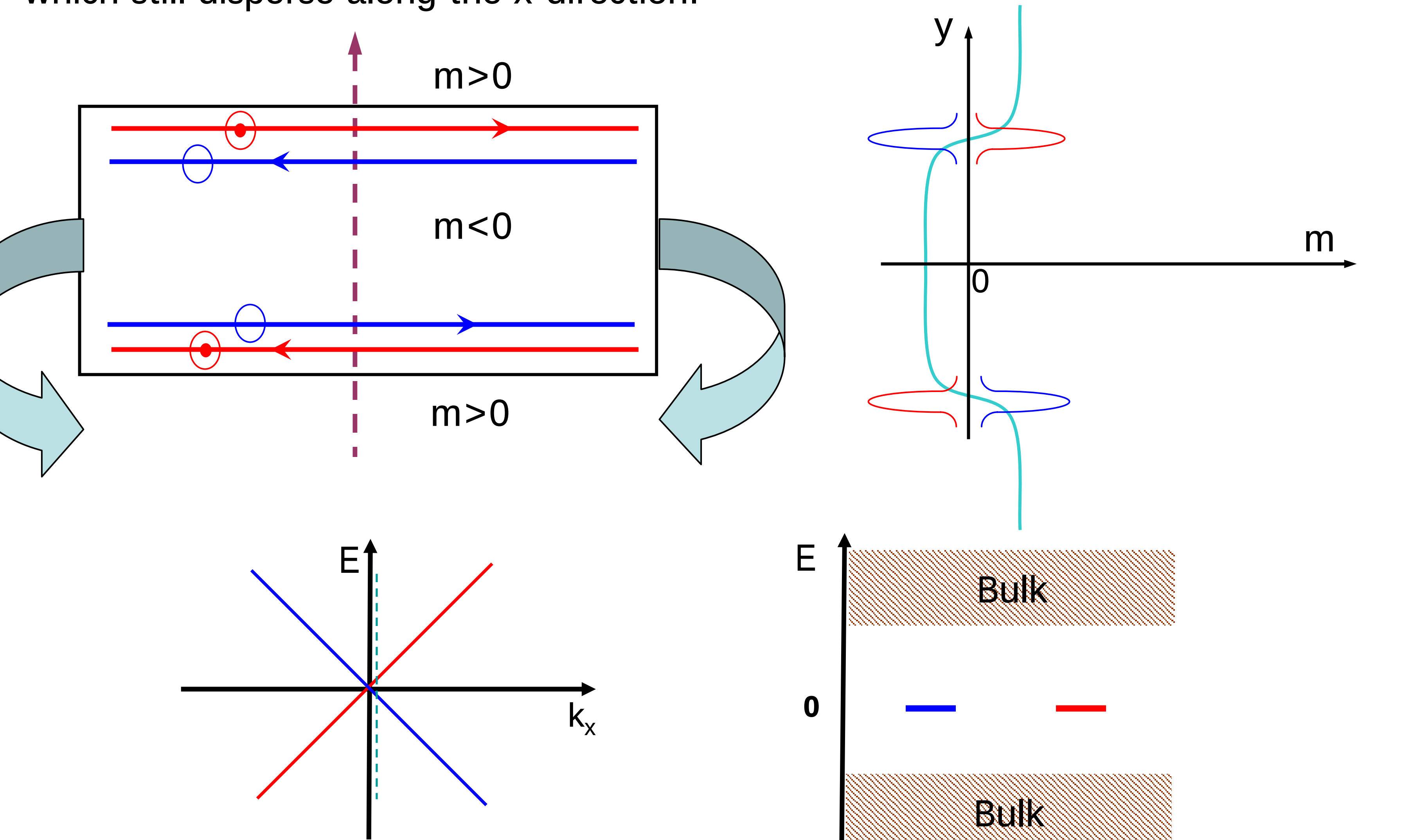
$$h(k)=\begin{pmatrix} m(k) & A(\sin k_x - i\sin k_y) \\ A(\sin k_x + i\sin k_y) & -m(k) \end{pmatrix} \equiv d_a(k)\tau^a$$

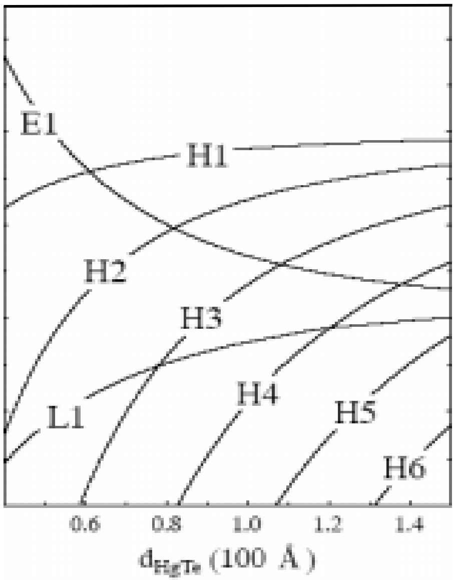
$$\Rightarrow \begin{pmatrix} m & A(k_x-ik_y) \\ A(k_x+ik_y) & -m \end{pmatrix} \qquad \Delta\sigma^{\uparrow}{}_{xy}=\frac{1}{2}\Delta\, sign(m) \qquad \Delta\sigma^{\downarrow}{}_{xy}=-\Delta\sigma^{\uparrow}{}_{xy}$$

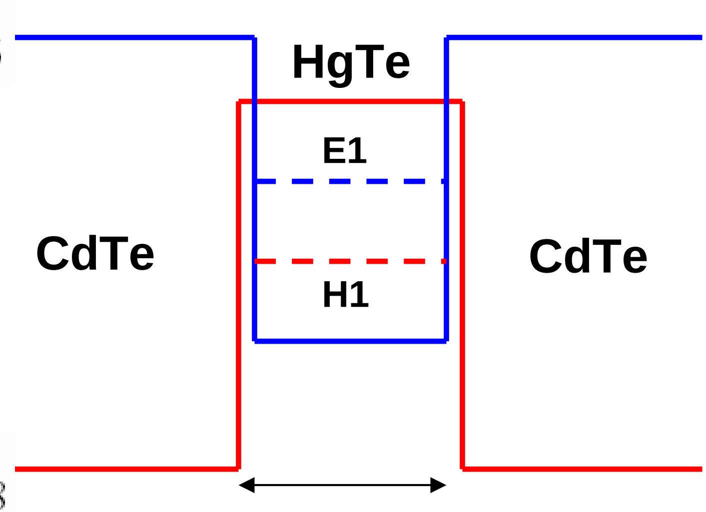
Relativistic Dirac equation in 2+1 dimensions, with a tunable mass term!

