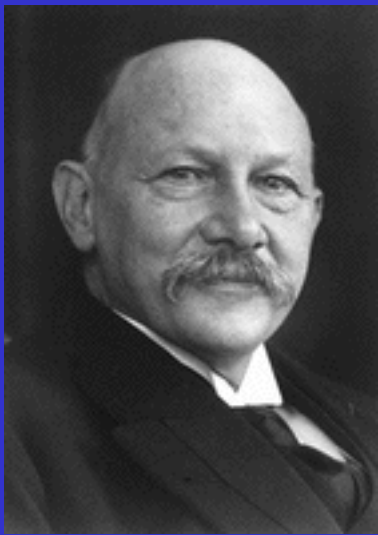


Hai-Hu Wen

**National Lab for Superconductivity, Institute of Physics,
Chinese Academy of Sciences, Beijing, China**



- IOP, CAS, China' Yue Wang, Lei Shan, Hong Gao, Zhiyong Liu, Fang Zhou
- MIT, USA: Xiao-Gang Wen
- Osaka Univ., Japan: Yoichi Ando's group (LSCO, Optimal doped)
- Tohoku Univ., Japan: Yoichi Tanabe, Tadashi Adachi, and Yoji Koike (LSCO, Overdoped)



1913

1908

Leiden

Kamerlingh Onnes

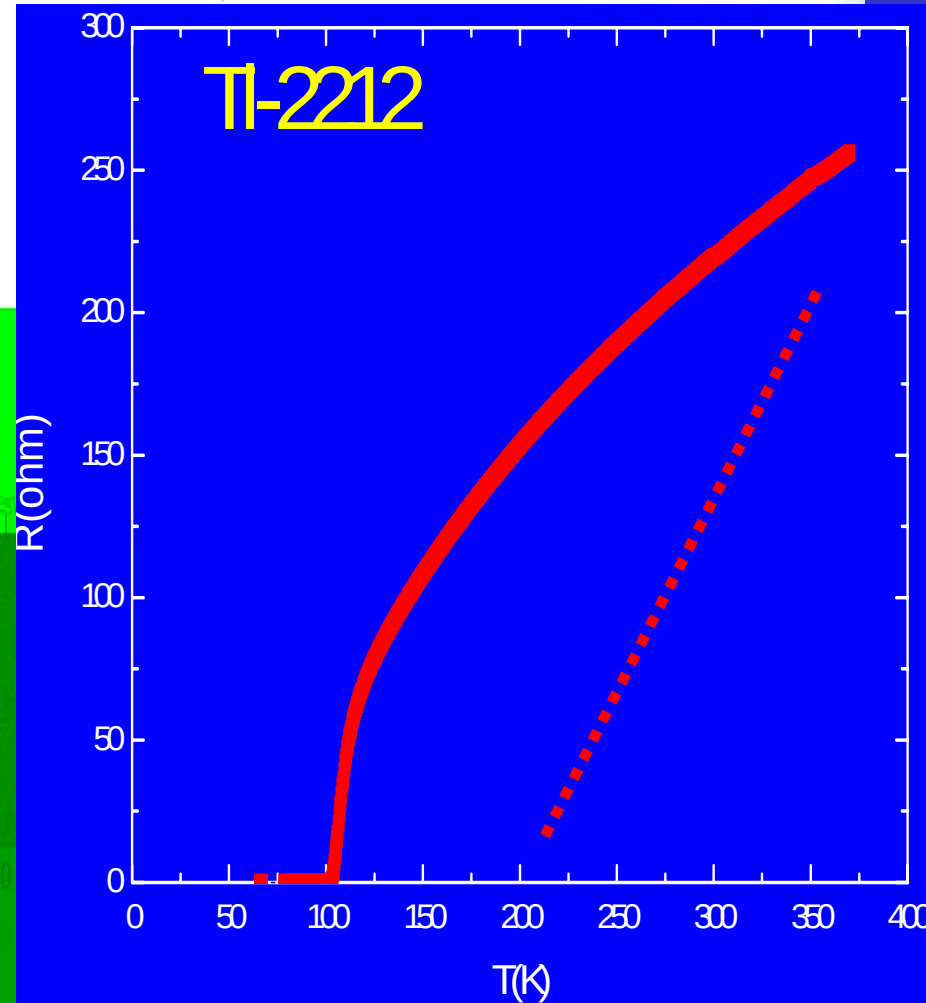
He

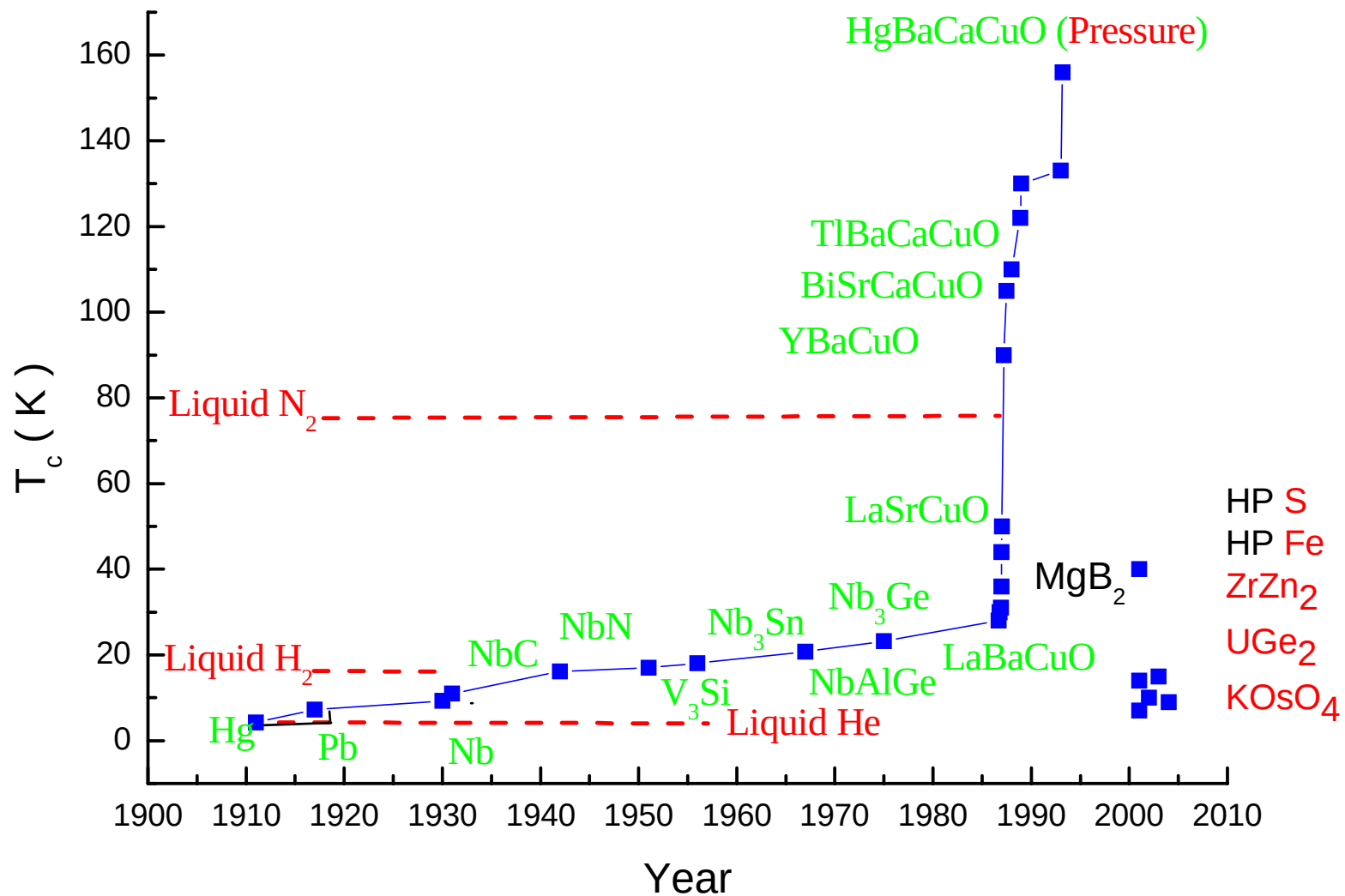
1911

Hg

4.2 K

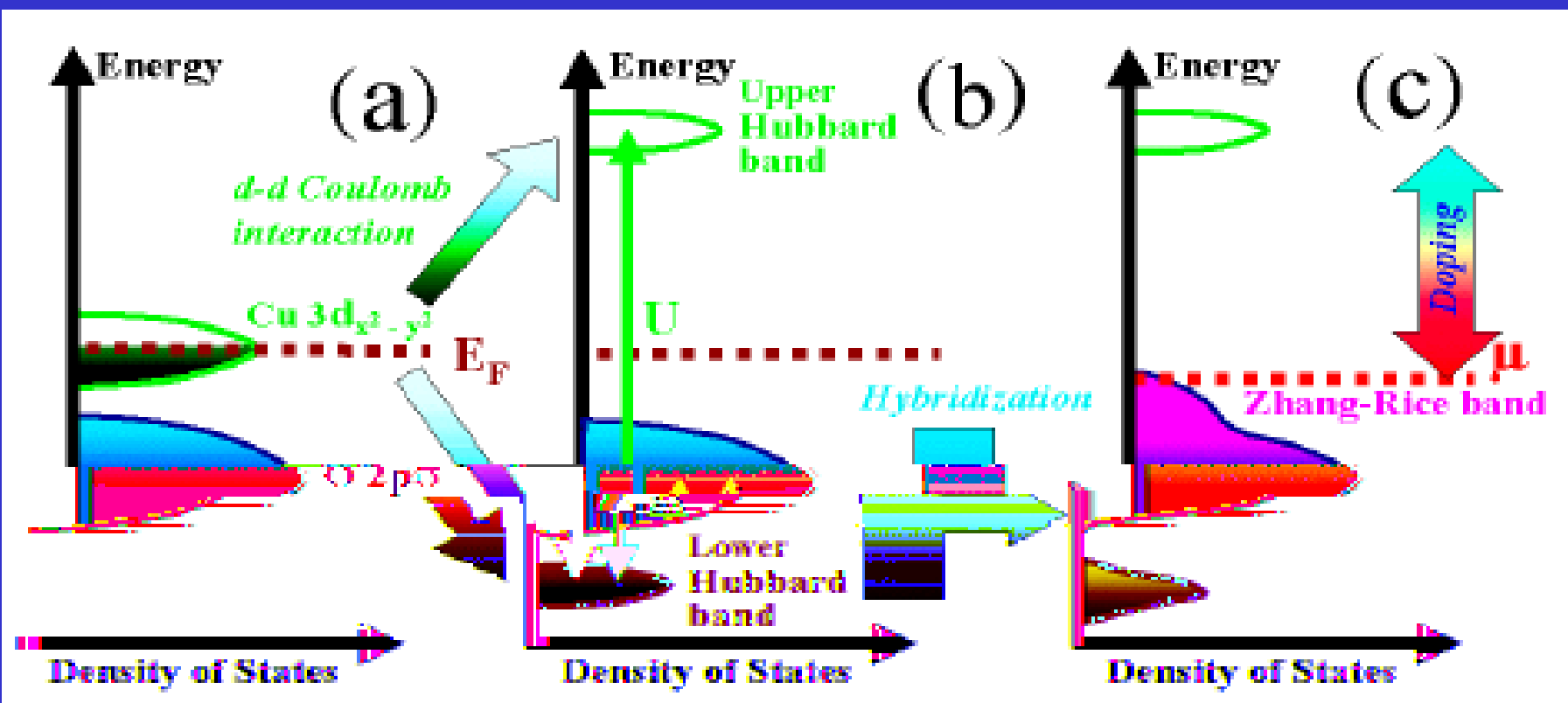
269



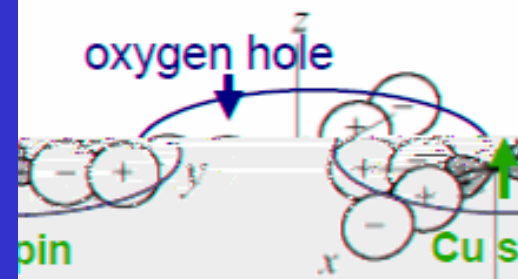


Structure of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$: Doped Mott Insulator



γ E 

Zhang-Rice singlet





Temperature

D-wave
or S-wave ?

