



The topological insulator with strong correlation effects

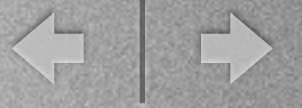
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Feng Lu, Jian Zhou Zhao, HongMing Weng, Zhong Fang

Lu et al, PRL 110, 096401 (2013)



outline

- Band inversion and TI
- Mixed valence compound: Band inversion between d and f bands; the important role of e-e interaction
- SmB₆, YbB₆, and YbBI₂
- Conclusion

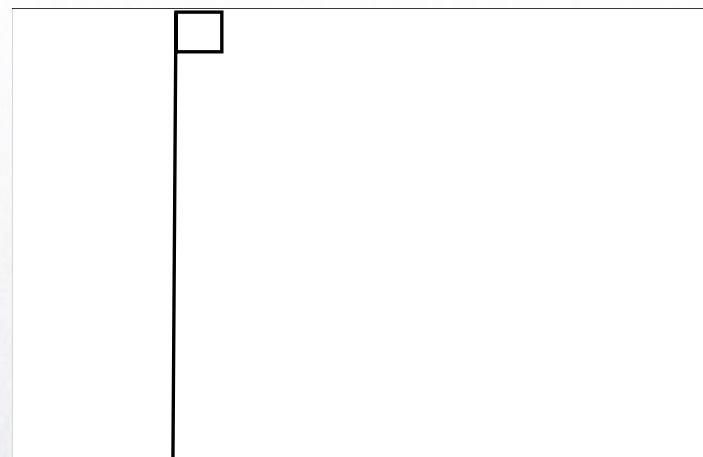


Topological classification of band insulators

- normal insulator: the electronic structure can be smoothly transformed to isolated atoms
- can not be smoothly transformed to isolated atoms without going through a phase transition, $\mathbb{Z}_2$ invariant is defined for non-interacting TI
- TI with interaction can be defined by the θ angle of topological magneto-electric effect



Evolution of band structure from solid to isolated atoms



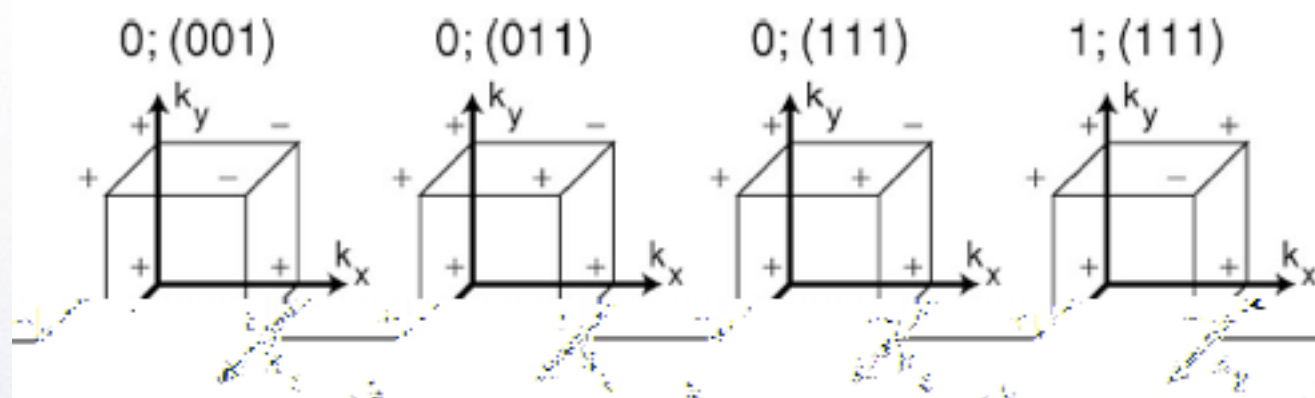


$\mathbb{Z}_2$ invariant for non-interacting TI with inversion symmetry

- Simple rules for TI with inversion symmetry:
strong index and weak indices; $\mathbf{K} = -\mathbf{K}$ high symmetry points

$$(-1)^{\nu_0} = \prod_{i=1}^8 \delta_i.$$

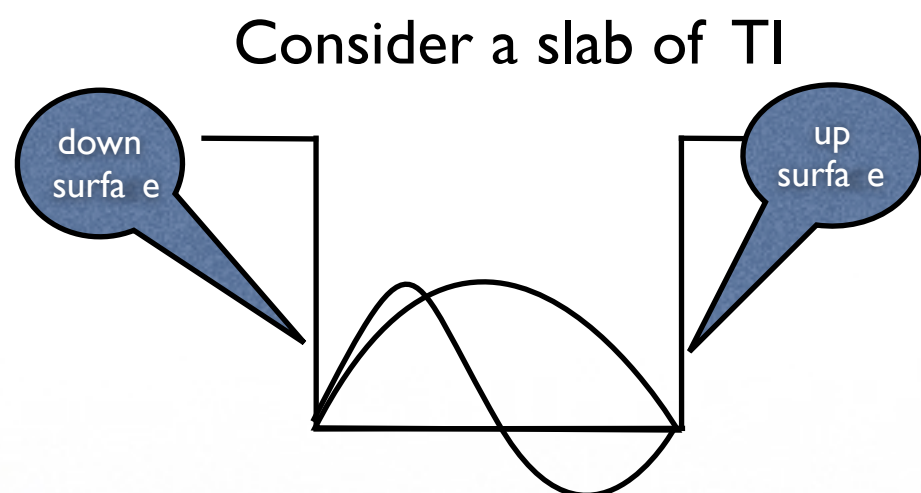
$$\delta_i = \prod_{m=1}^N \xi_{2m}(\Gamma_i).$$