Mass domain wall

Cutting the Hall bar along the y-direction we see a domain-wall structure in the band structure mass term. This leads to states localized on the domain wall which still disperse along the x-direction.




E1
H1
H2
H3
H4
H5
H6
L1
0.6
0.8
1.0
1.2
1.4
d_HgTe (100 Å)

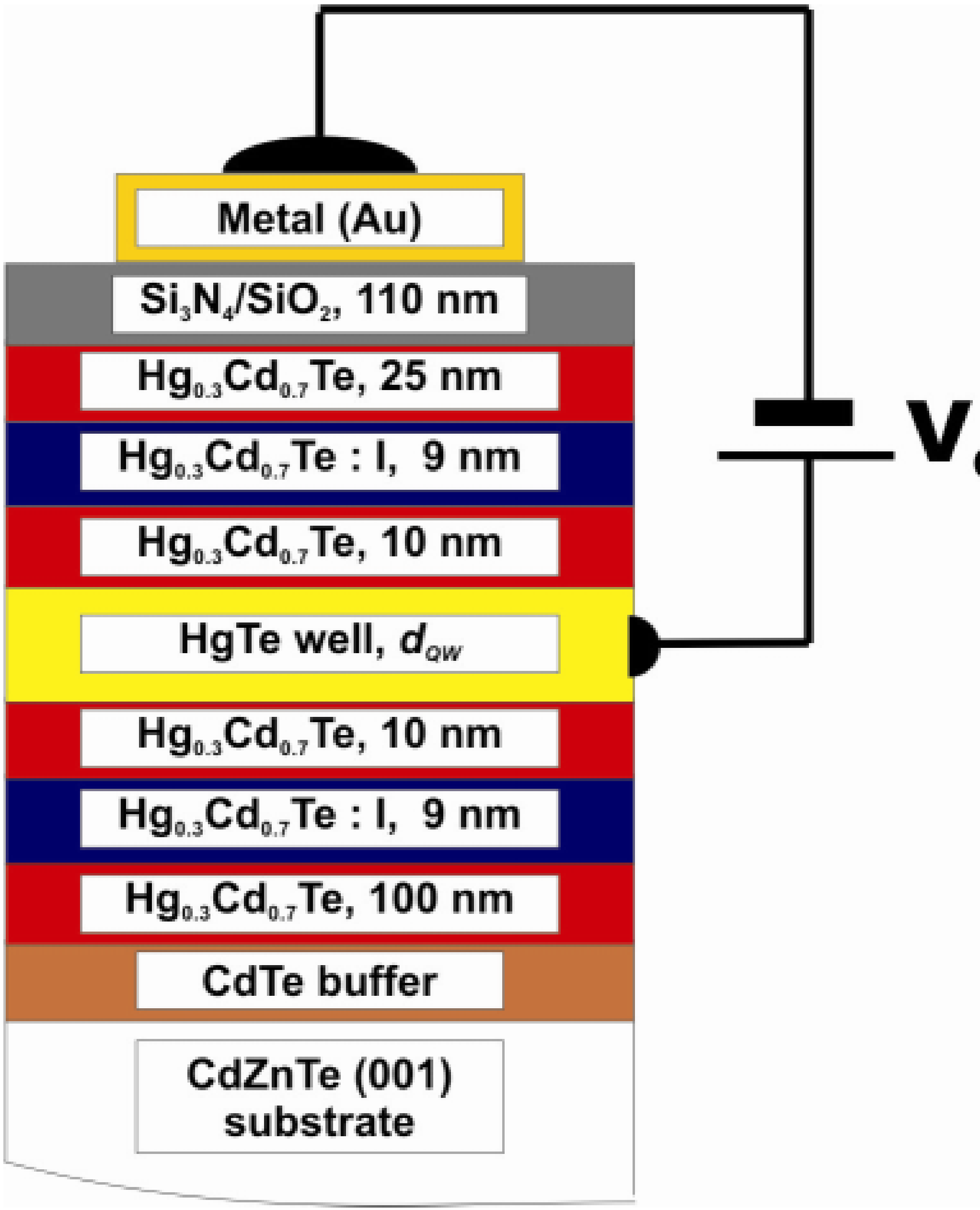

HgTe
E1
H1
CdTe
CdTe

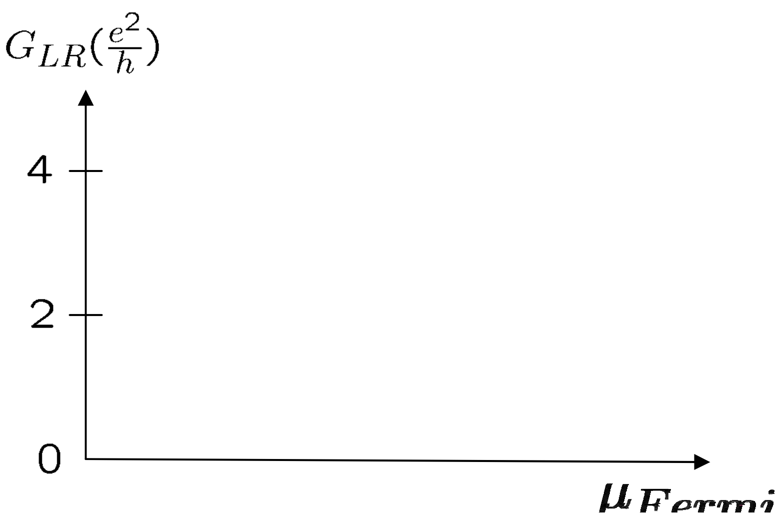
$d < d_c$

Experimental setup

High mobility samples of HgTe/CdTe quantum wells have been fabricated.