D-wave pairing

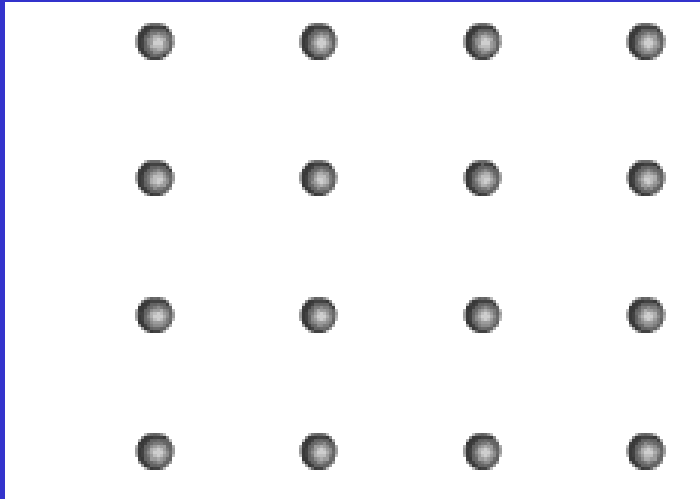
AF

Electron

doping

Hole



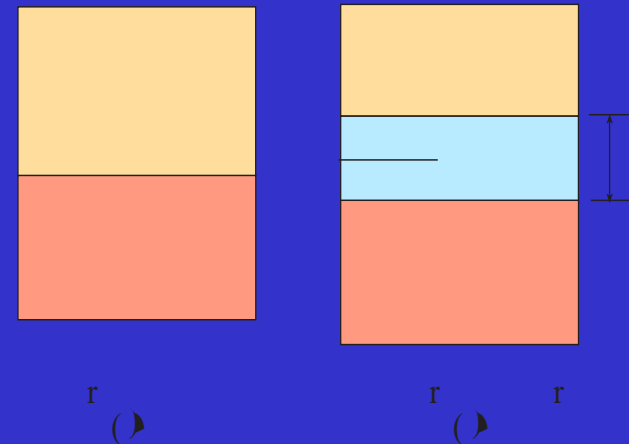


$$\approx 2 - 2\hbar\omega^{-2/3} \quad (0)$$

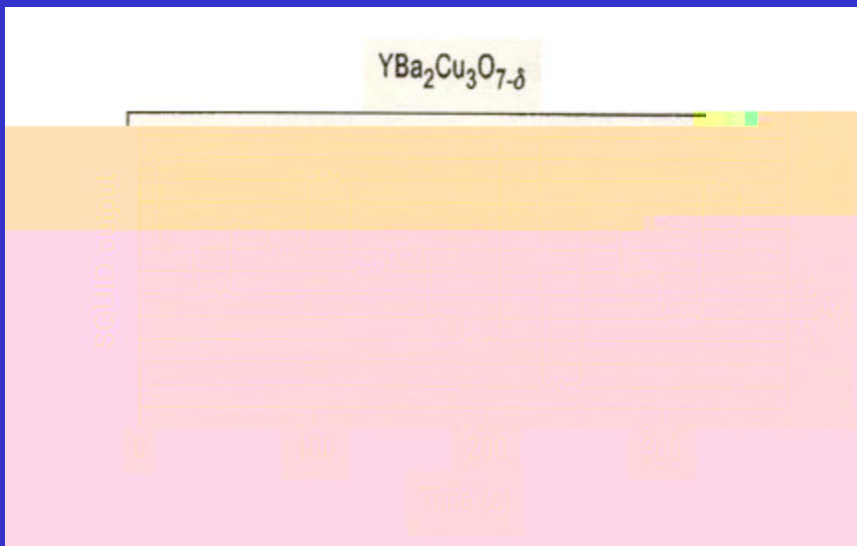
$$0 - 0 = -\frac{1}{2} \quad (0)\Delta^2$$

$$\Delta/\quad = 1.76 \quad (\text{weak coupling s-wave})$$

$$\Delta/\quad = 2.14 \quad (\text{weak coupling d-wave})$$



- High T_c superconductors: Cooper pairs: single flux with $2e$ as carriers in the superfluid condensate:



$$\Phi = \quad / 2$$

C. E. Gough, et al.,
Nature 326, 855 (1987).

- Pauli exclusion principle: two electrons cannot occupy the same quantum state.



Spin State	S	Description	Orbital Number
Spin Singlet	0	$\uparrow(1)\downarrow(2) - \uparrow(2)\downarrow(1)$	0 (s-wave) 2 (d-wave)
Spin Triplet	1	$\uparrow(1) \uparrow(2)$ $\downarrow(1) \downarrow(2)$ $\uparrow(1)\downarrow(2) + \uparrow(2)\downarrow(1)$	1 (p-wave)



Possible symmetries of curate high- T_c superconductors:

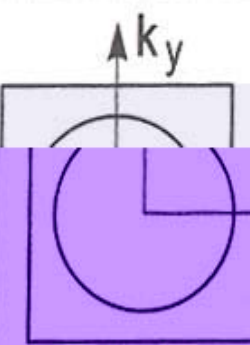
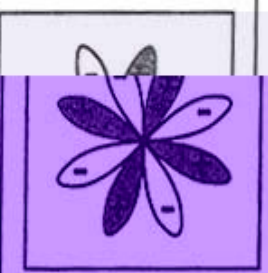
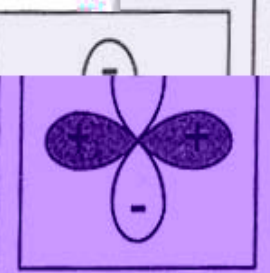
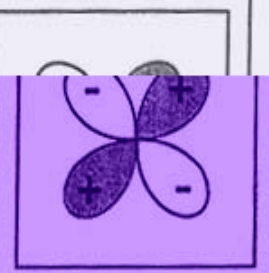
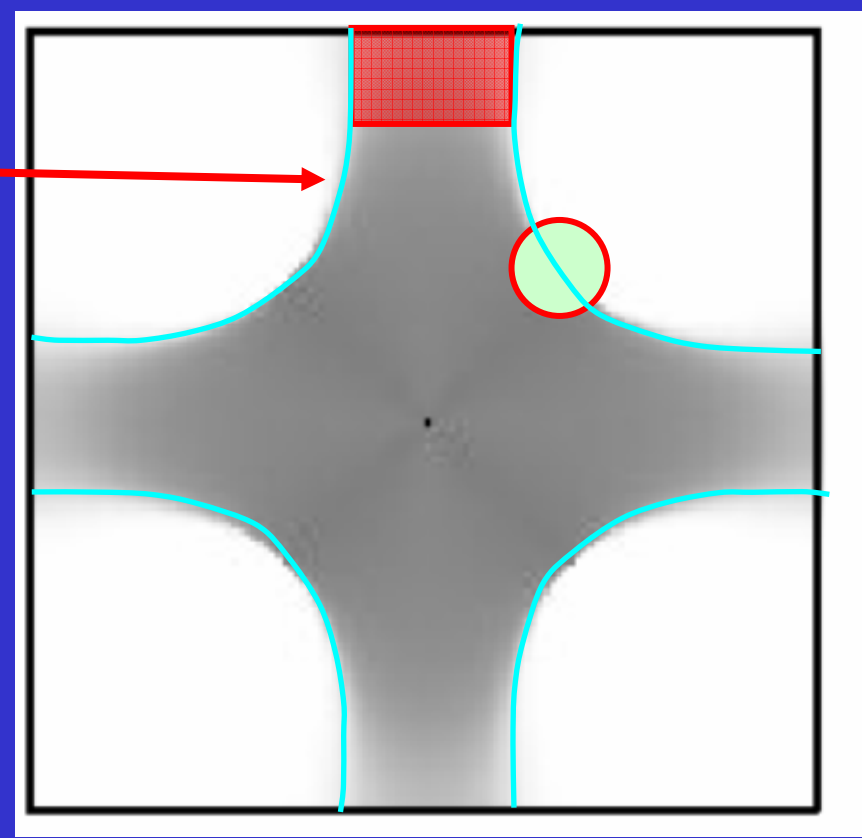
Group-theoretic notation	A_{1g}	A_{2g}	B_{1g}	B_{2g}
Order Parameter basis function	constant	$xy(x^2-y^2)$	x^2-y^2	xy
Wavefunction name	s-wave	g	$d_{x^2-y^2}$	d_{xy}
Schematic representation of $\Delta(k)$ in B.Z.				