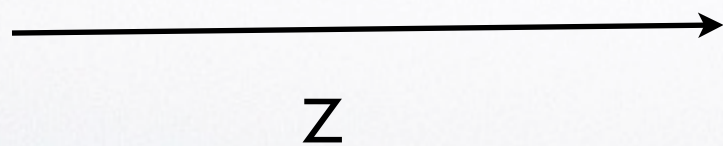
L. Fu and C. Kane, PRB 76,045302



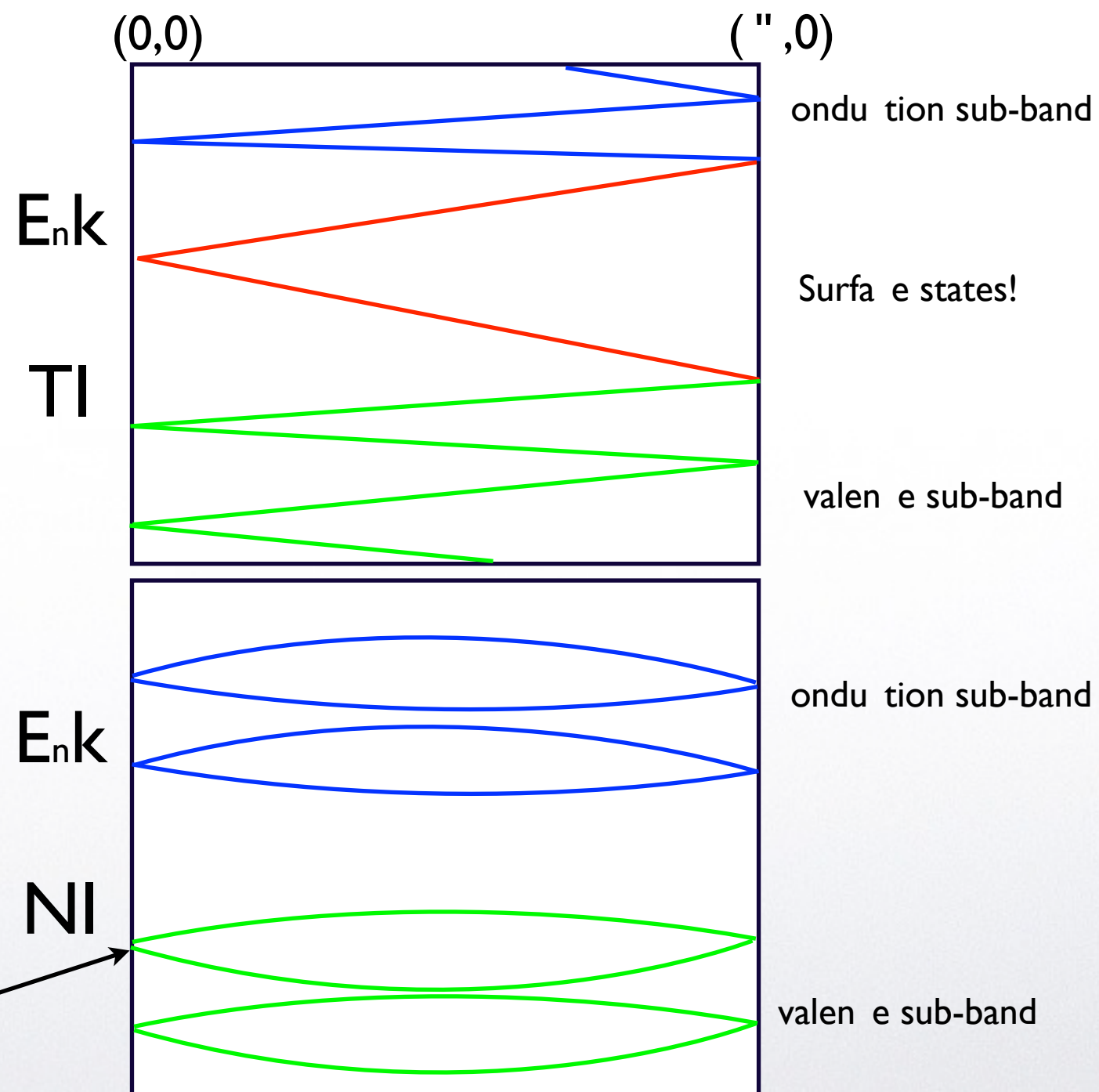
TI and surface states



$$\psi_n(k_x, k_y)$$

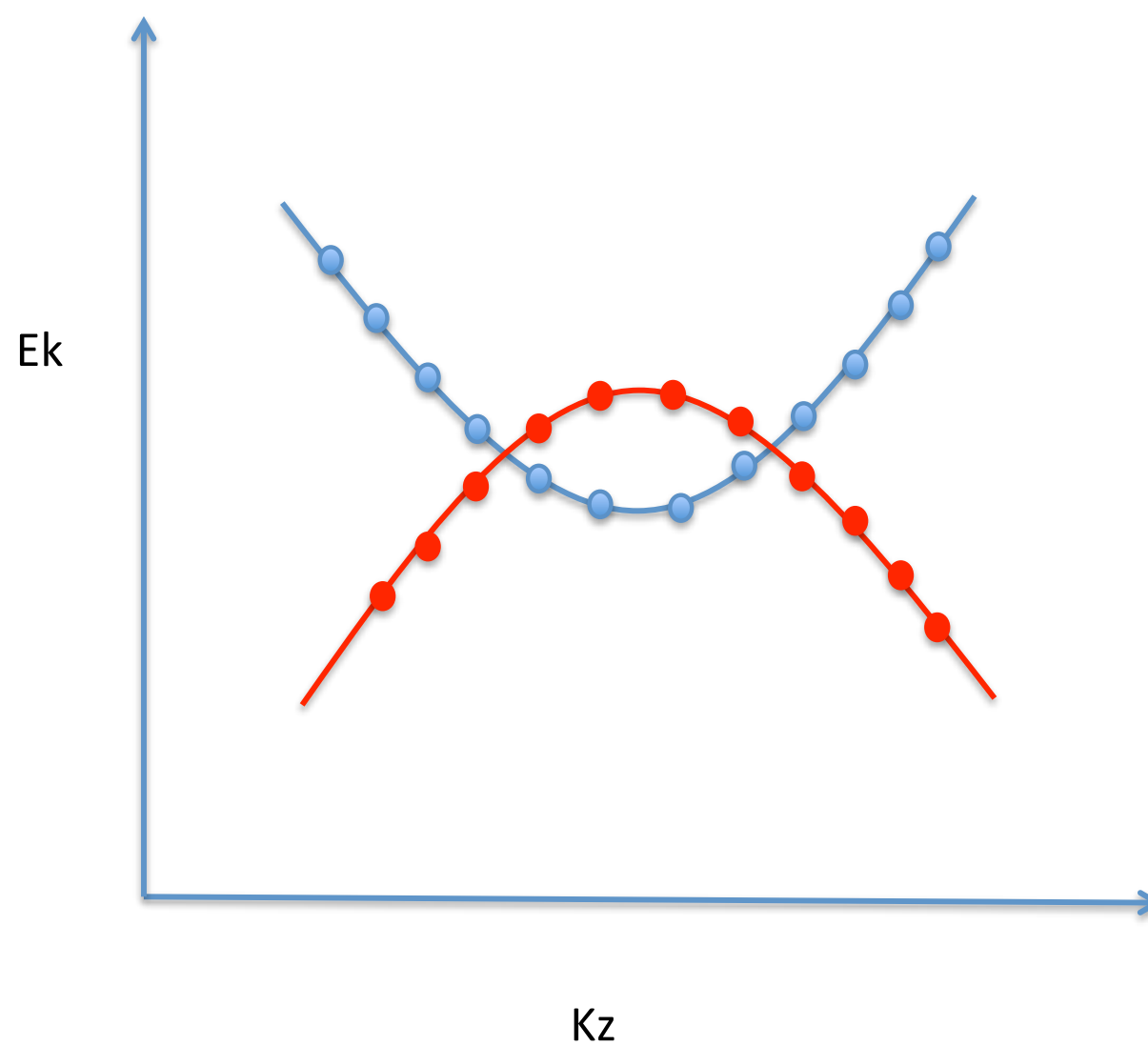


$k = -k + G$; Kramers degeneracy





I d c : Ba d I e a d TI





The different types of band inversion

- Band inversion between s and p bands: HgTe
- Band inversion between bonding and anti-bonding p-bands: Bi₂Se₃, Sb
- Band inversion generated by valence fluctuation of 4f/5f bands
- Very strong correlation effects in f-electron materials:
- Does these materials topologically non-trivial?

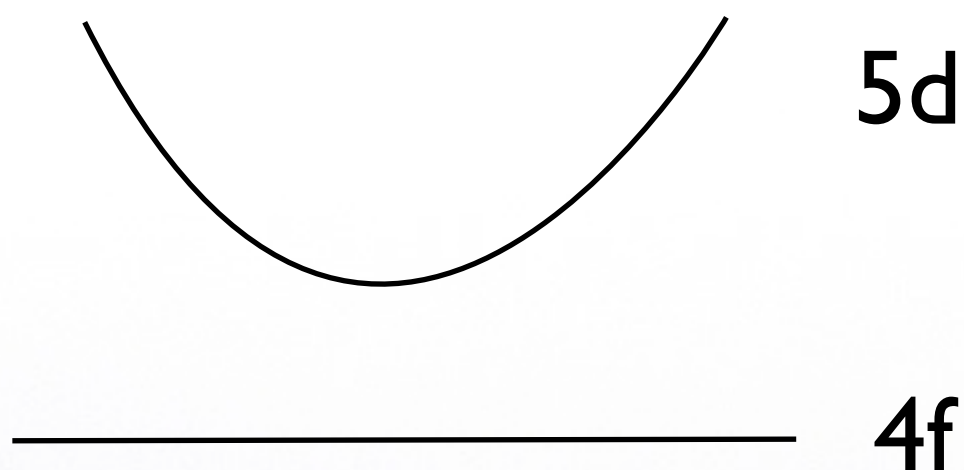


4f/5f compounds with intermediate valence

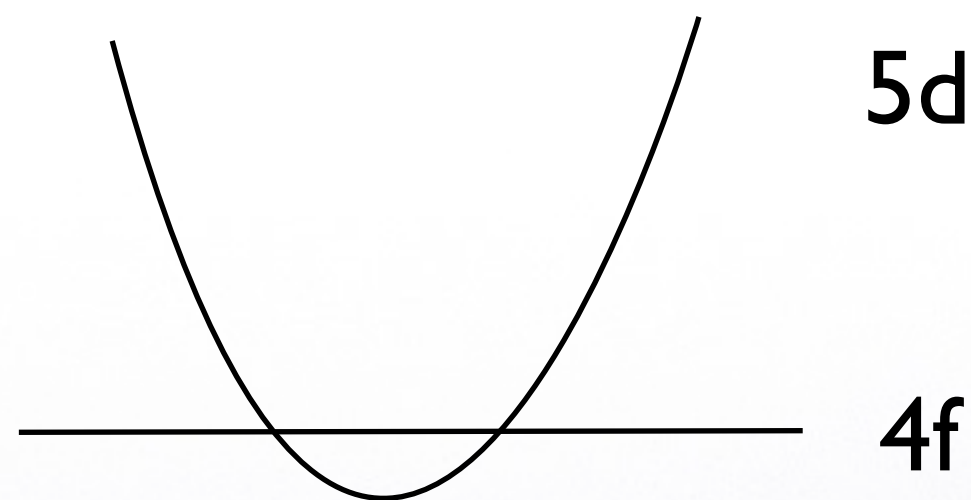
- golden phase of SmS
- SmB₆, YbB₆, YbBI₂
- PuTe and PuSe
- Are these materials topological Kondo insulator? M. Dzero et al, Phys. Rev. Lett. **104**, 119901, 2010



Intermediate Valence and band inversion: from the band theory point of view



Rare earth compounds
with divalent (not very stable),
Sm, Eu, Yb.....



intermediate valence state
band theory point of view



Is the band description correct for these compounds?

- from non-interacting band insulator to strong coupling “Kondo insulator”
- how to capture the correct electronic structure?
- how to describe its topological nature?



How to compute the Z2 invariant for interacting system?