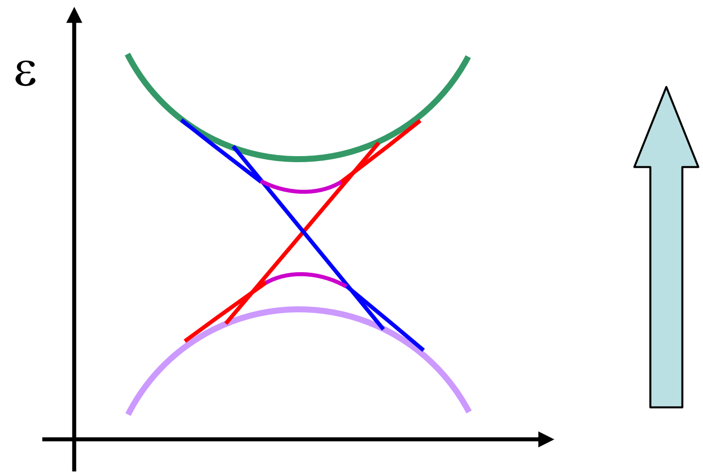
- Because of the small band gap, about several meV, one can gate dope this system from n to p doped regimes.
- Two tuning parameters, the thickness d of the quantum well, and the gate voltage.




$G_{LR}(\frac{e^2}{h})$
4
2
0
μ_{Fermi}

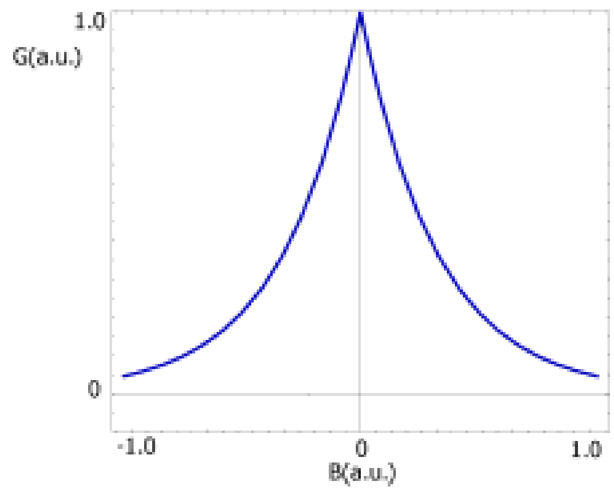
Smoking gun for the helical edge state: Magneto-Conductance

The crossing of the helical edge states is protected by the TR symmetry. TR breaking term such as the Zeeman magnetic field causes a singular perturbation and will open up a full insulating gap:



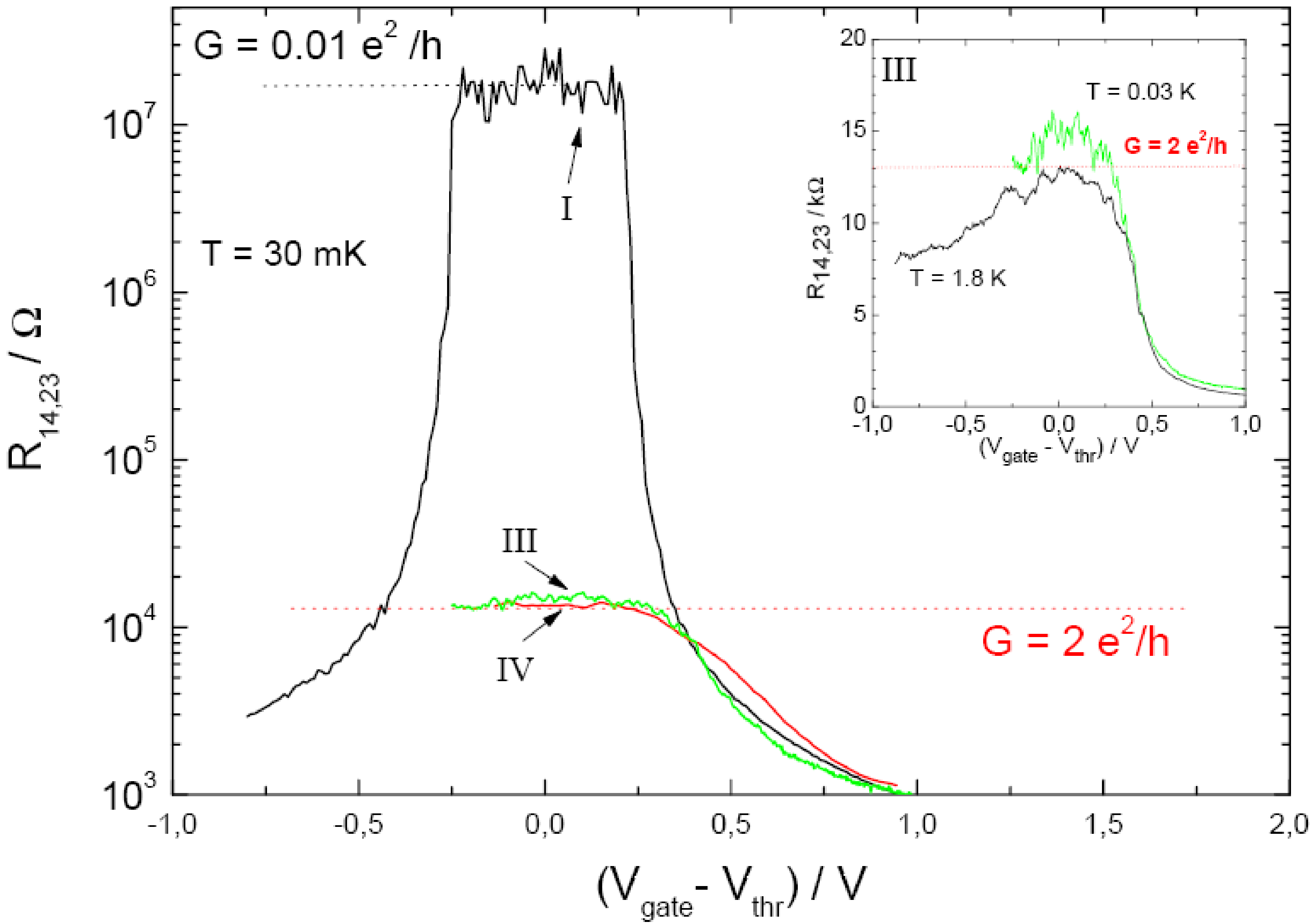
$$E_g \propto g|B|$$

Conductance now takes the activated form:

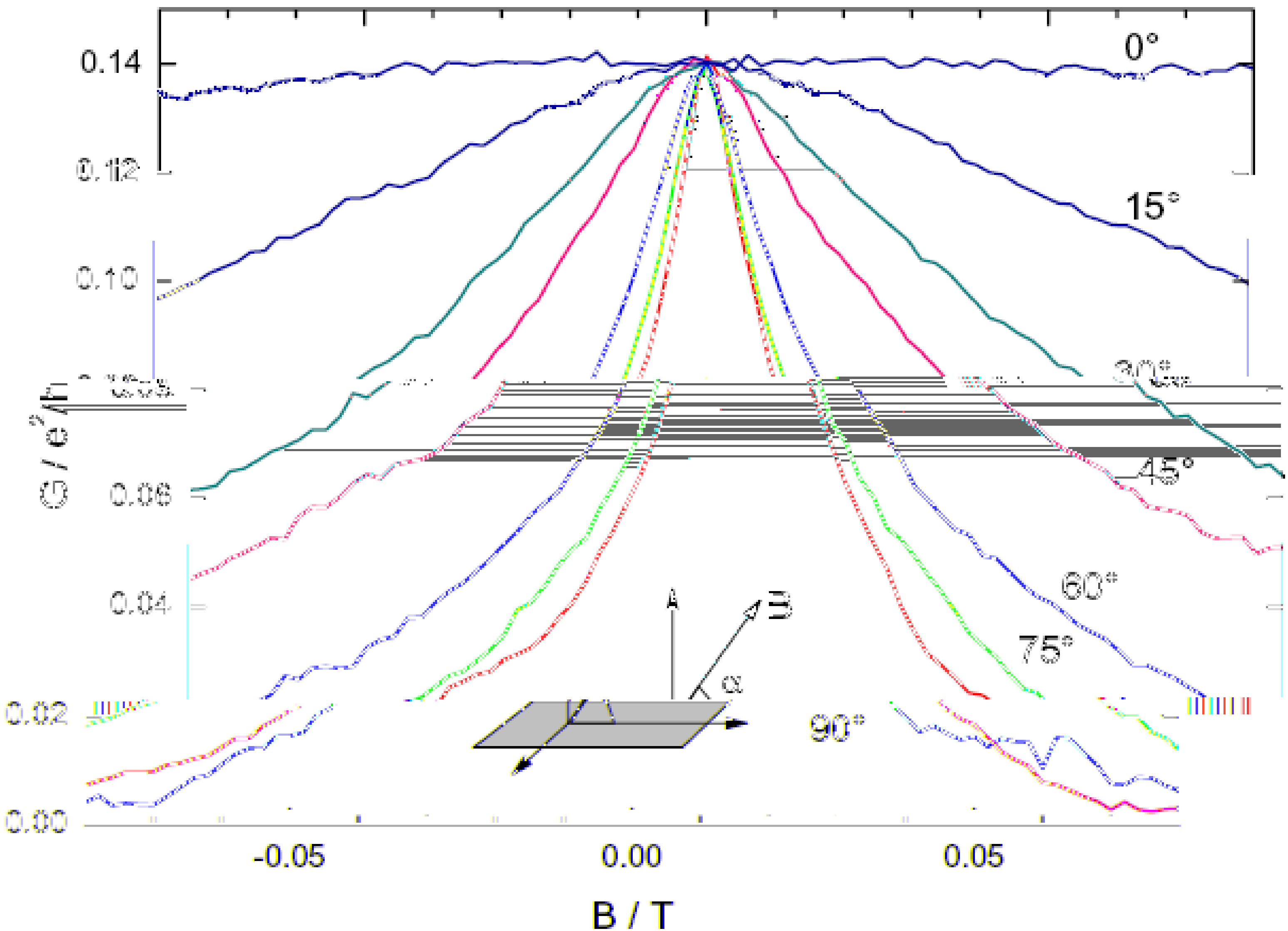


$$\sigma \propto f(T) e^{-g|B|/kT}$$

Experimental evidence for the QSH state in HgTe

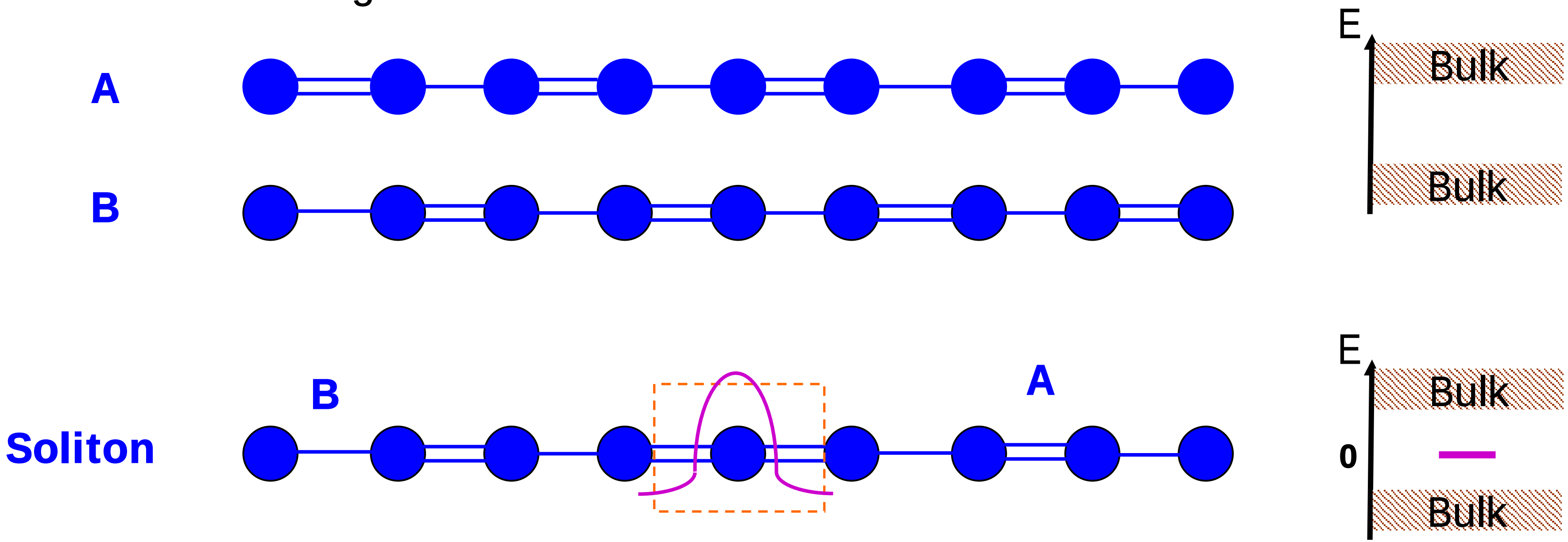


Magnetic field dependence of the residual conductance



A brief history of fractional charge

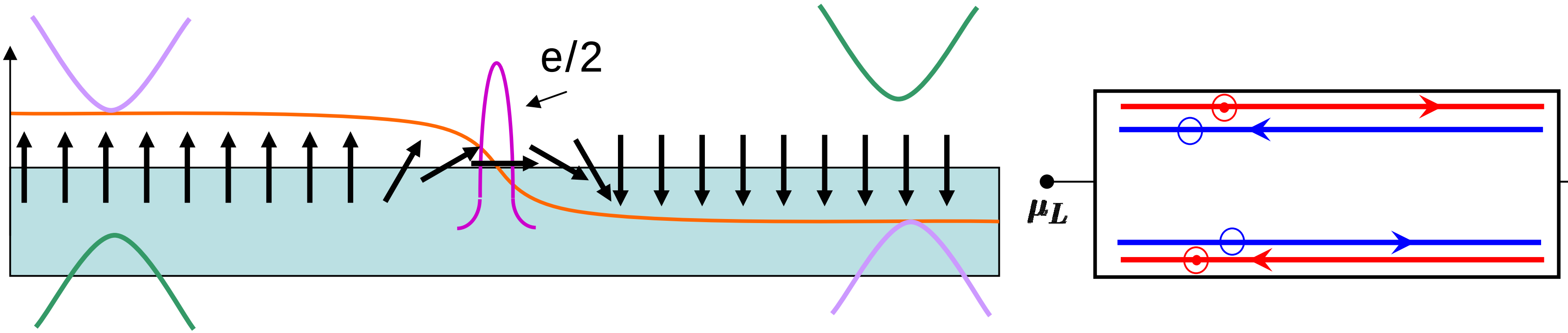
- Jackiw & Rebbi predicted that a fractional charge e/2 is carried by the mass domain wall (soliton) of 1-d Dirac model.