Table 1

Possible even-parity singlet pair states in a square lattice (point group C_{4v})



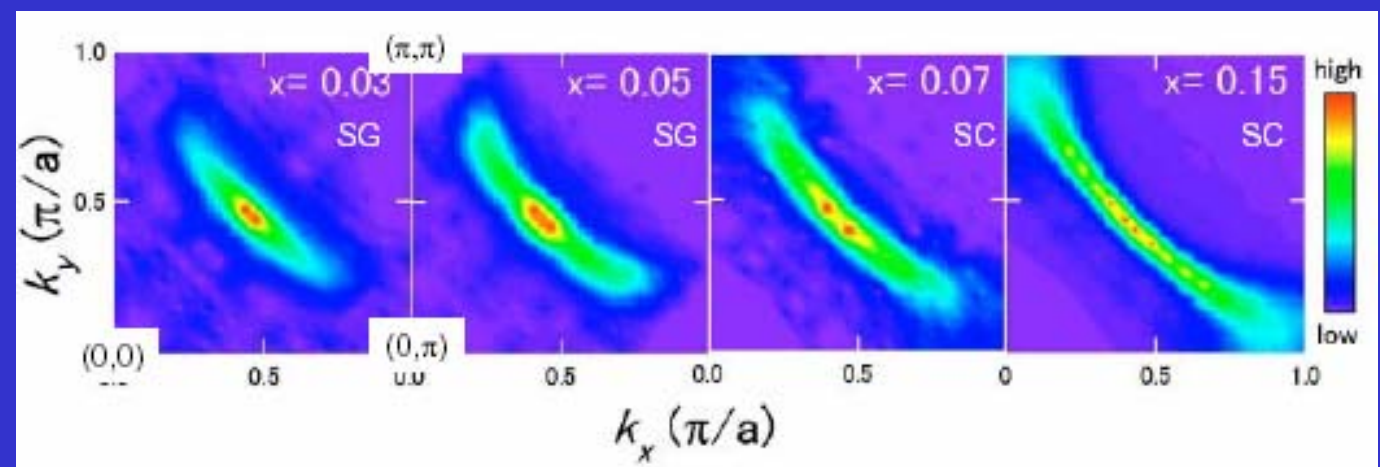
Fermi surface

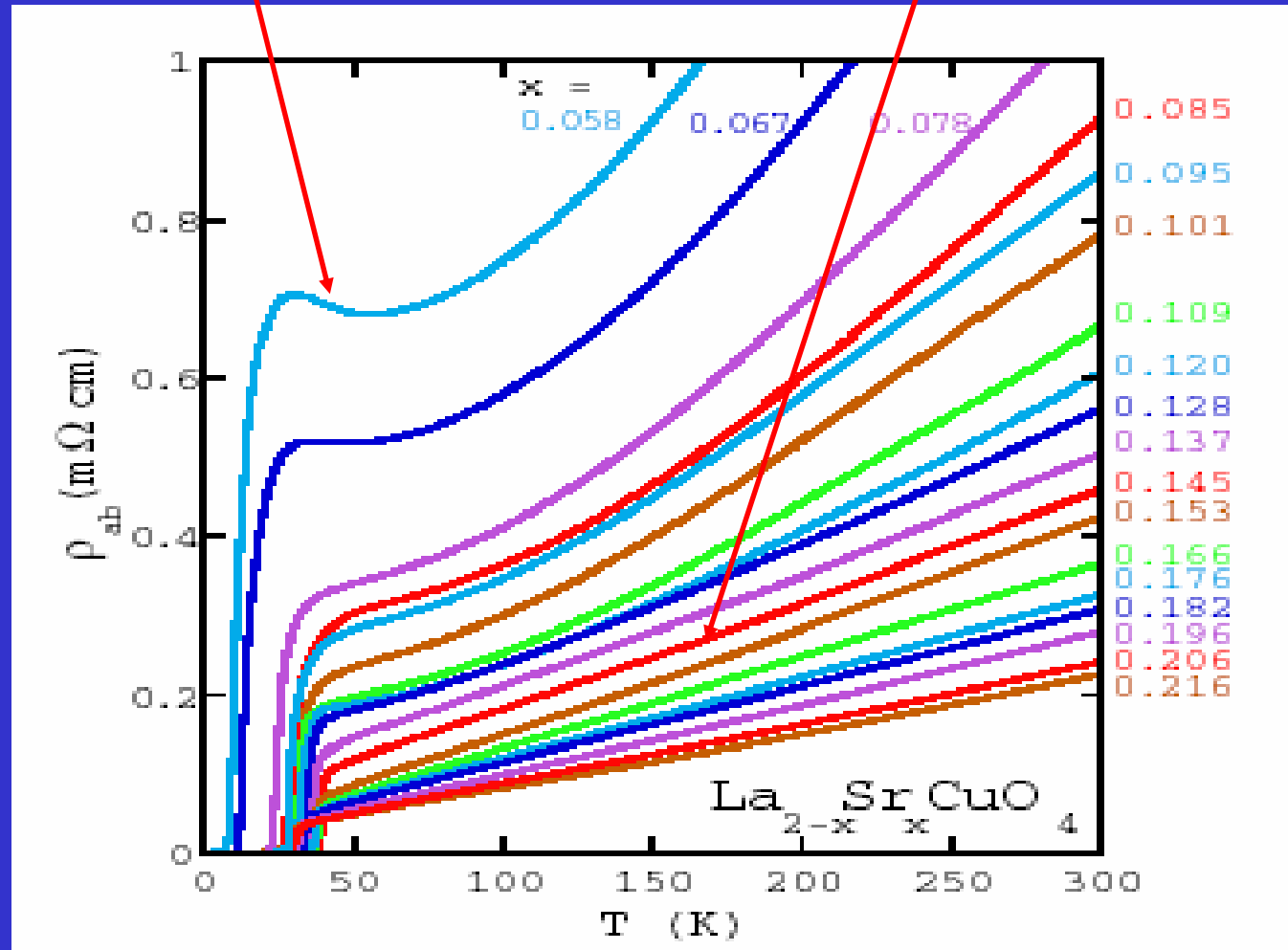


Nodal region

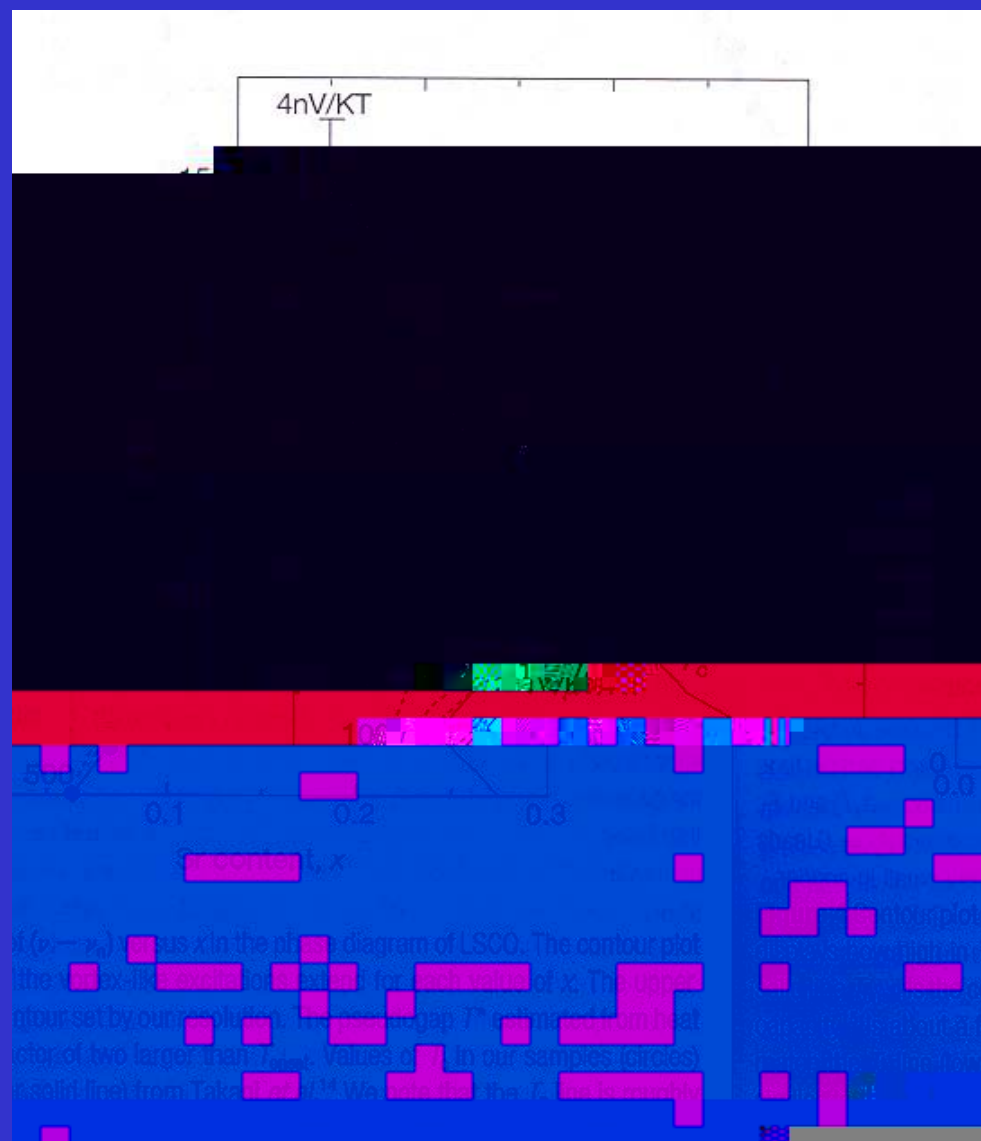
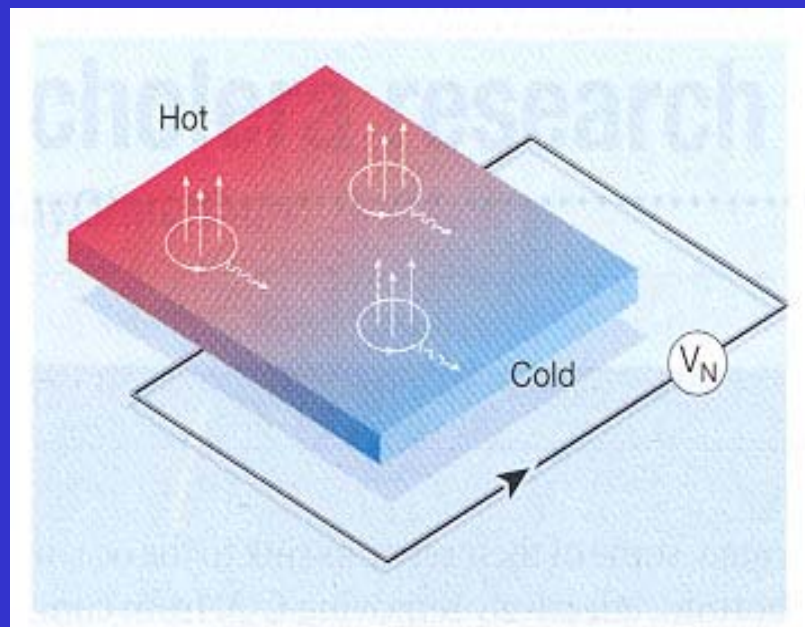
k_x

T. Yoshida et al.
LaSrCuO

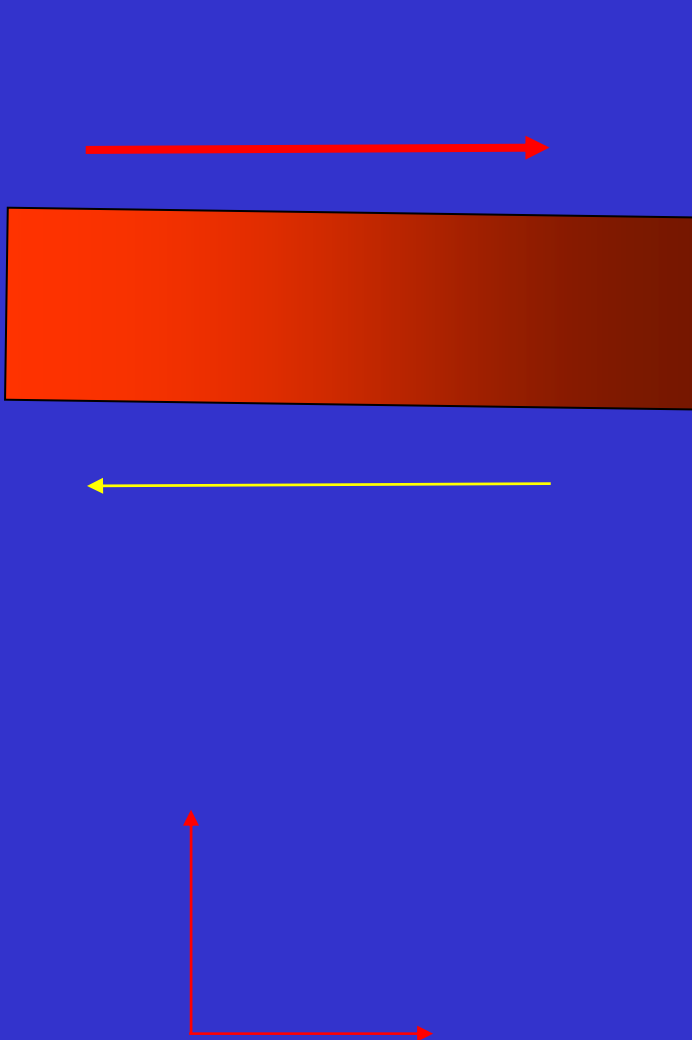


ψ Θ Λ 

Komiya, ..., Ando, PRL94, 207004(2005)