Formula derived from the topological field theory:
[PRL.105,256803 \(2010\)](#)

$$P_3 = \frac{\pi}{6} \int_0^1 du \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \epsilon^{\mu\nu\rho\sigma} [G \partial_\mu G^{-1} G \partial_\nu G^{-1} \\ \times G \partial_\rho G^{-1} G \partial_\sigma G^{-1} G \partial_u G^{-1}] \quad (1)$$

- Pole expansion of the self energy without k dependence [PRB](#) ,235135 (2012); [EPL98 \(2012\) 57001](#)
- Using the eigenstate of $H_0 + \Sigma(0)$, condition: there is no singularity along the imaginary axis of self energy, [PRB 85, 165126 \(2012\)](#)



Our method: LDA+Gutzwiller

- Gutzwiller type of ground state wave function
- Combined with first principle code
- Suitable for the study of f-electrons
- Has been applied to many correlated systems:
LaOFeAs; Na_xCoO_2 ; Ce; Pu....PHYSICAL REVIEW B, 075114, 2009



from PHYSICAL REVIEW B **66**, 165209, 2002

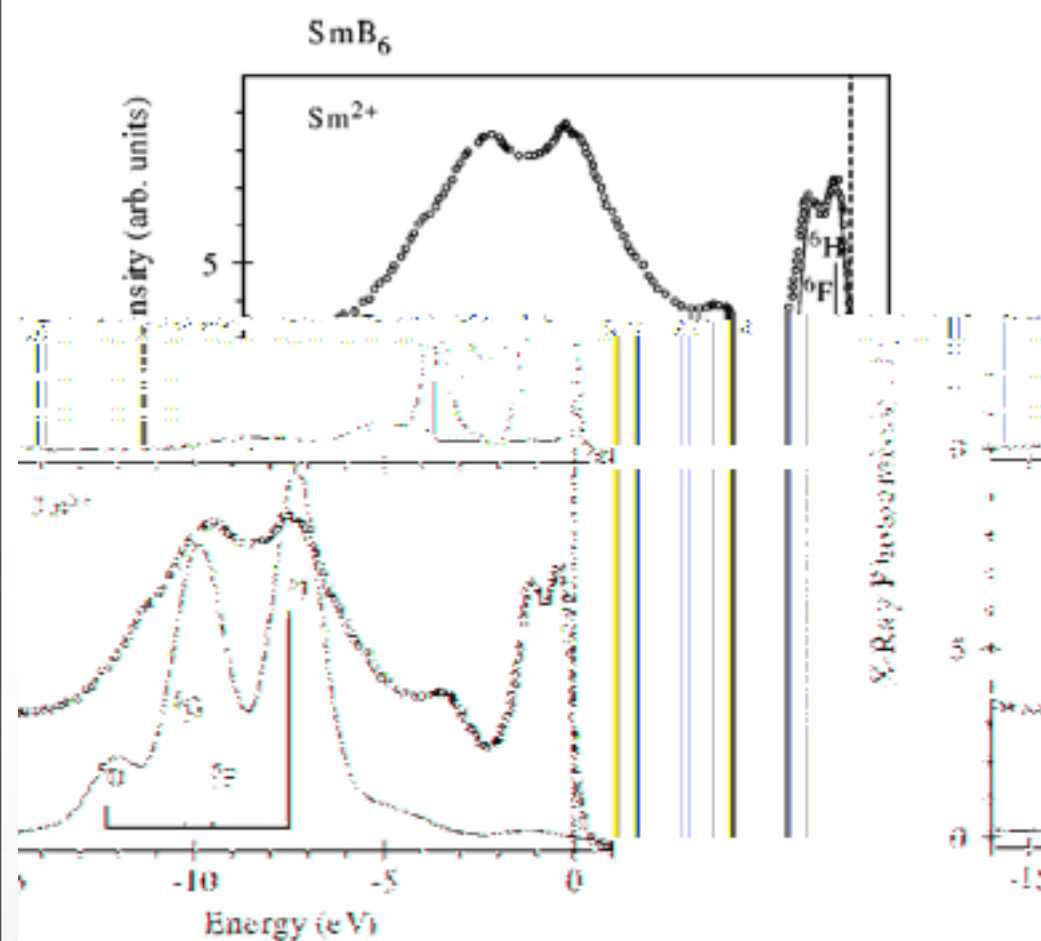
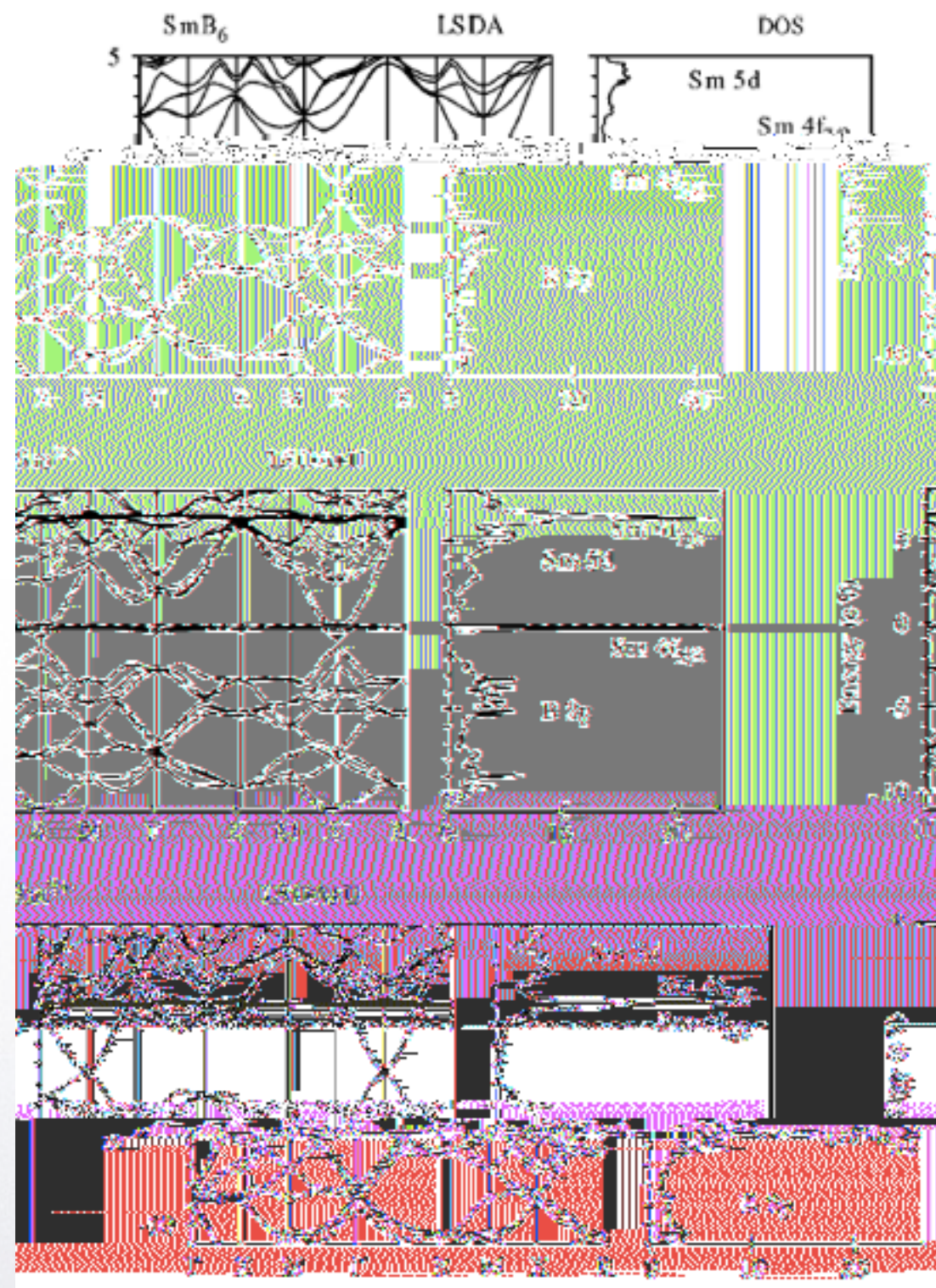


FIG. 3. Comparison of the calculated 4f DOS of SmB_6 using the LDA+U approximation with the experimental XPS spectra (see explanation in the text).

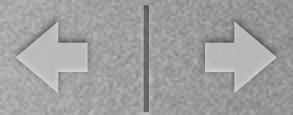
compare to XPS

The valence determined by XPS is 2.54



band structure
obtained by LDA
and LDA+U

Assuming some
kind of orbital
ordering from Sm^{3+}
phase



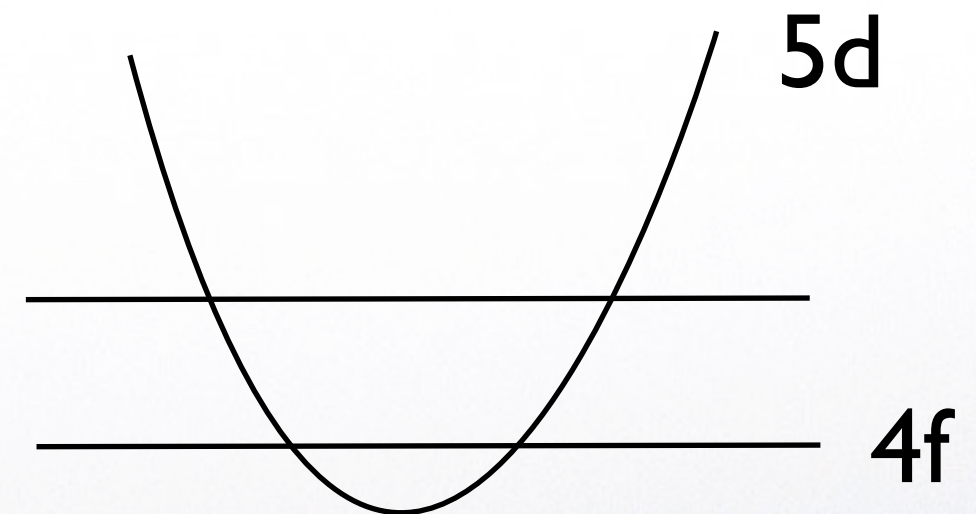
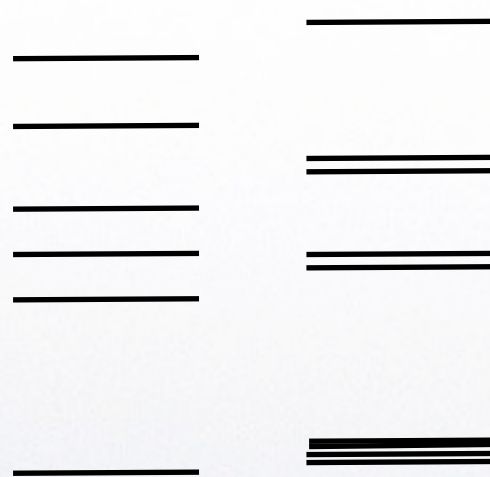
Atomic multiplets or Band states?