- Su, Schrieffer and Heeger presented a model of polyacetylene with two-fold degenerate ground states. A domain wall defect carries fractional charge e/2.



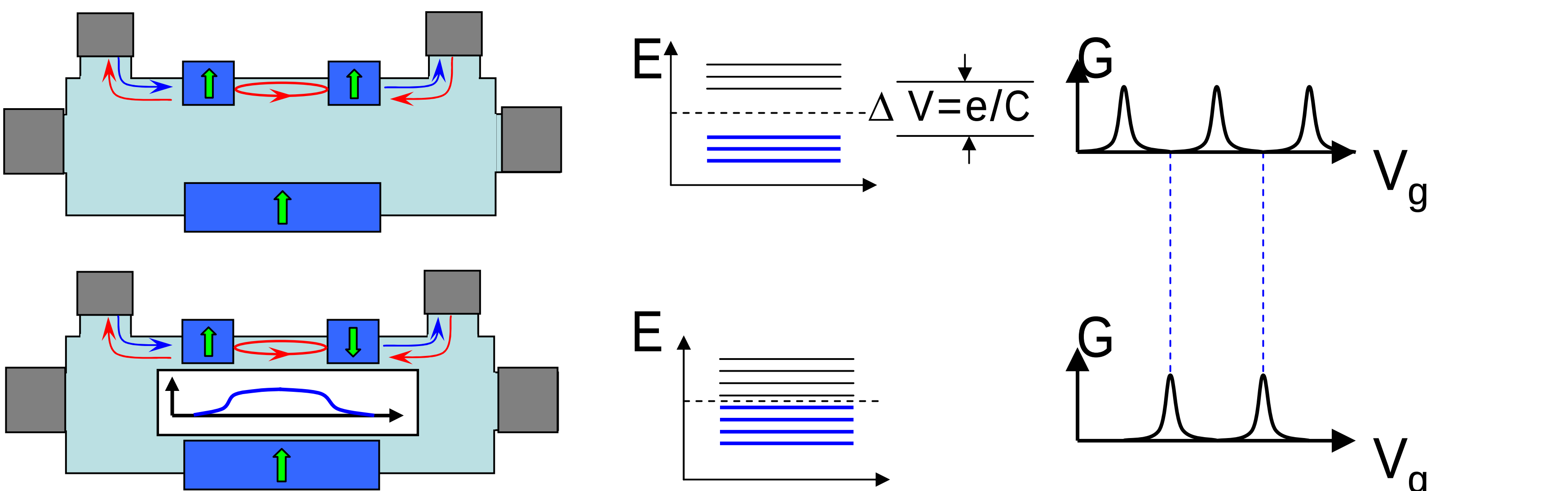
- Because of both up and down spin components carry fractional charge e/2, the net system only carries integer charge. Fractional charge has never been observed in any 1D system!

Fractional charge in the QSH state

- Since the mass is proportional to the magnetization, a magnetization domain wall leads to a mass domain wall on the edge.