Nernst signal due to normal electrons is almost zero



$$= \sigma + \alpha(-\nabla)$$

$$= -(\alpha / \sigma)(-\nabla)$$

$$\nu = \frac{|\nabla|}{|\nabla|} = \left[\frac{\alpha}{\sigma} - \frac{\sigma}{\sigma} \right] /$$

$$= \left[\frac{\alpha}{\sigma} - \tan \theta \right] / = 0$$

*$H \neq 0$, No electrons flow,
thus NO transverse voltage.*



Nernst effect due to *flux motion*

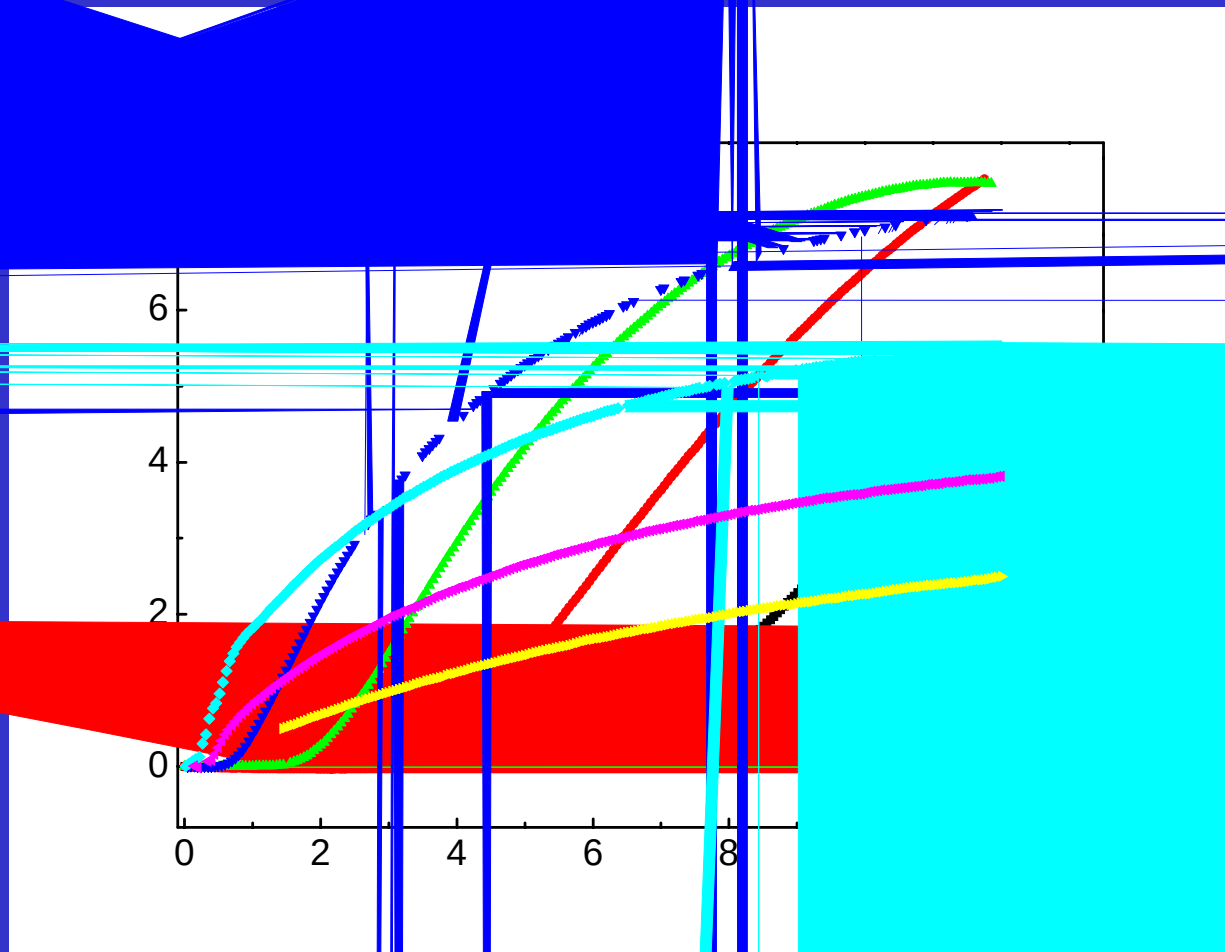
$$\begin{aligned} &= -\phi \nabla \\ \eta &= -\eta \phi \\ &= \phi \end{aligned}$$

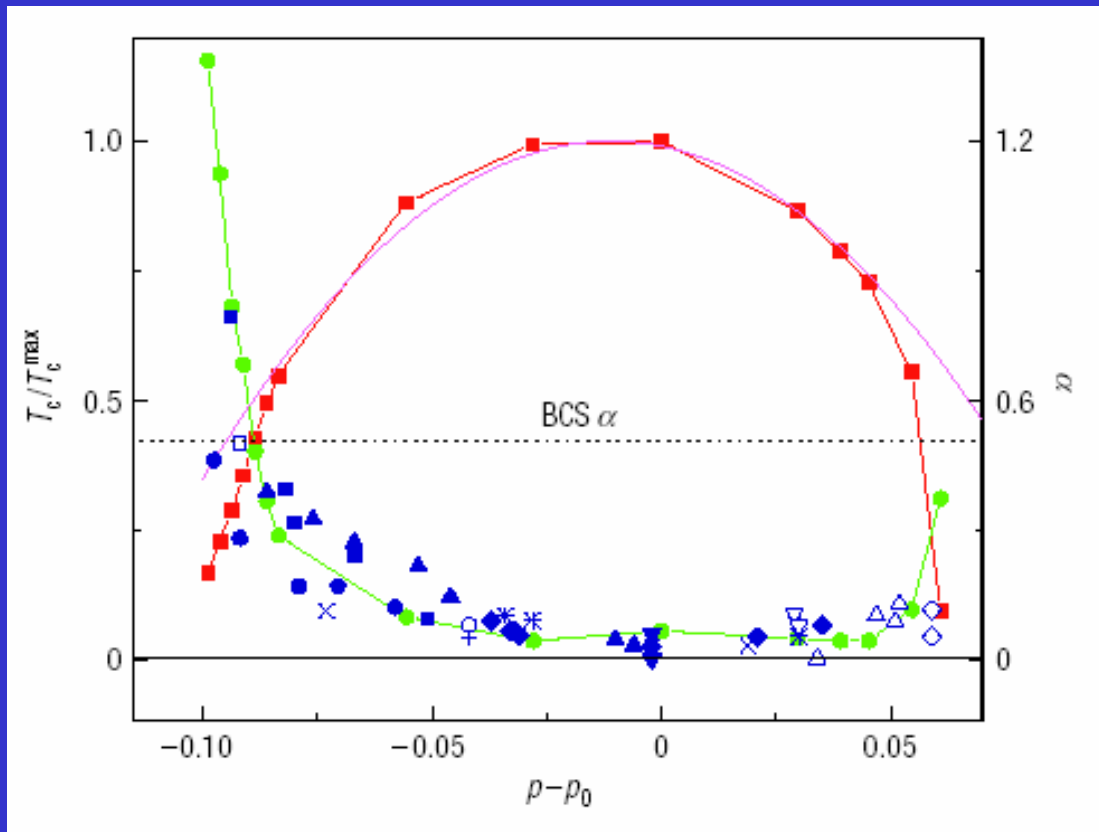
$$= \frac{\nabla \phi}{\phi_0} \rho = \frac{\nabla \phi}{\phi_0} \rho \frac{1}{2} \left(\right)$$

$$\phi = \left(\frac{\phi_0}{4\pi} \right) \left(\frac{1}{2} \left(\right) - \right) \frac{1}{1.16(2\kappa^2 - 1)}$$









BCS

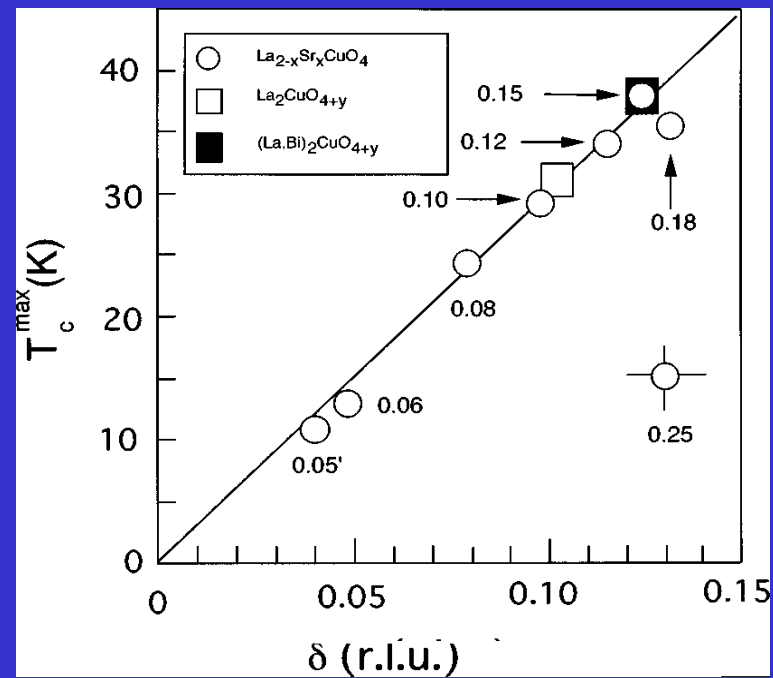
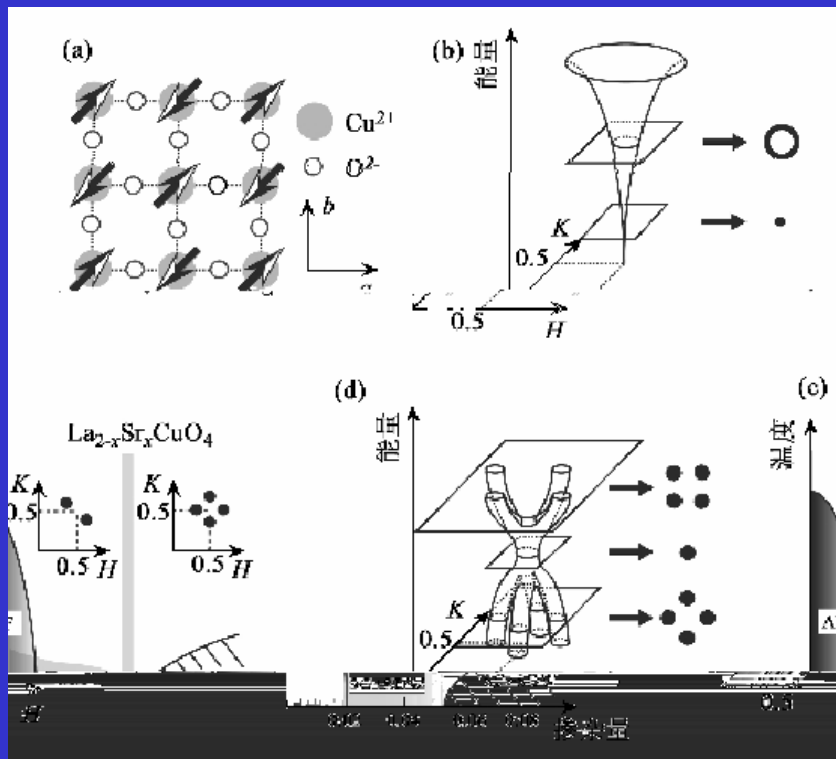
$$T_c M^\alpha =$$