= +

$$= \frac{(-1)^0}{2} + \sum_{\alpha, \beta, \gamma, \delta} \frac{t_{\alpha\beta\gamma\delta}}{2} f_{\alpha}^{\dagger} f_{\beta}^{\dagger} f_{\delta} f_{\gamma} + \sum_{\alpha} \epsilon_{\alpha} f_{\alpha}^{\dagger} f_{\alpha}$$

$$H_{\text{hopping}} = \sum_{ij\alpha\beta} t_{ij}^{\alpha\beta} f_{i\alpha}^{\dagger} f_{j\beta} + h.c.$$

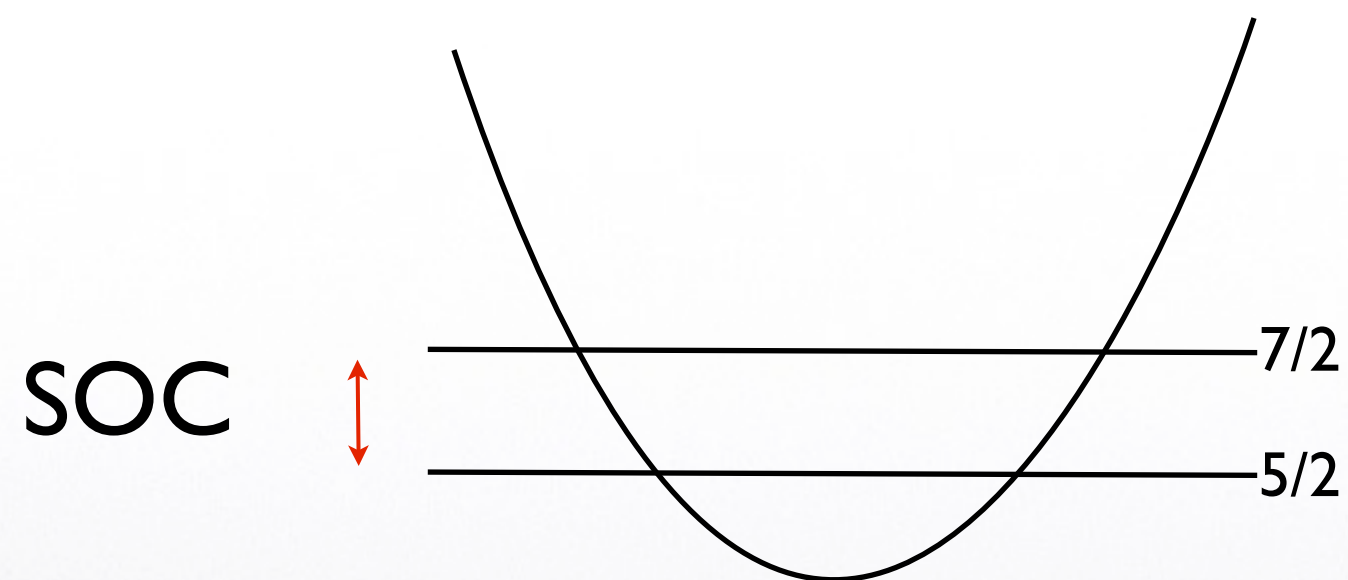
... nf=6 nf=5 ...



Both are non-free states!



Problem of Hartree-Fo k treatment

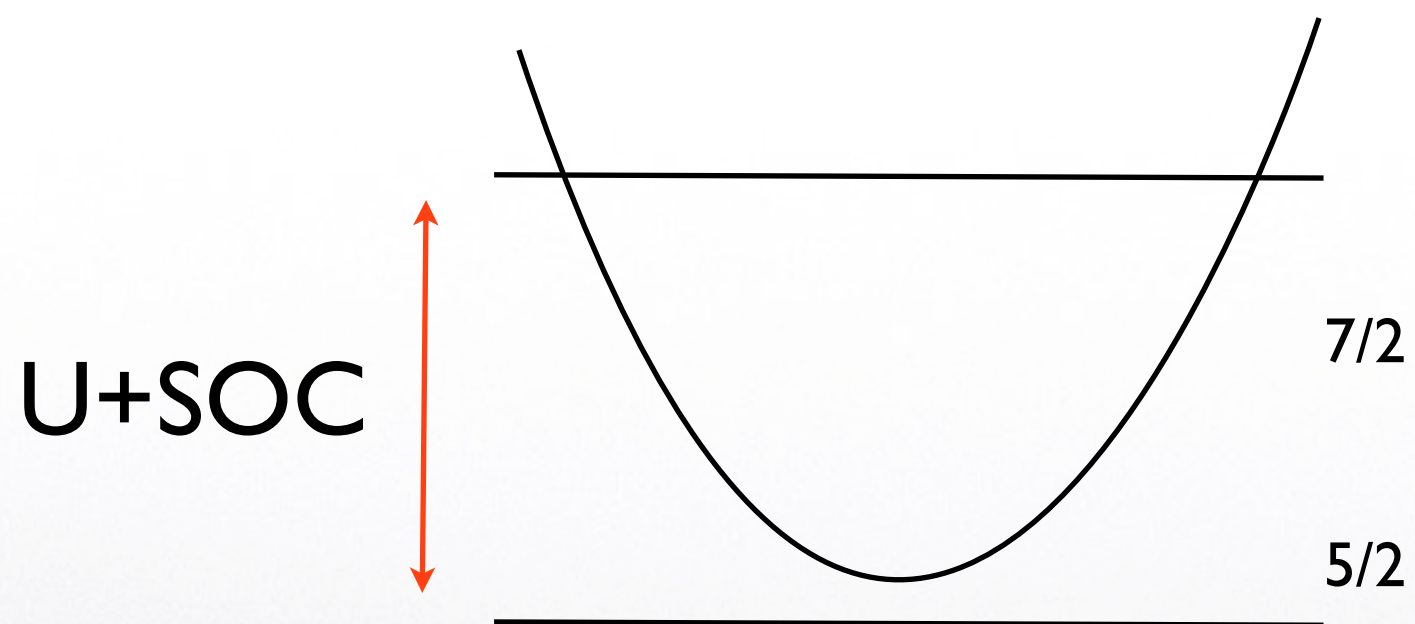


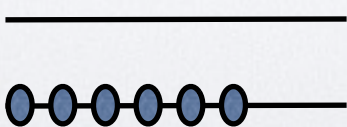
MF atomi state:

Diagram showing the many-body state of the atom. The $j=7/2$ level is empty, and the $j=5/2$ level is filled with six blue spheres representing electrons.



Problem of Hatree-Fo k treatment



MF atomi state:  7/2

5/2



The Gutzwiller Trial wave function used in this study

Rotational Invariant Gutzwiller Approximation

Gutzwiller variational wavefunction:

$$|\Psi_G\rangle = \mathcal{P}|\Psi_0\rangle = \prod_{\mathbf{R}} \mathcal{P}_{\mathbf{R}} |\Psi_0\rangle$$

$$\mathcal{P}_{\mathbf{R}} = \sum_{\Gamma\Gamma'} \lambda(\mathbf{R})_{\Gamma\Gamma'} |\Gamma, \mathbf{R}\rangle \langle \Gamma', \mathbf{R}|$$

The eigenstates of atomic Hamiltonian H_{atom}^i are denoted by $|\Gamma, \mathbf{R}\rangle$

$|\Psi_0\rangle$: uncorrelated wave-function (Wick's Theorem holds)

$\mathcal{P}_{\mathbf{R}}$: projector operator modify weight of local configuration

Gutzwiller Constraints:

$$\langle \Psi_0 | \mathcal{P}^\dagger \mathcal{P} | \Psi_0 \rangle = 1$$

$$\langle \Psi_0 | \mathcal{P}^\dagger \mathcal{P} n_{i\alpha} | \Psi_0 \rangle = \langle \Psi_0 | n_{i\alpha} | \Psi_0 \rangle$$

We only apply truncation respect to the occupation number, for SmB6, we keep all the atomic states with $n_f=5,6,7$, about 8000 variational parameters!