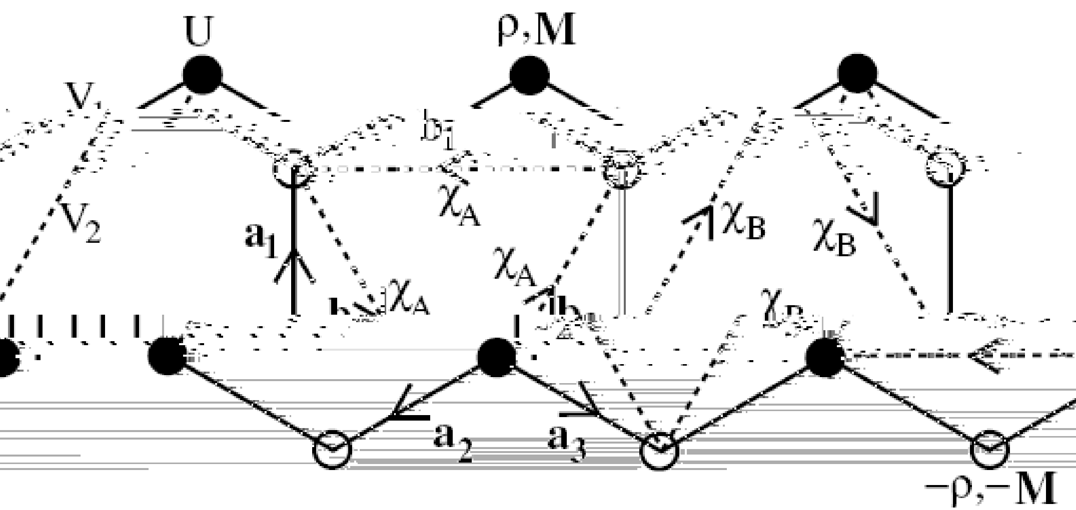


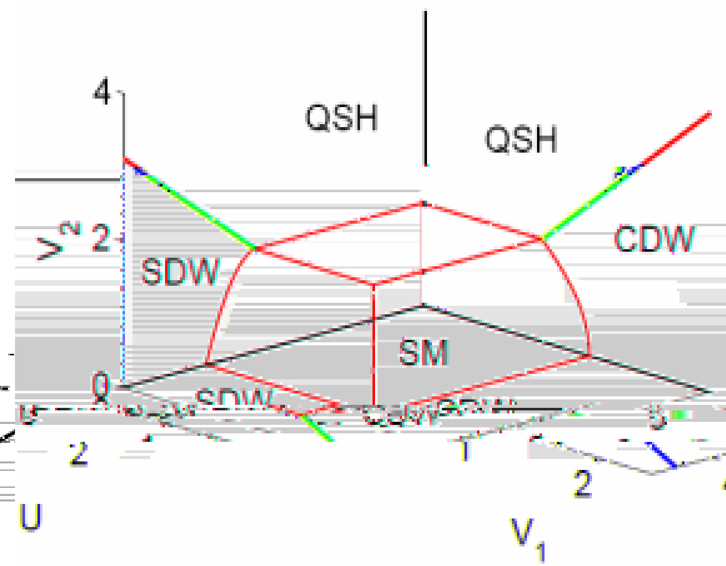
- The fractional charge e/2 can be measured by a Coulomb blockade experiment, one at the time!

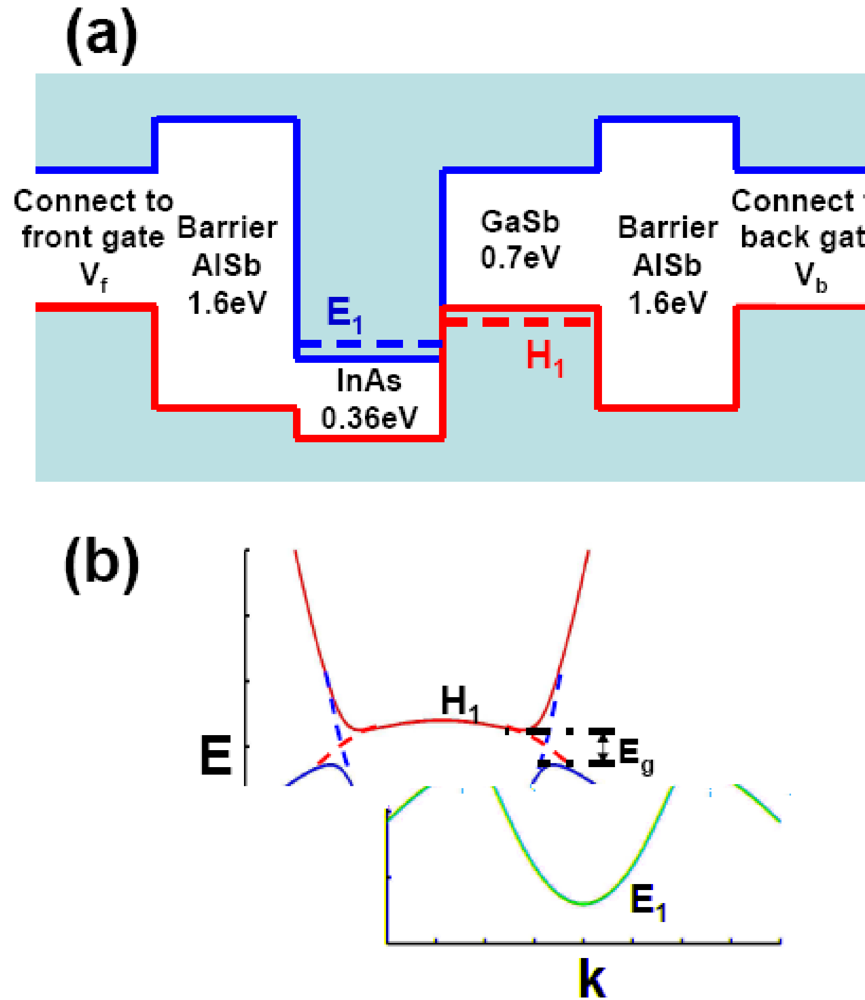


Topological Mott insulators

- So far, the QSH insulator is a topologically non-trivial band insulator. Can we have a topological Mott insulator, where the topologically non-trivial gap arises from interactions, not from band structure?
- Yes, on a honeycomb lattice with U, V1 and V2, one can obtain a TMI phase in the limit of V2>>U, V1. (Raghu et al, arXiv:0710.0030)
- This model provides an example of dynamic generation of spin-orbit coupling. (Wu+Zhang, PRL 2004).