$$\alpha, \quad 0.5$$

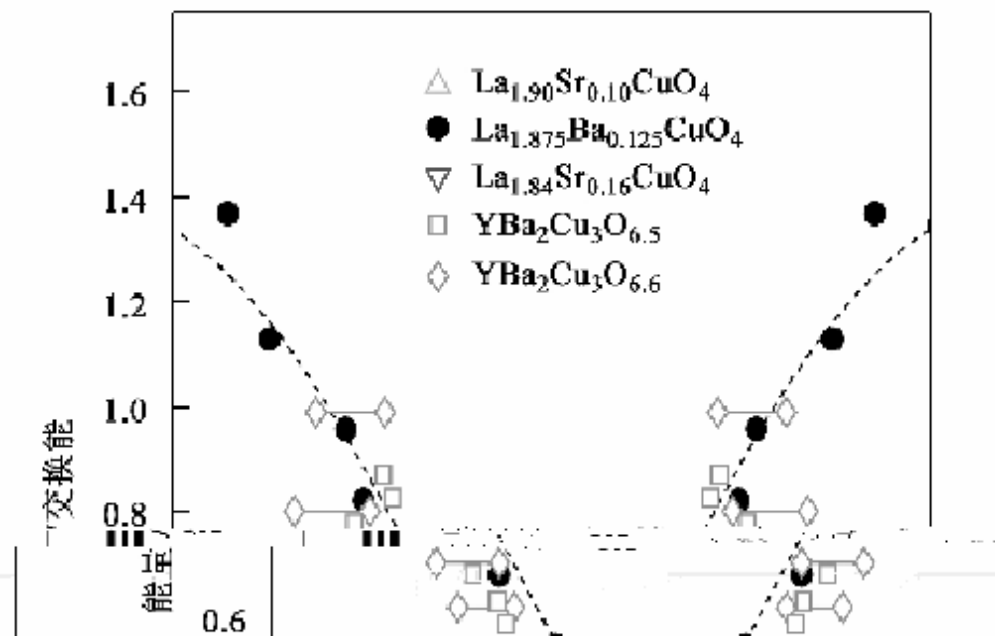
↓ ↓ $\propto$

↓ D. M. Newns and C. C. Tsuei, Naturephys.
3, 184 (2007).

E è

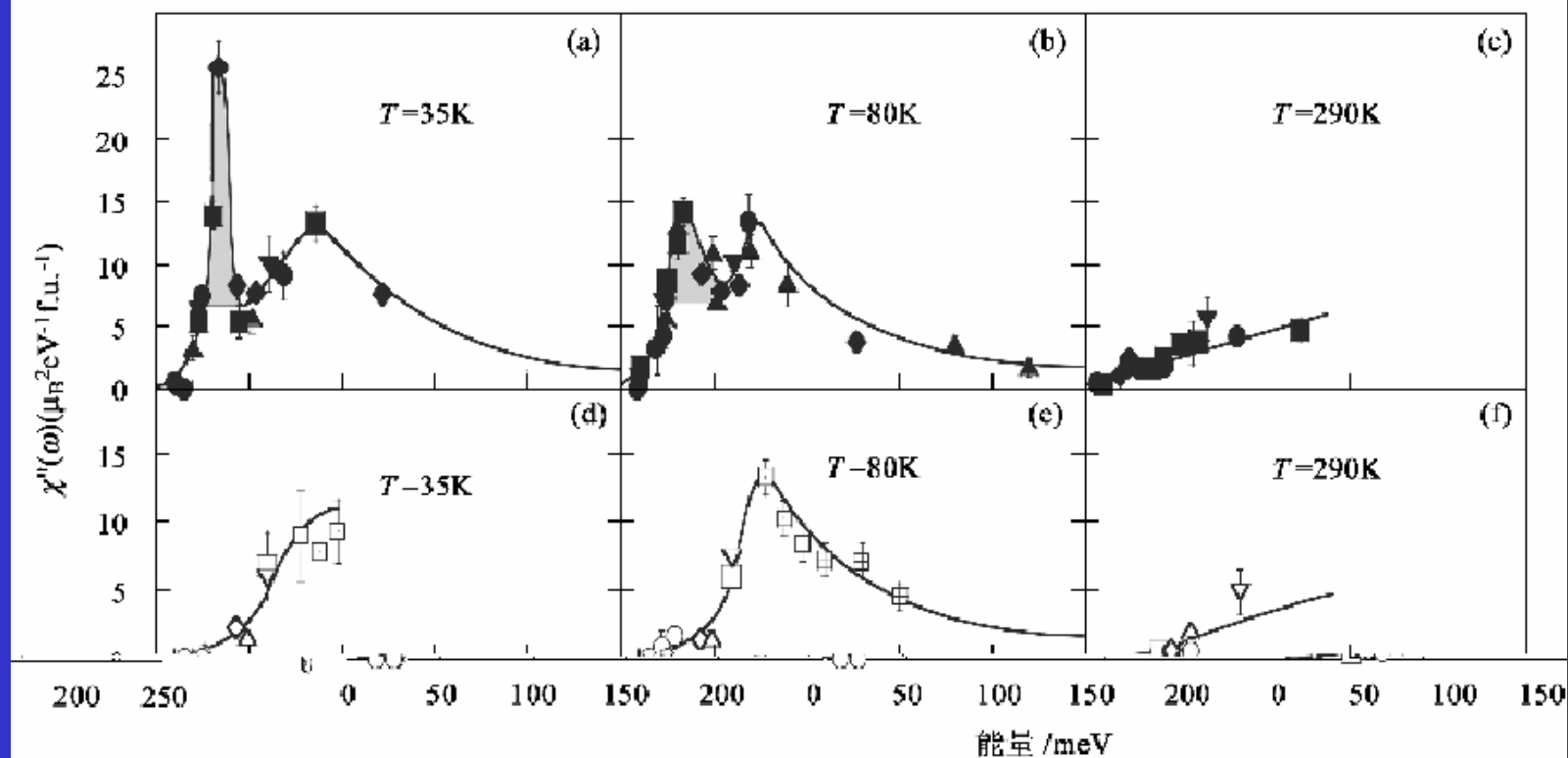


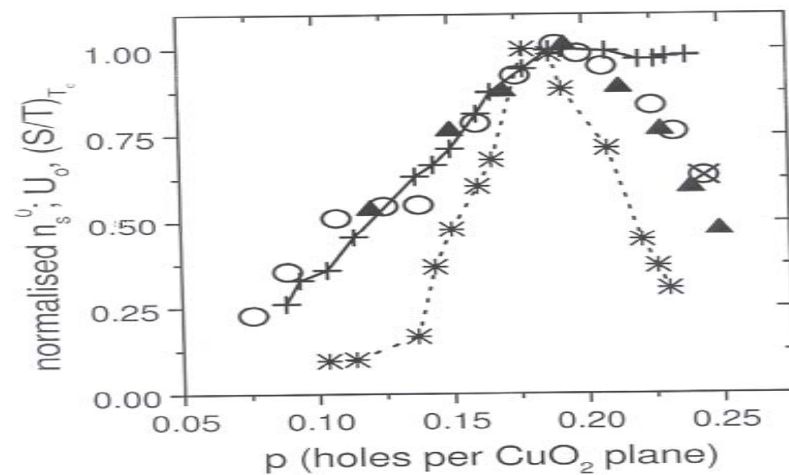
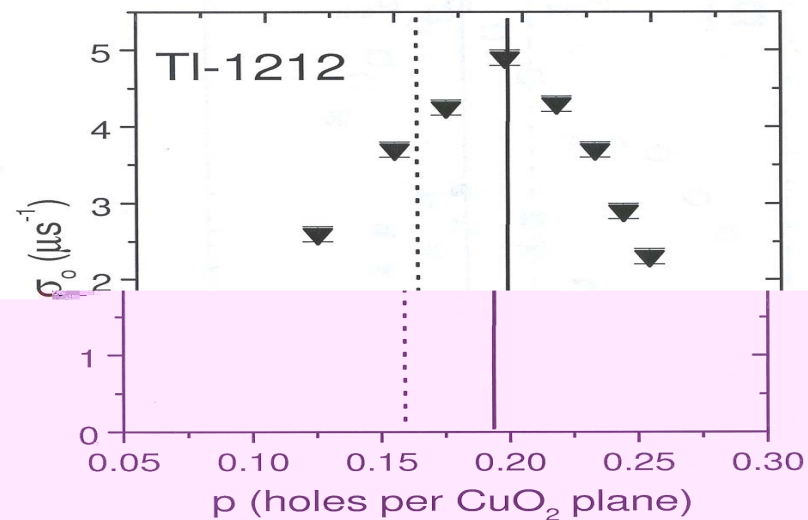
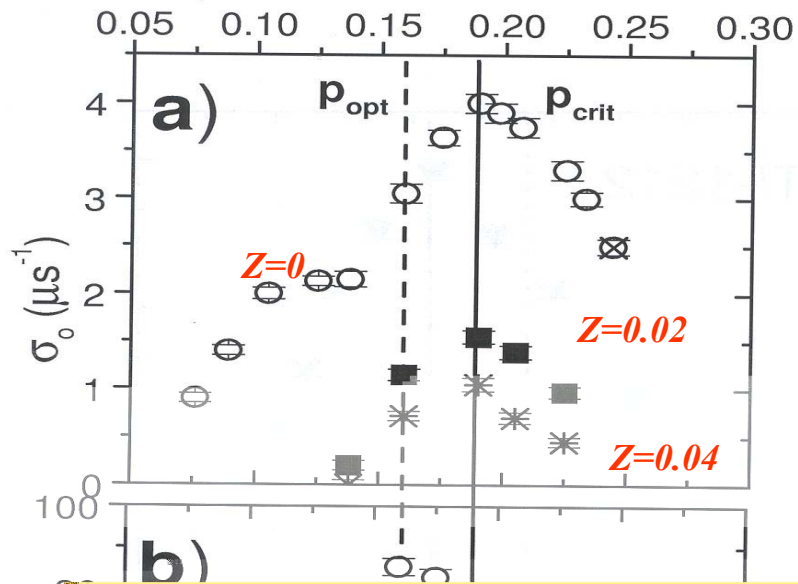
й ә ъ
2006



J. Tranquada, Nature 2006

$\text{YBa}_2\text{Cu}_3\text{O}_{6.6} (T_c = 62.7\text{K})$





BE-BCS Condensation Scheme

- Y. J. Uemura, *PRL* 66, 2665(1991).
Nature 364, 605(1993). *Physica C* 282-287, 194(1997).