Effective Hamiltonian in Gutzwiller approximation

- Under Gutzwiller approximation, we can define

$$E_G = \langle 0 | P H_{LDA} P | 0 \rangle + E_{int} \approx \langle 0 | H_{eff} | 0 \rangle + \sum_{\Gamma} \lambda_{\Gamma, \Gamma} E_{\Gamma}$$

It can be easily proved that H_{eff} is equivalent to $H_0 + \Sigma(0)$ by comparing the Green's function in low frequency limit

$$G(i\omega) = \frac{z}{i\omega - H_{eff}} = \frac{1}{i\omega/z - H_{eff}/z}$$

$$H_0 + \Sigma(0) = -G^{-1}(0) = H_{eff}/z$$



Main difficulties for the electronic structure calculation for f-electron systems

- Strong interactions among f-electrons, which can be expressed in terms of Slater integrals: F_0, F_2, F_4, F_6
- for SmB₆ $F_0=5.8\text{eV}$, $F_2=9.9\text{eV}$, $F_4=7.09\text{eV}$, $F_6=4.99\text{eV}$
- Typical interaction strength is one order bigger than the f-band width
- Multiplet state VS the band state



The double counting problem

- The total Hamiltonian treated in LDA+Gutzwiller

$$H_{total} = H_{LDA} + H_U + H_{DC}$$

$$H_{DC} = V_{DC} \sum_{k\sigma} f_{k\sigma}^\dagger f_{k\sigma}$$

- A commonly used form for DC term:

$$V_{ab}^{DC} = \delta_{ab} \left[\bar{U} \left(\bar{n}_c - \frac{1}{2} \right) - \bar{J} \left(\bar{n}_c^\sigma - \frac{1}{2} \right) \right].$$

- Kotliar et al, RMP78, 865,2006



Valence of Sm determined by exp

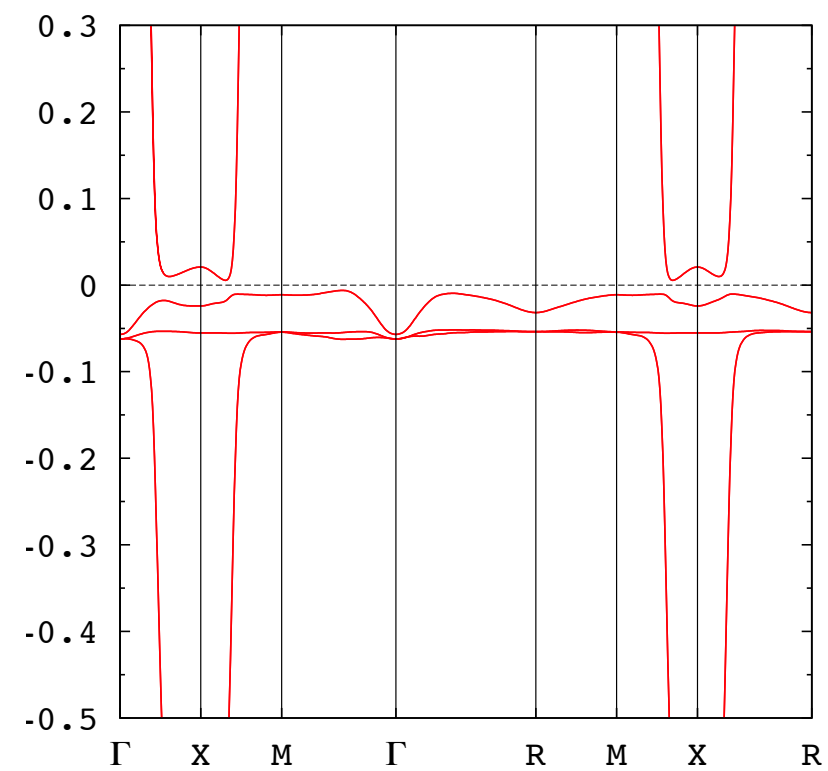
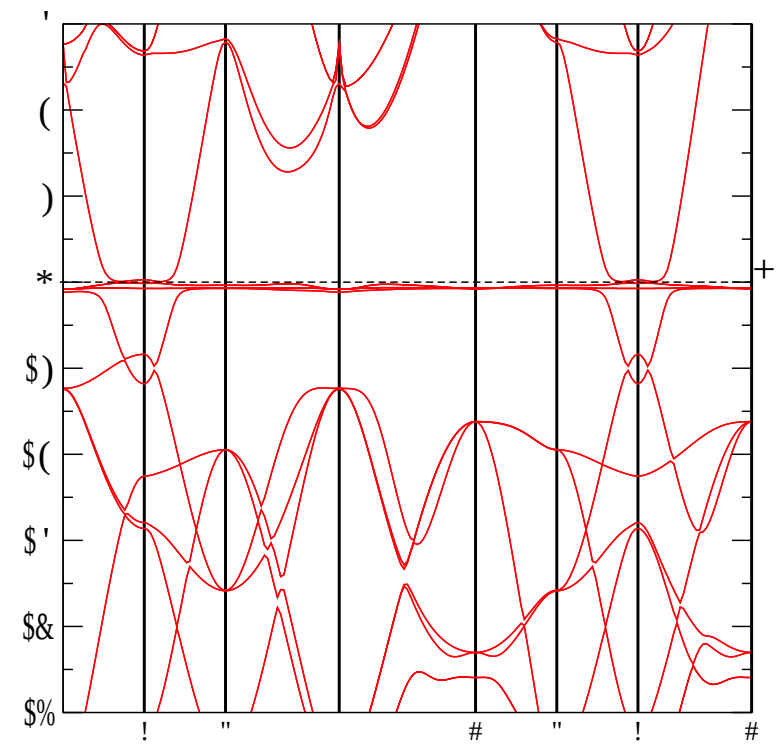
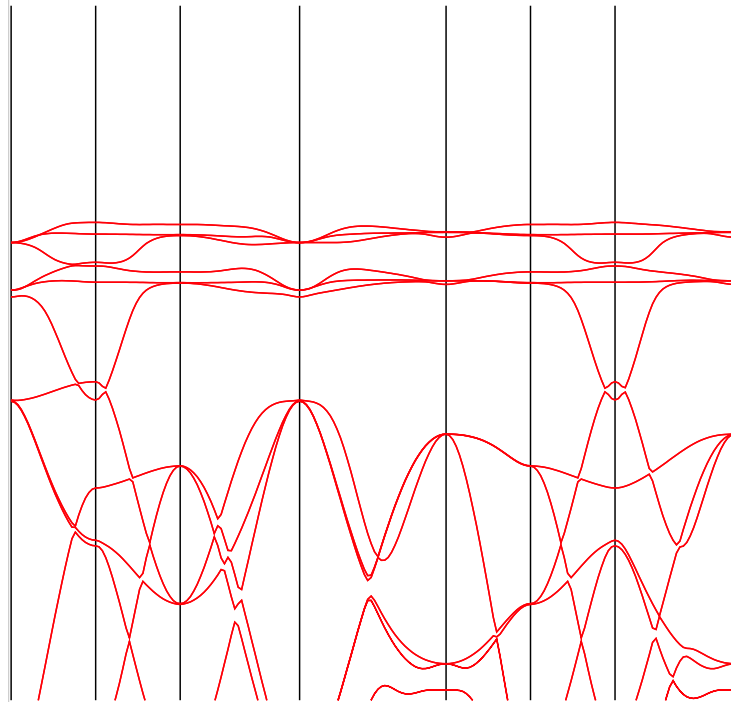
	paper	experiment	nf	Average valence
! " # \$	%&#'() * + \$ (,) , (, . / \$	0 % !	.12	31 +
! " # \$	45 % ' () * + 6 (,) (- / * /	7 899 : ; < = > (= ? = @ A	.1 ,	31 \$
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! " # \$	4 % H ! ' (3 6 6 * () + \$ - (6) 3 6 2 ,	0 5 !	.1 , +	31 . 2