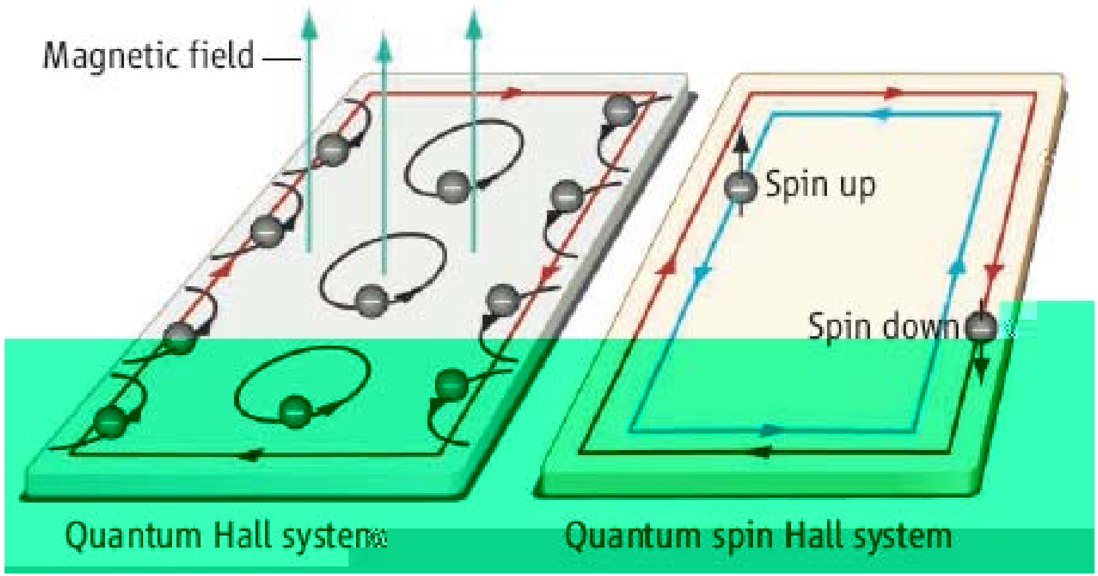



(a)
Connect to front gate V_f
Barrier AlSb 1.6eV
E_1
InAs 0.36eV
GaSb 0.7eV
H_1
Barrier AlSb 1.6eV
Connect to back gate V_b
(b)
E
H_1
E_g
E_1
k

A New State of Quantum Matter

Experiments show that electron spins can flow without dissipation in a novel electrical insulator.

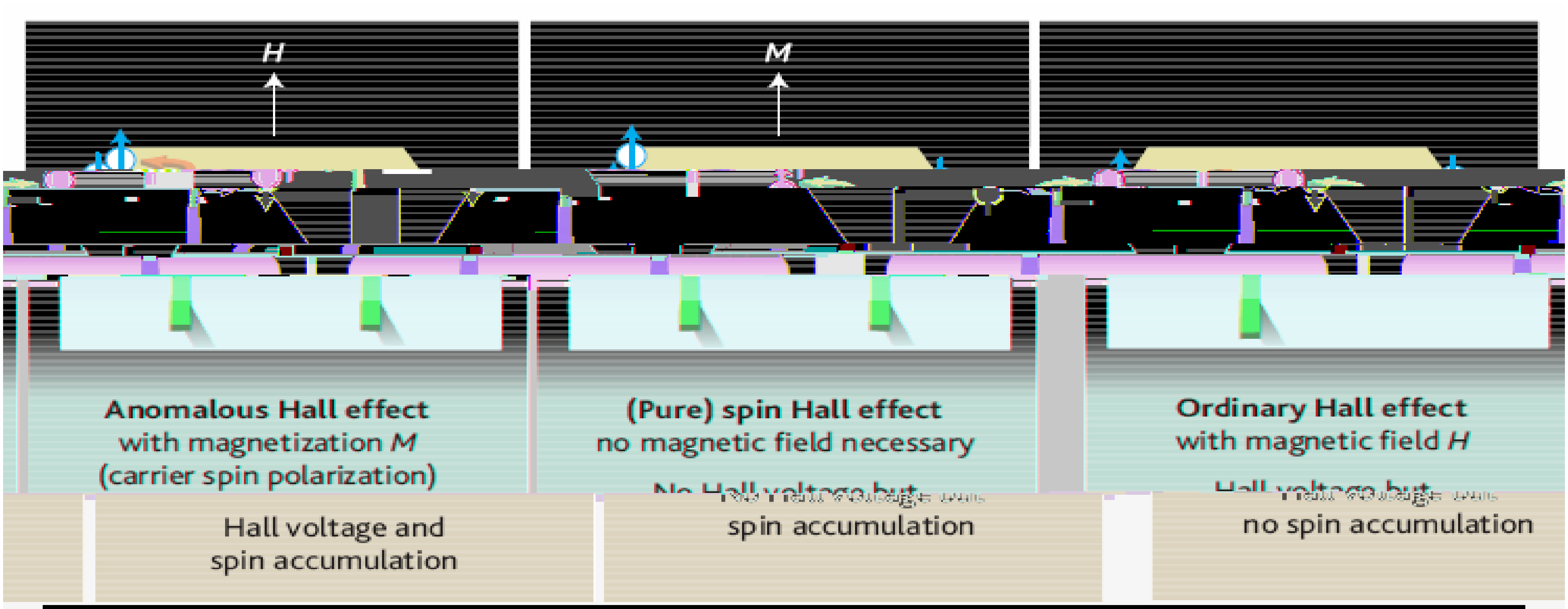
aoto Nagaosa



ust as predicted

e effect, which occurs without a magnetic field, is a new and topological- distinct electronic state.