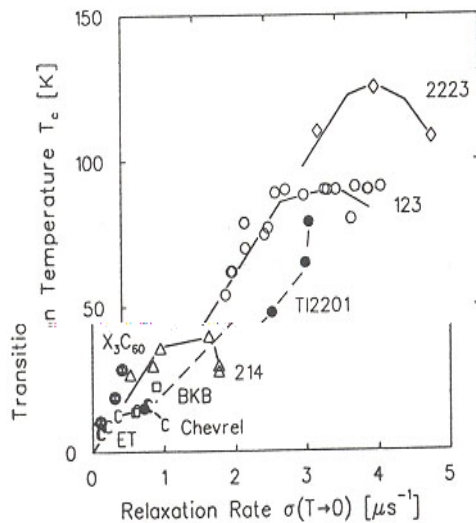
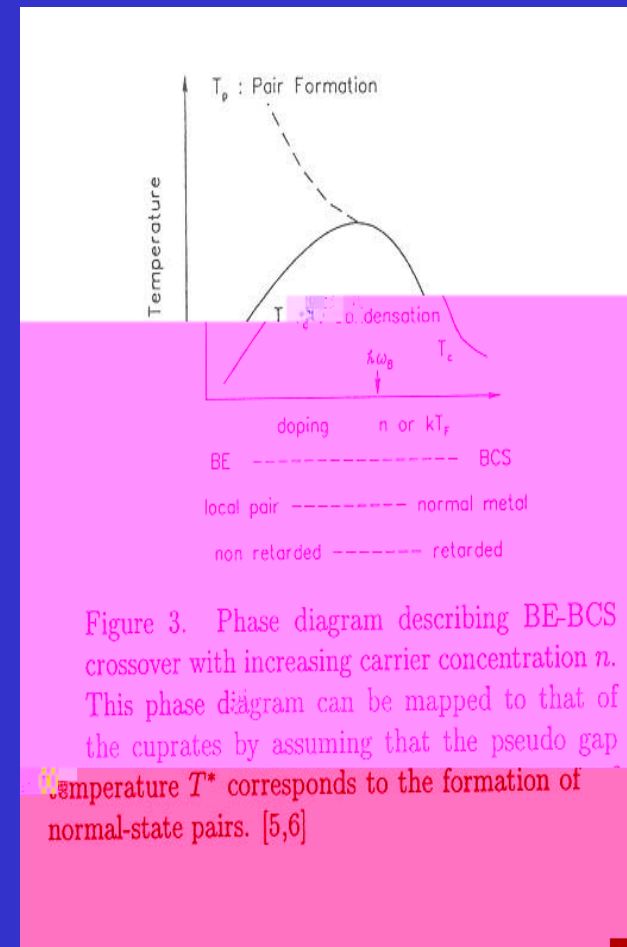
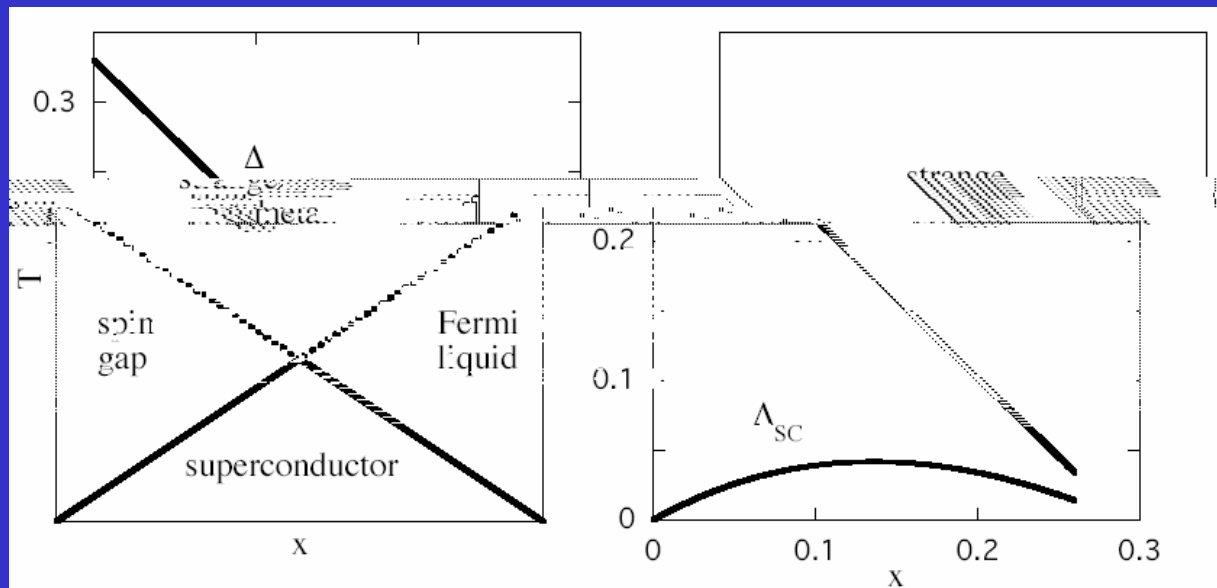
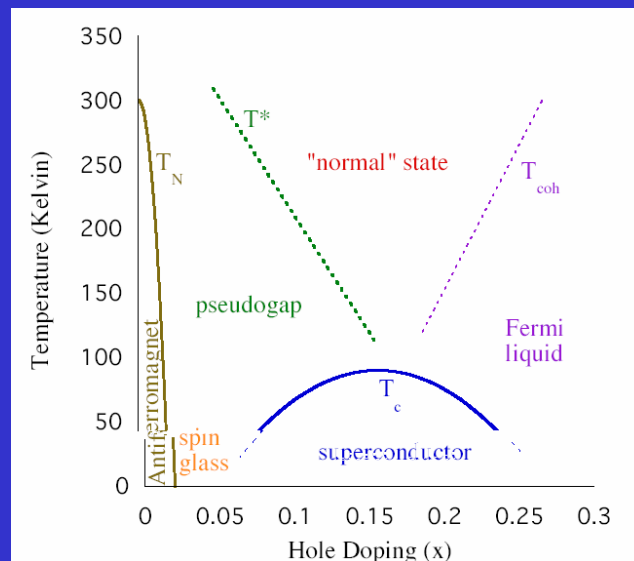


Figure 1. Muon spin relaxation rate $\sigma(T \rightarrow 0)$ in various superconductors plotted versus T_c [1-3].

Overdoped
Regime:
Swiss Cheese
Model





V. J. Emery and S. A. Kivelson, Nature 374, 434 (1995).

H₂

↑

T_c vs. p

BCS

A powerful tool for low energy excitation

$$= \text{---} = + +$$

$$\Delta = \Delta \left[1 - \exp\left(-\frac{t}{\tau}\right) \right]$$

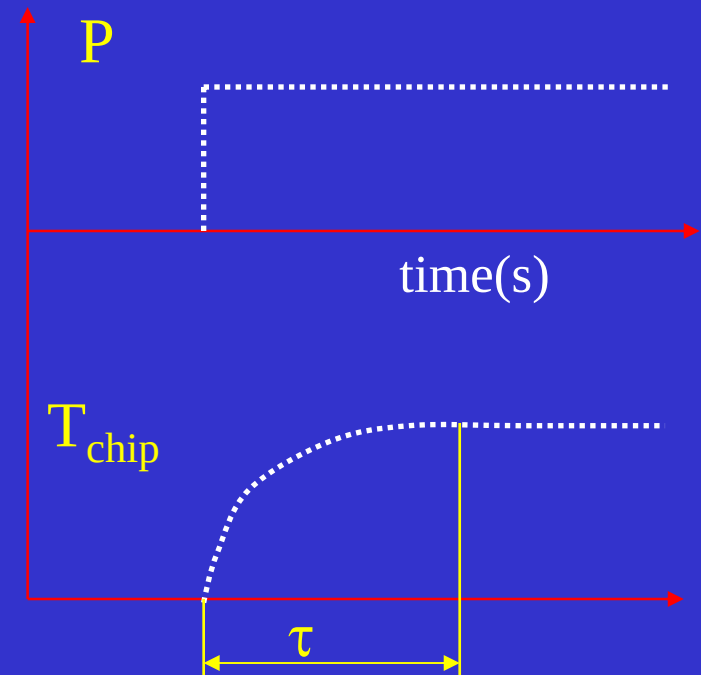
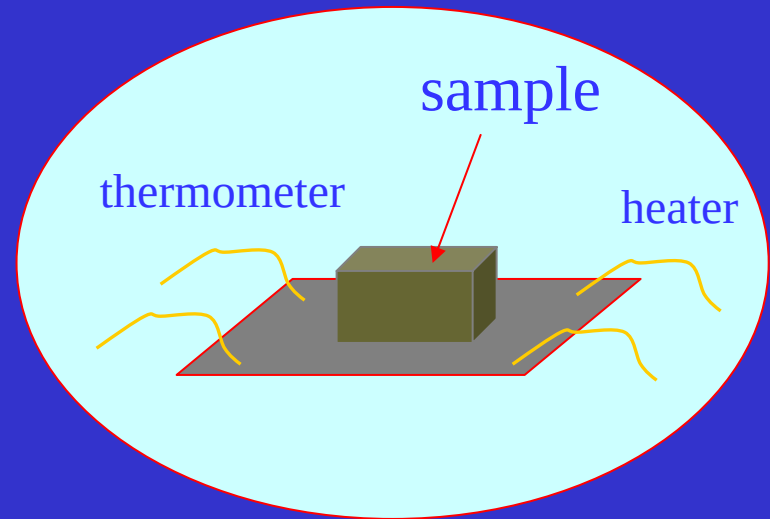
$$\tau = \frac{\left(\frac{C}{K} \right)}{K}$$