SmB6: Z, nf, Gap vs Parity

		3_4	5	26	(	27	89:
; * <=	>?@	) . * AB	C	D	C	C	D
	+		C	D	C	C	

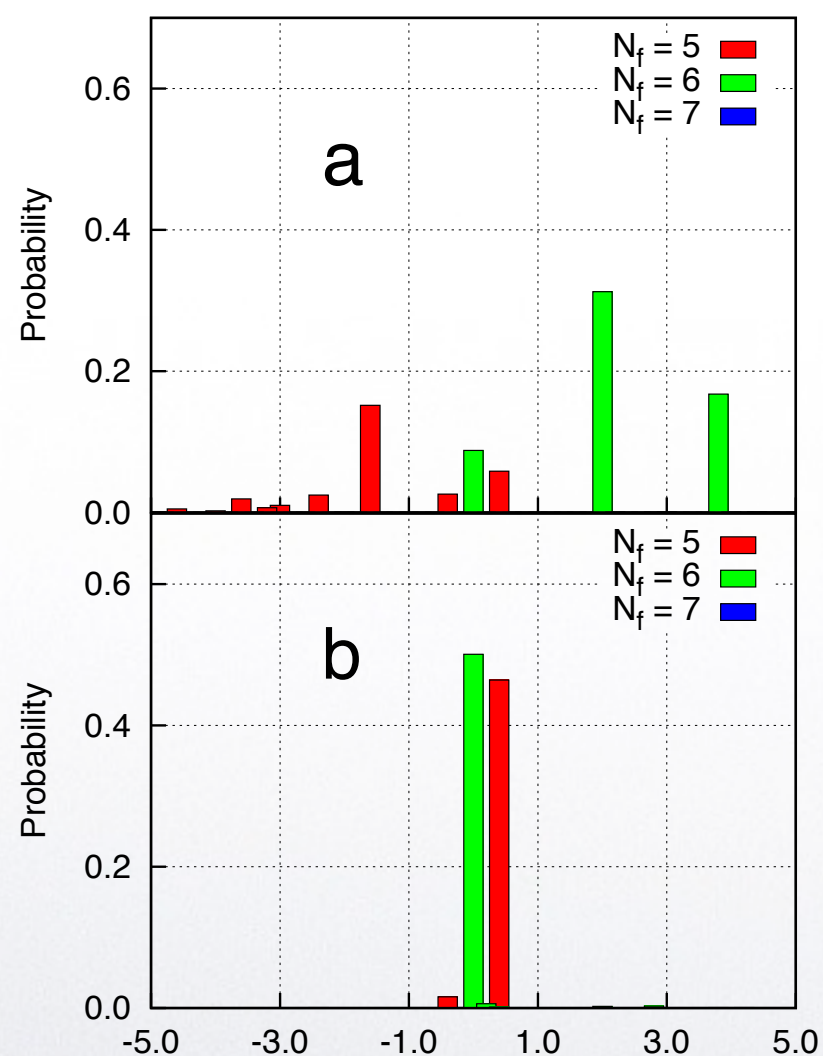
! " #	\$ %		&' () * + , -	
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Band structure obtained by LDA and LDA+G