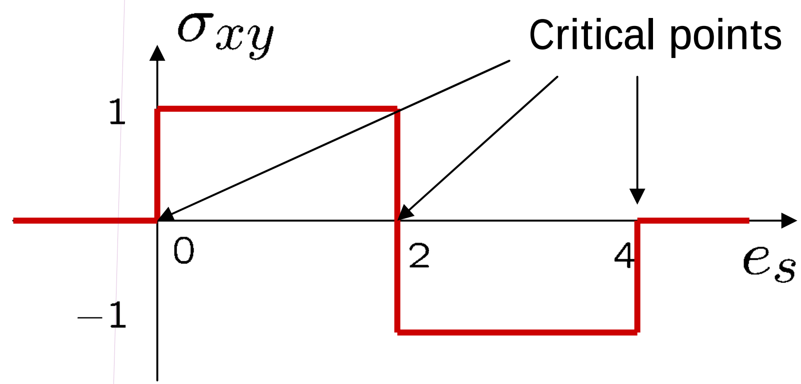
Completing the table of Hall effects

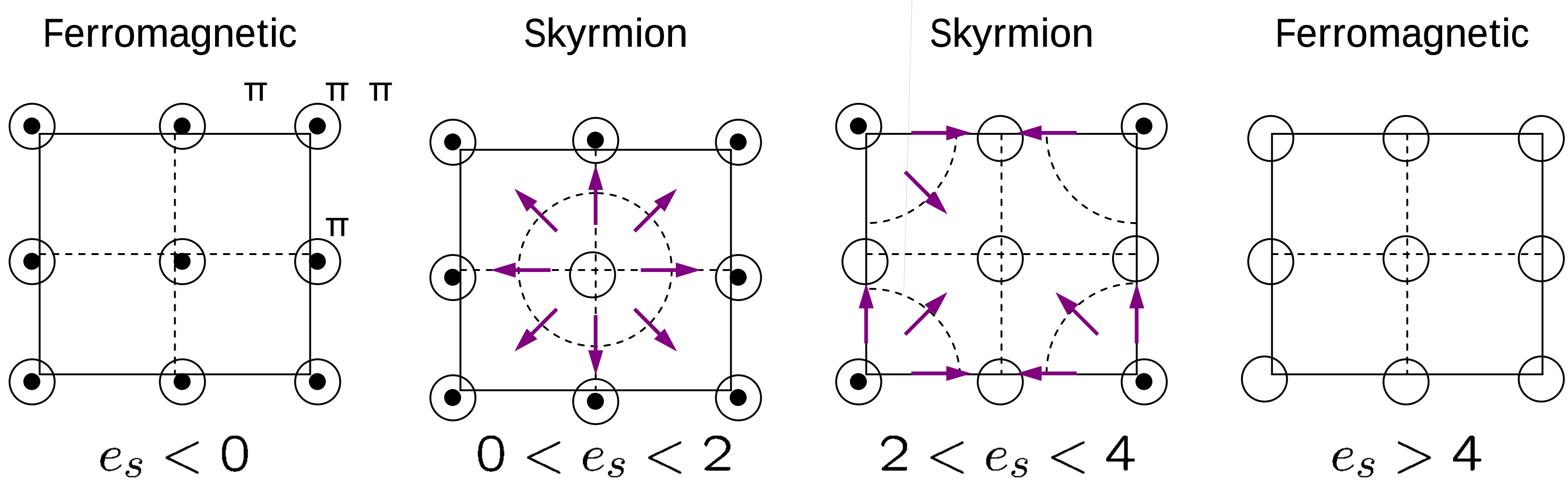


Hall **1879**	**Anomalous Hall** **1889**	**Spin Hall** **2004**
QHE **1980**	**QAHE** **2008?**	**QSHE** **2007**

Momentum space topology of the tight-binding model

$$\sigma_{xy} = -\frac{1}{8\pi^2}\int\int dk_x dk_y \hat{\mathbf{d}}\cdot\partial_{\mathbf{x}}\hat{\mathbf{d}}\times\partial_{\mathbf{y}}\hat{\mathbf{d}}$$





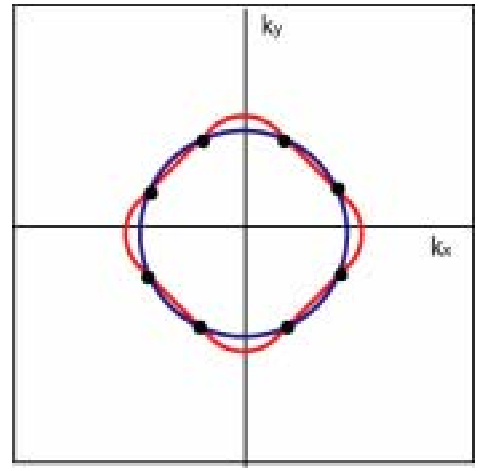
Topological quantum phase transition

Meron in
continuum

Inversion symmetry breaking in zincblend lattices

Inversion breaking term comes in the form:

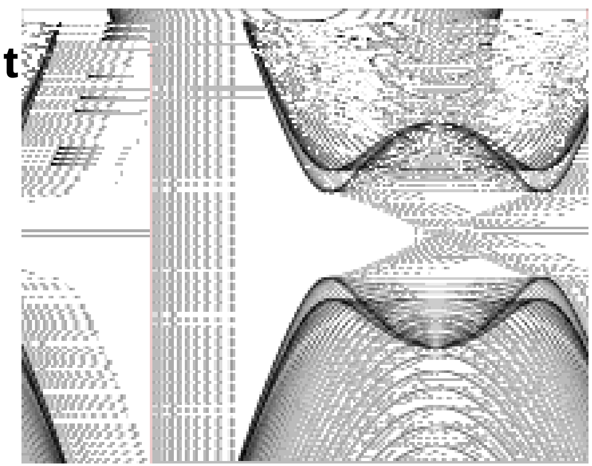
$$C(<k_z> + \ldots)\{J_z, J_x^2 - J_y^2\} \qquad J_x,\ J_y,\ J_z$$



which couples E1+, H1- and E1-,H1+ states and is a constant in quasi-2d systems

$$H_\Delta^{eff} = \begin{pmatrix} 0 & 0 & 0 & -\Delta \\ 0 & 0 & \Delta & 0 \\ 0 & \Delta & 0 & 0 \\ -\Delta & 0 & 0 & 0 \end{pmatrix}$$

E/t



k

Gap closes at nodes away from k=0, gap reopens at non-zero value of M/2B.
In the inverted regime, the helical edge state crossing is still robust.

Tight-binding model by X Dai, Z Fang, ...

Quantum control of the electron spin