The probability of atomic eigenstates for SmB6



LDA

LDA+G

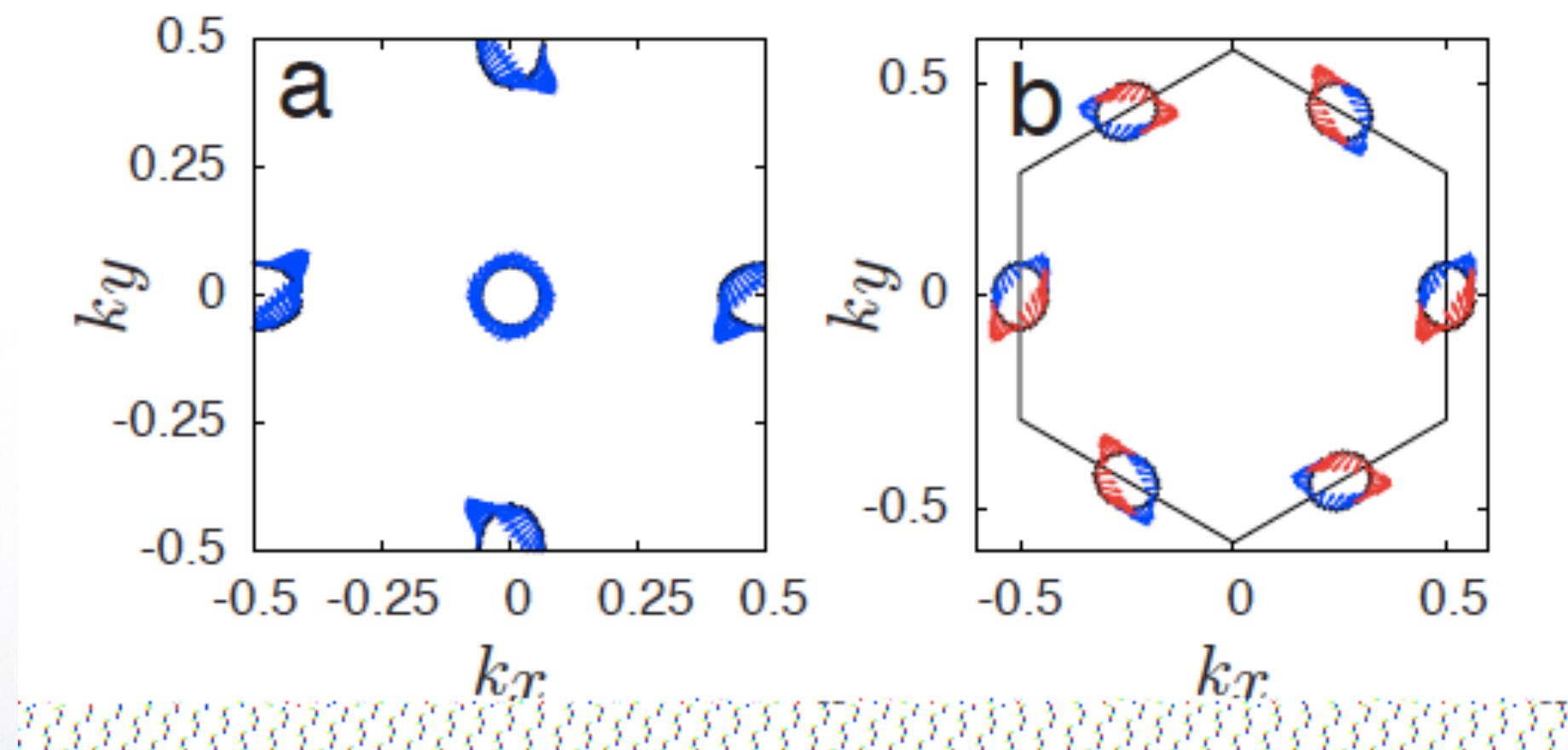


The unique surface states of SmB₆ on (001) surface





Spin texture for surface states

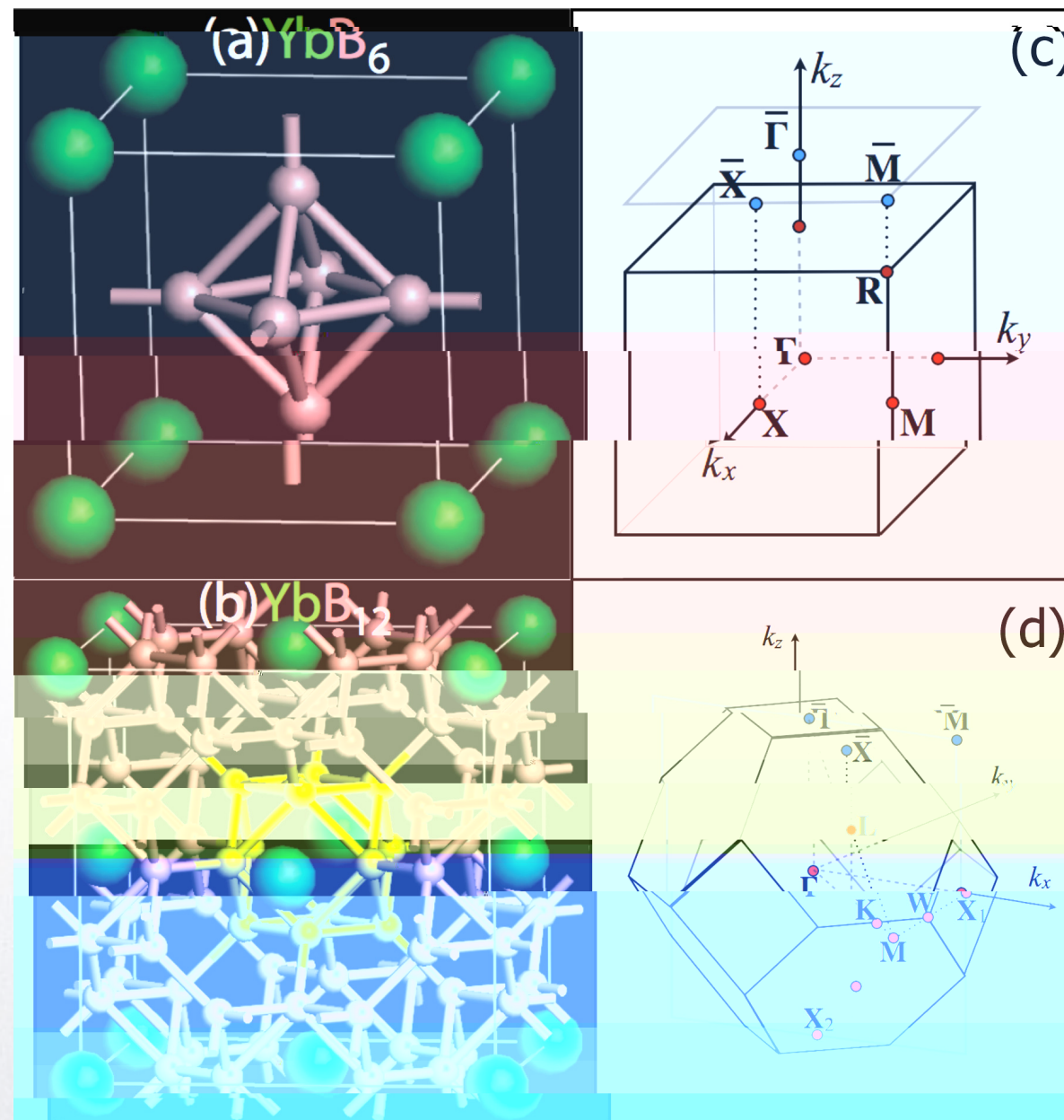


001 surface

111 surface

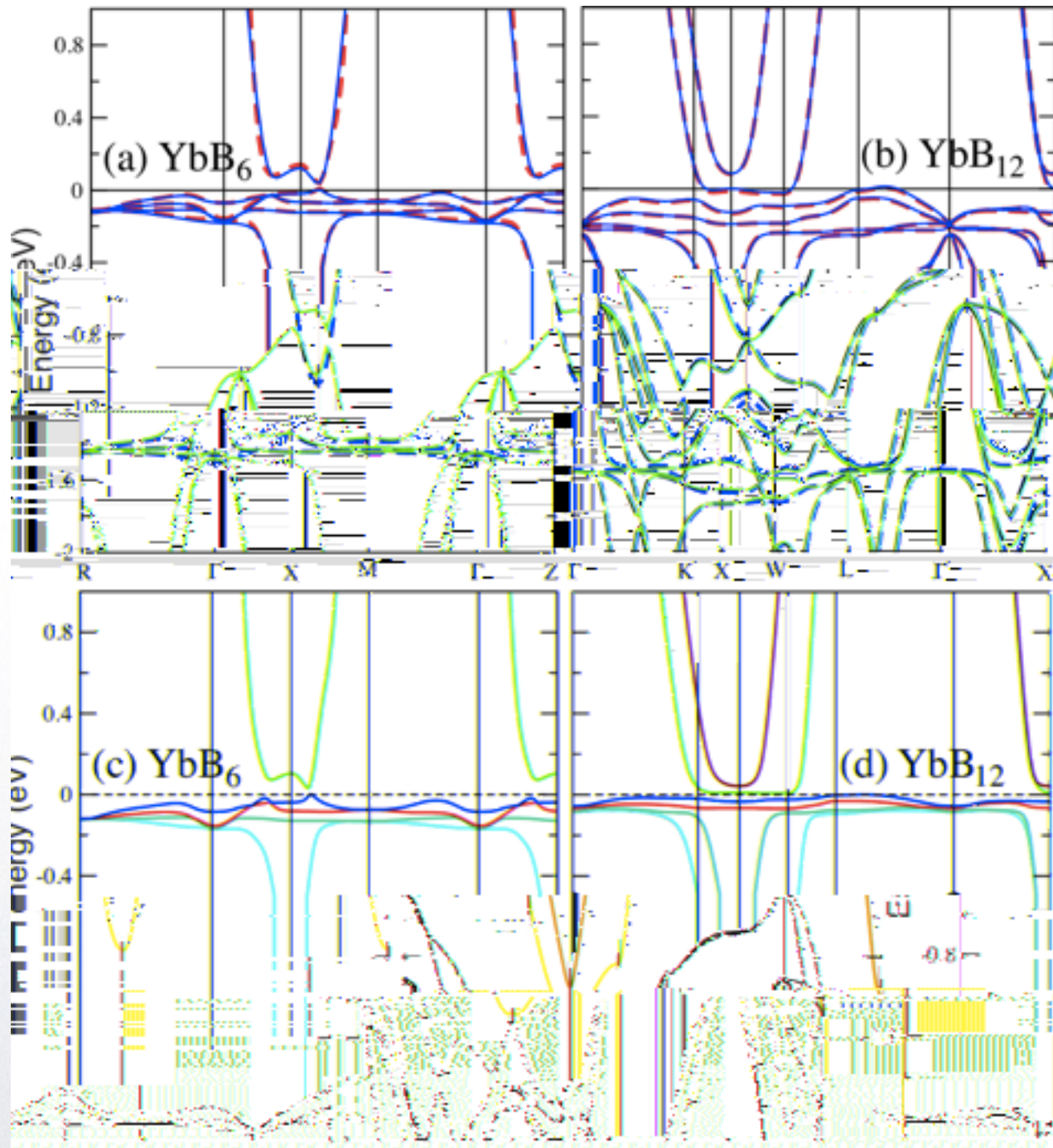


Structure of YbB₆ and YbB₁₂

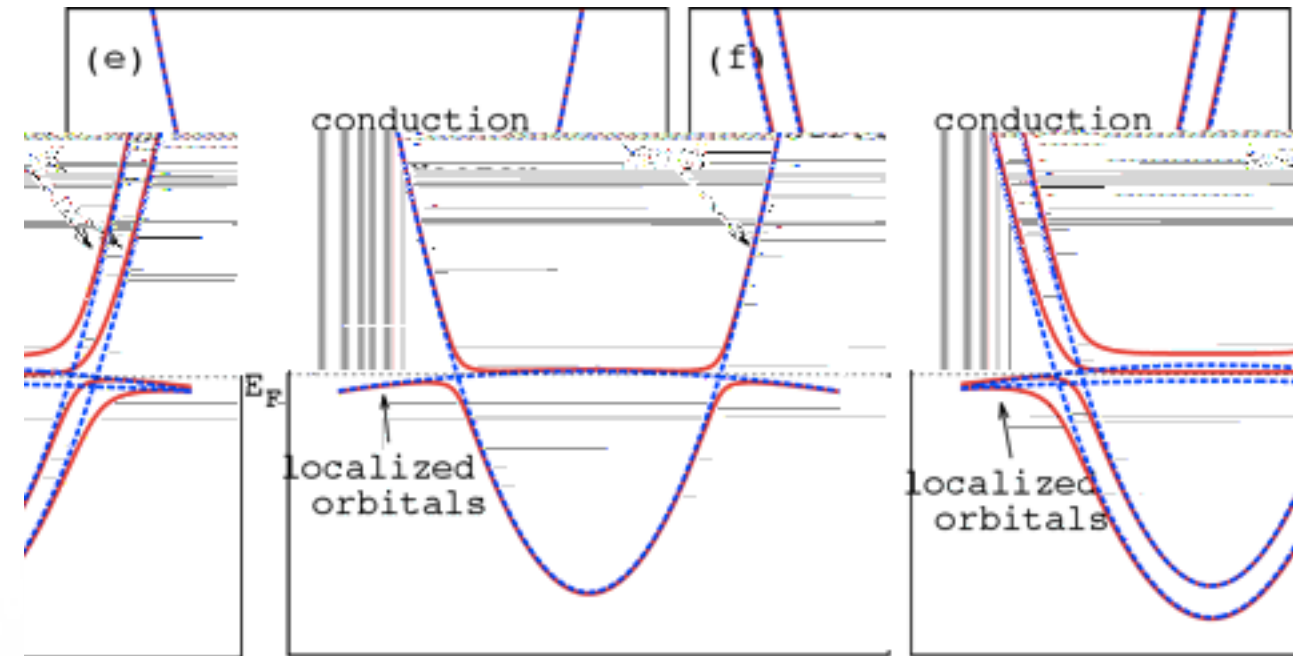


Renormalized Band structure

LDA



LDA+Gutzwiller



	Γ	3X	3M	R	n_f	z
YbB_6	-12.80	-12.58	-12.58	-12.58	0.87	0.87
YbB_{12}	-12.80	-12.58	-12.58	-12.58	0.87	0.87



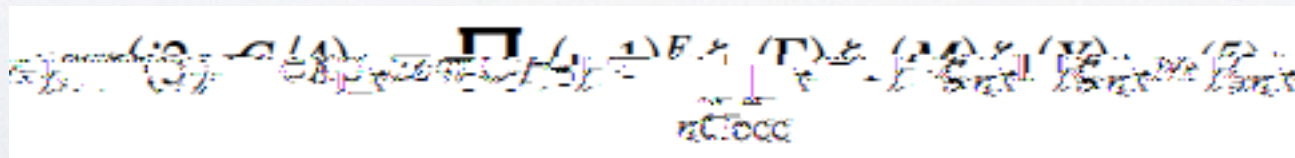
Determine the Mirror Chern number



By the eigenvalue of the C_n rotational
at high symmetry point

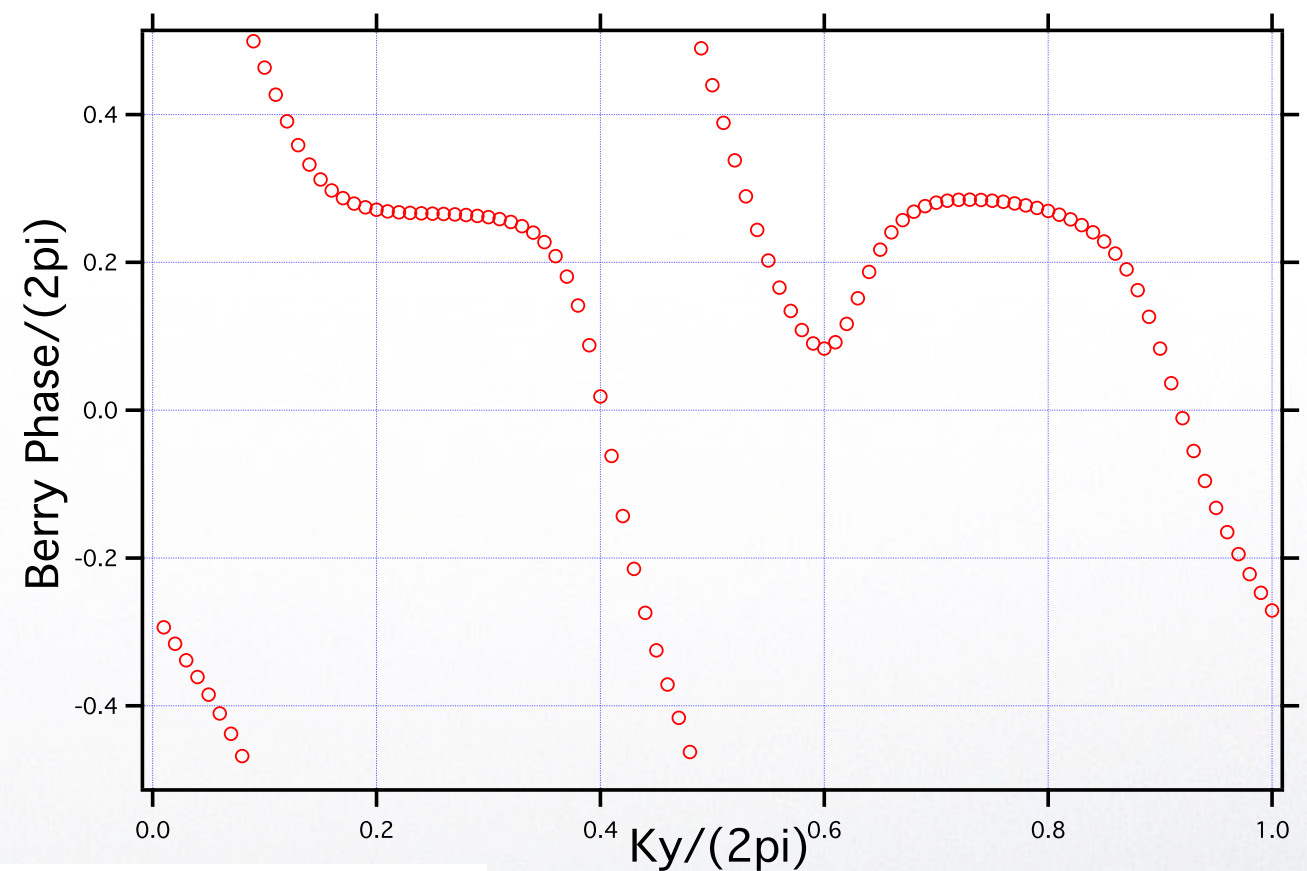
	C4	C2	M
4f7/2 $j_z=1/2$	$\exp(i\pi/4)$	$\exp(i\pi/2)$	$-i$
5d5/2 $j_z=-1/2$	$\exp(-i\pi/4)$	$\exp(-i\pi/2)$	$-i$
4f7/2 $j_z=-3/2$	$\exp(-i3\pi/4)$	$\exp(-i3\pi/2)$	$-i$
5d5/2 $j_z=3/2$	$\exp(i3\pi/4)$	$\exp(i3\pi/2)$	$-i$

$$i^C = -1$$



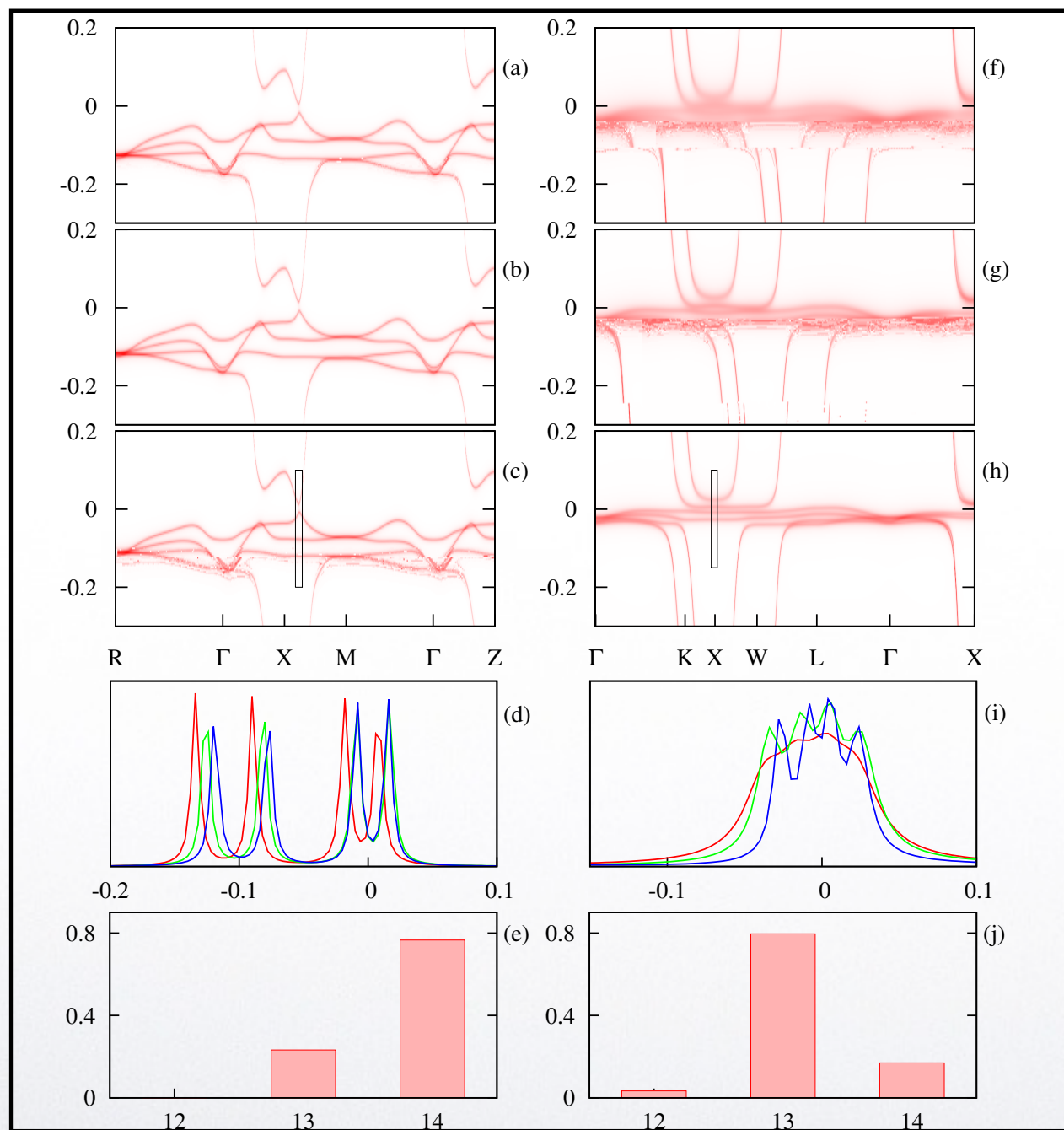
C. Fang et al, PRL 108, 266802 (2012); R. Yu, et al, Phys. Rev. B **84**, 075119 (2011)

By Wilson Loop method



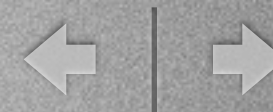


Temperature dependence of the spectral function



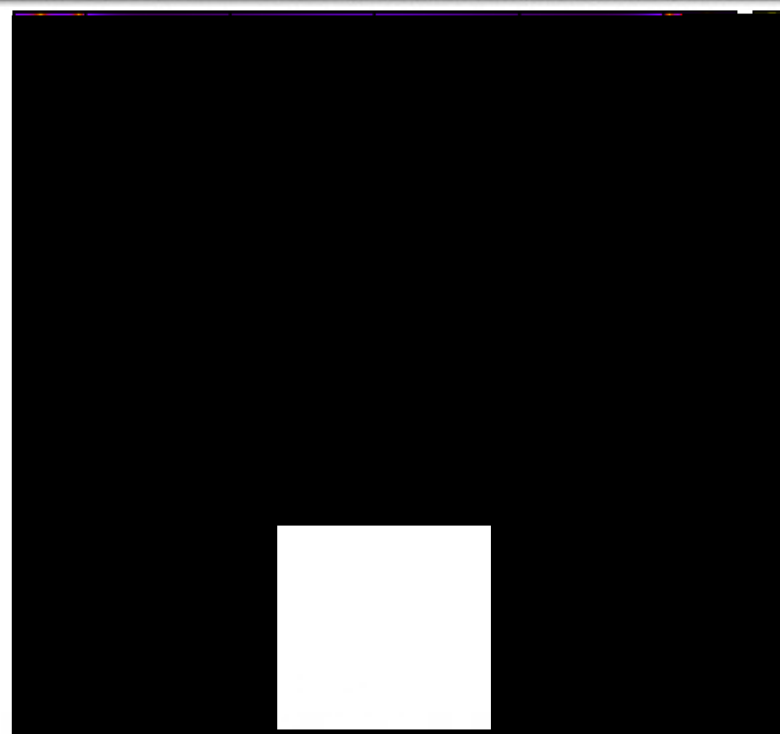


Surface states for YbB6





Surface states for YbBI₂





Con lusions

- Mix valen e Tl SmB₆, and YbB₆
- Topologi al Crystalline Kondo Insulator with mirror Chern number 2 :YbBI₂
- The topologi al phases survive from the Strong e-e intera tion among f ele trons
- More similar materials!