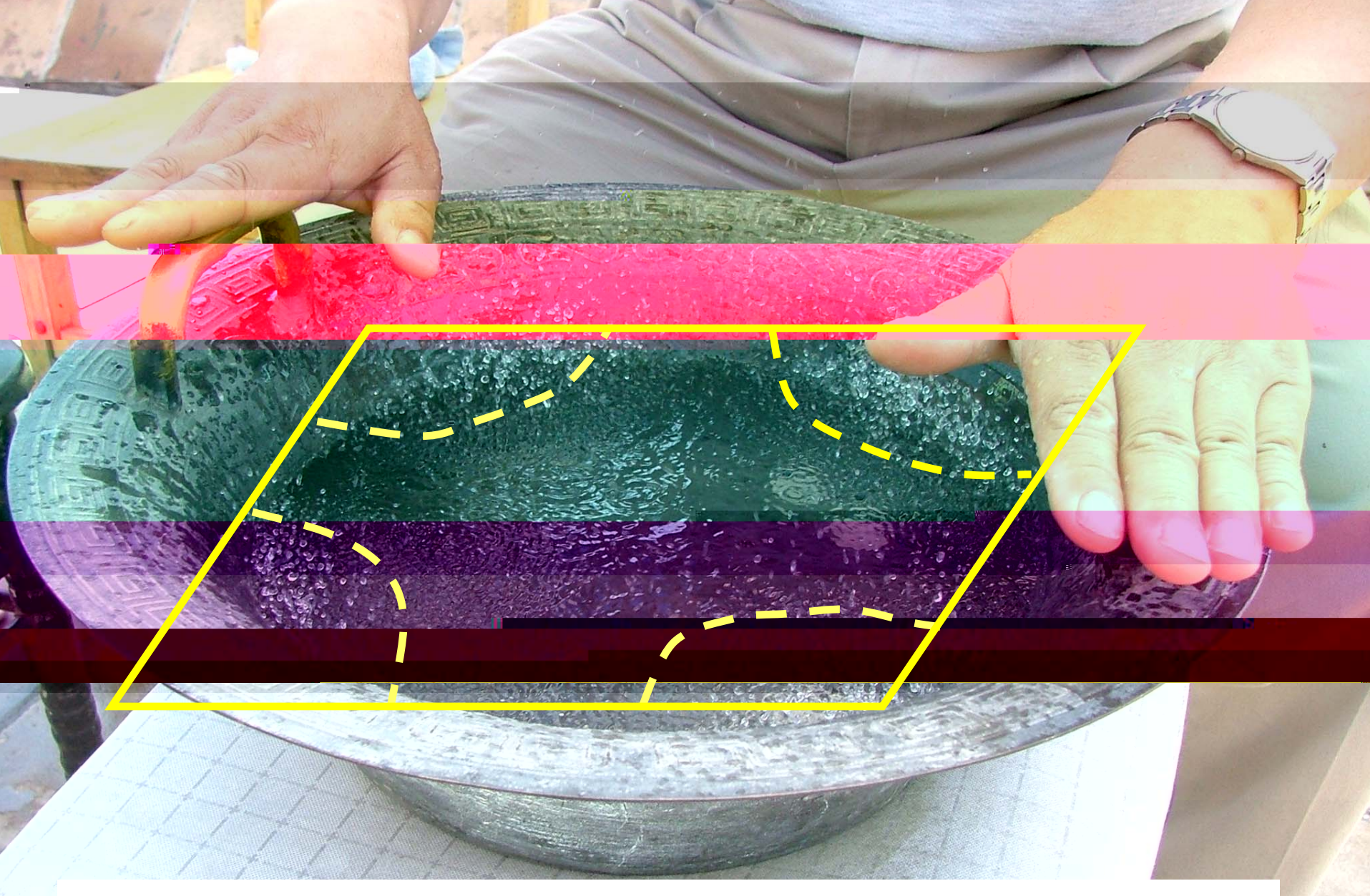
Sample: 1 to 50 mg.

T: 1.8 K to 400 K

H: 0 to 12 Tesla

C: 10 nJ / K

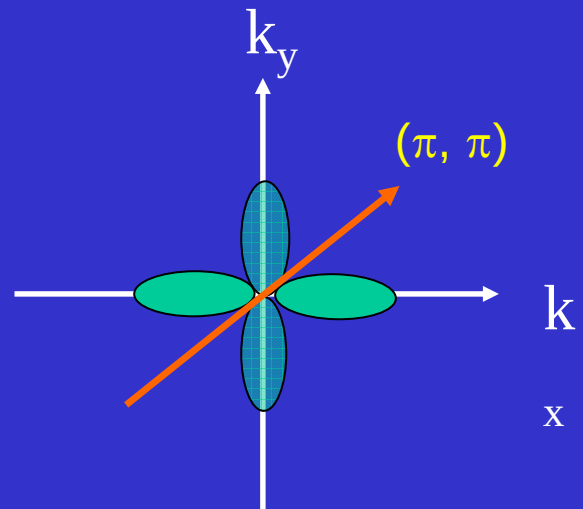


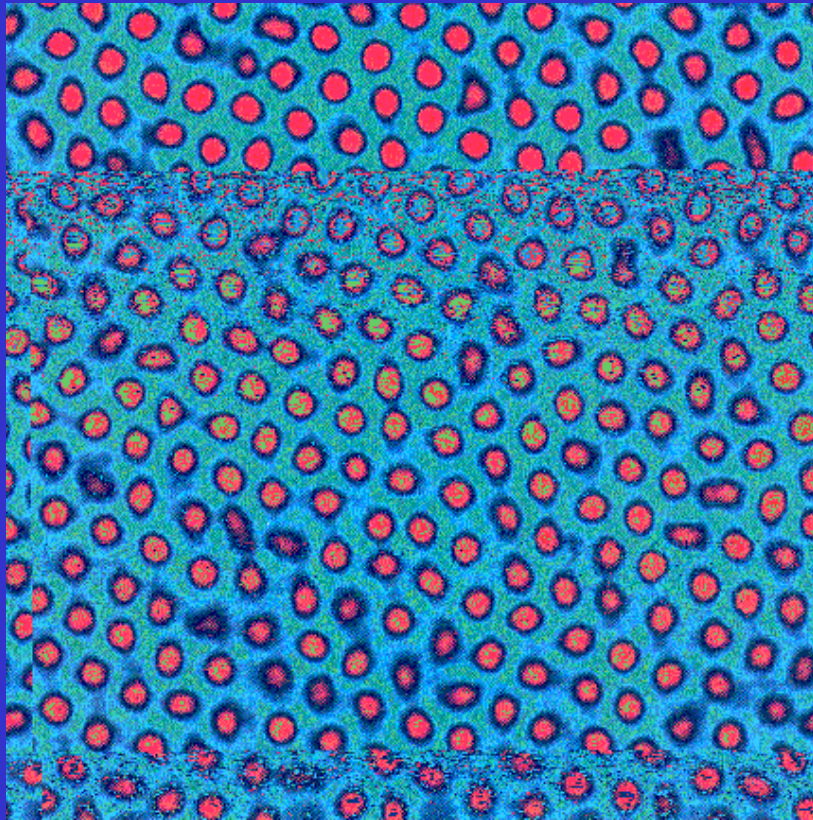


Quasiparticle excitations from Fermi arcs found by Chinese
2000 years ago!

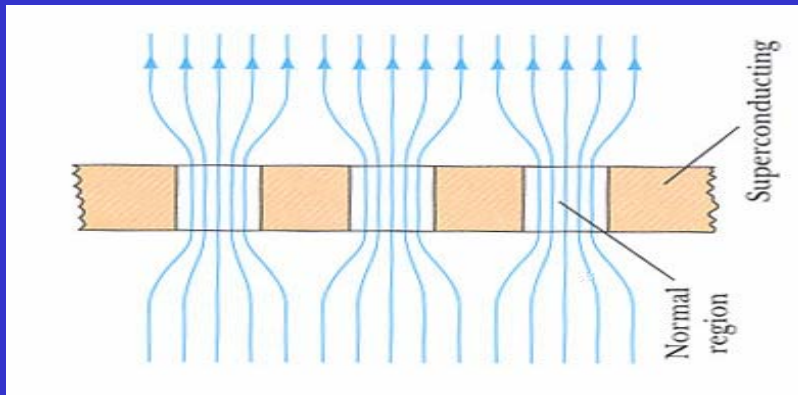
$$(\chi) = \chi(0) \operatorname{Re} \frac{1}{\sqrt{k^2 - \Delta^2}}$$

$$(\chi) = \chi(0) \int \frac{\theta}{2\pi} \operatorname{Re} \frac{1}{\sqrt{k^2 - \Delta_0^2 \cos^2(2\theta)}}$$





1. **S-wave:** superconductor, vortex cores play as potential wells for low energy quasi-particles.
One component: localized core state.
2. **D-wave:** two components for quasiparticles:
 Localized core state and de-localized extended states due to the nodes.

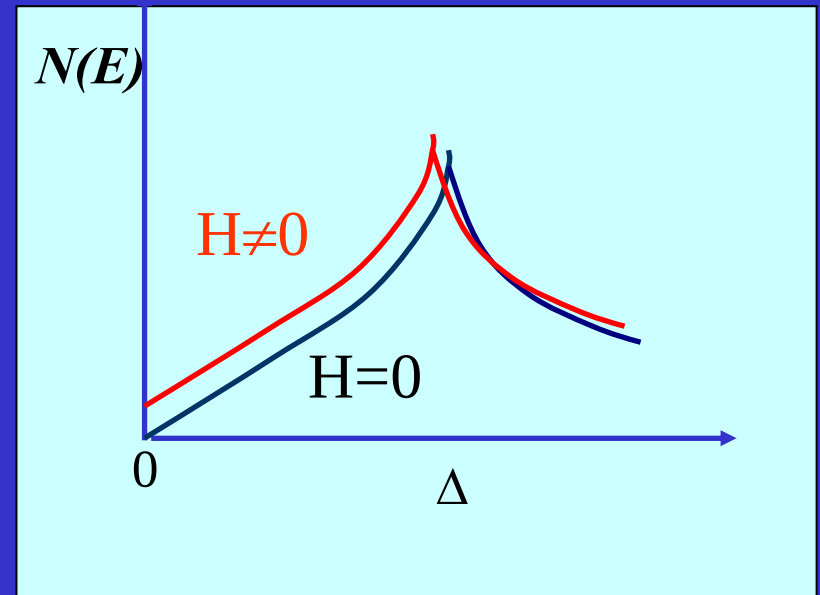




1. S-wave superconductor:

$$\begin{aligned} \chi(\vec{r}, \vec{r}') &\approx \gamma \frac{1}{r^2} \\ \chi(\vec{r}, \vec{r}') &\approx \gamma \frac{1}{r^2} \end{aligned}$$

2. D-wave superconductor:



$$\chi(\vec{r}, \vec{r}') \propto \sqrt{r/r_0}$$

$$\chi(\vec{r}, \vec{r}') = \int \frac{d^3k}{(2\pi)^3} \int d\Omega \delta[\vec{k} \cdot (\vec{r} - \vec{r}') - \vec{k} \cdot \vec{r} \Omega]$$

$\propto 1/\sqrt{r}$ Quasiparticle outside core

Doppler effect:

$$\delta = \vec{v} \cdot \vec{r}$$

$$\chi(\vec{r}, \vec{r}') = \gamma \sqrt{\frac{r}{r_0}}$$

G. E. Volovik, JETP Lett. 58, 469(1993).

By solving the BdG theory for a d-wave superconductor, Simon and Lee predict a scaling law for the quasiparticle spectrum for d-wave superconductor:

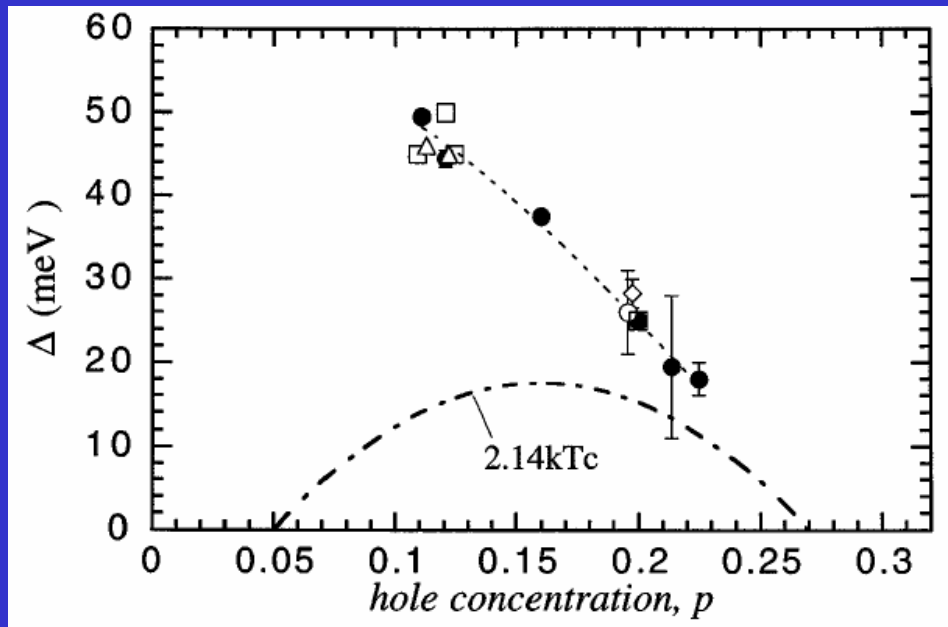
S. H. Simon, P. A. Lee, PRL78, 1548 (1997).

$$U = \sum_n \epsilon_n^H f(\epsilon_n^H / T) \\ = [H/H_0]^{1/2} \sum_n \epsilon_n^{H_0} f(\epsilon_n^{H_0} [H/H_0]^{1/2} / T),$$

$$= \frac{1}{2} \left(\frac{H}{H_0} \right)^{1/2} = \left(\frac{H}{H_0} \right)^{1/2}$$

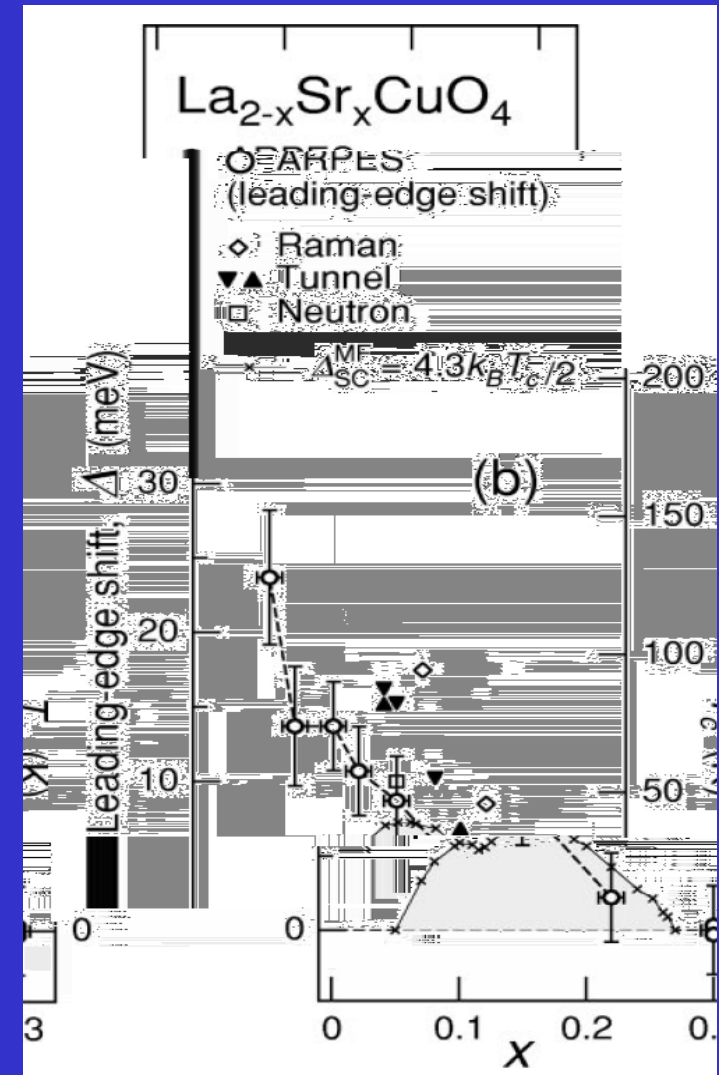
This scaling law contains cases in two limits: Volovik 7.18 145.3705 Tm(





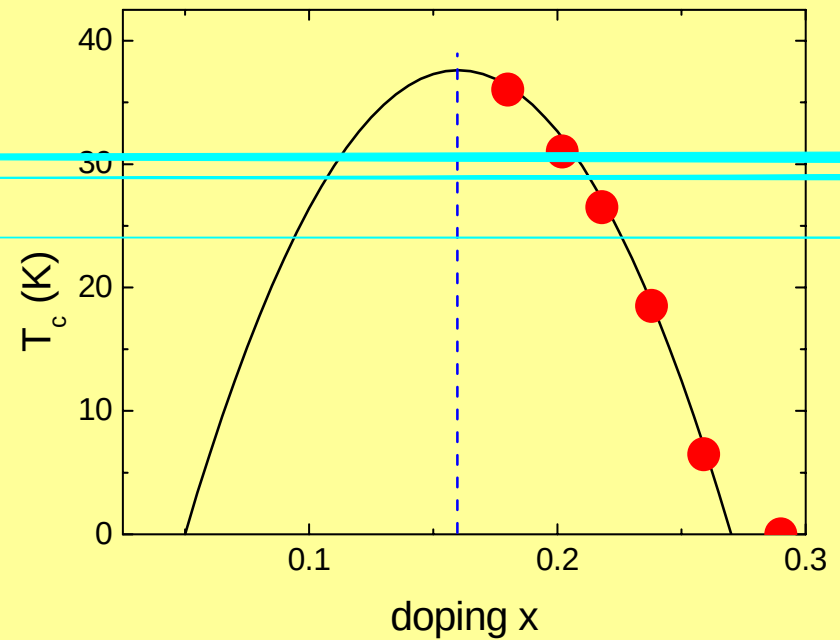
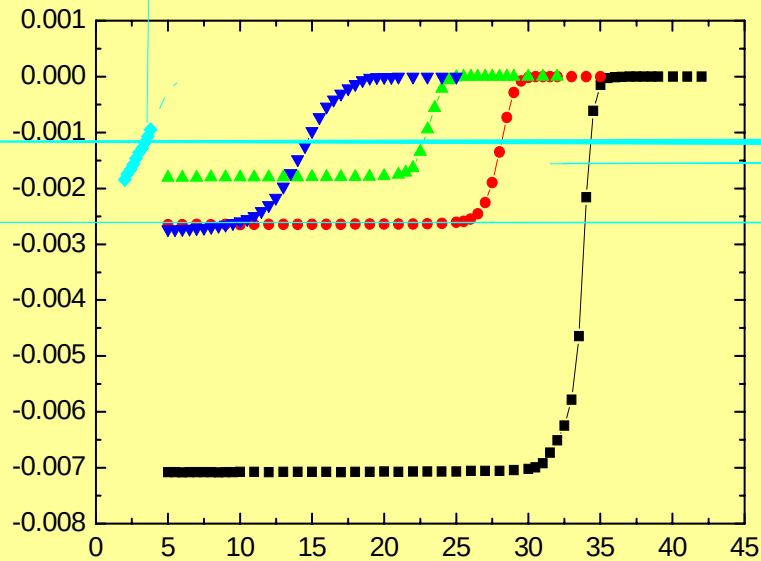
N. Miyakawa, et al., *PRL* 80, 157 (1998).

Break junction tunneling



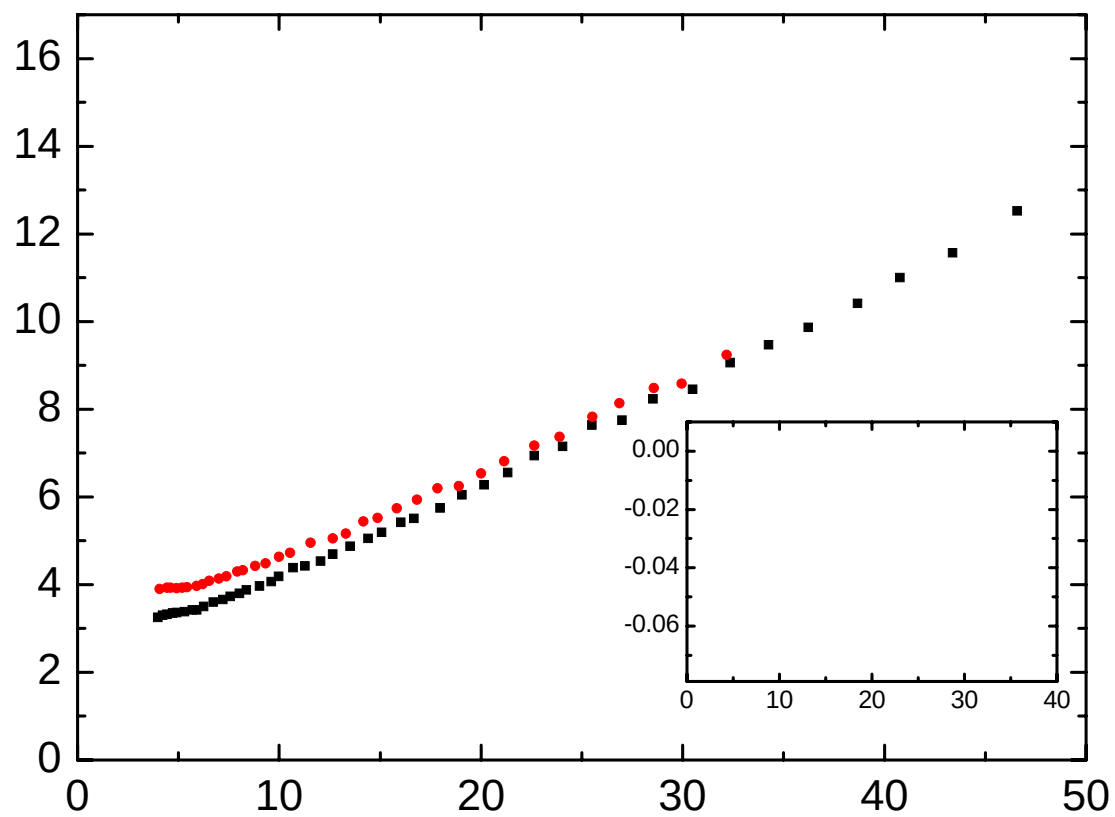
A. Ino, et al., (2002)

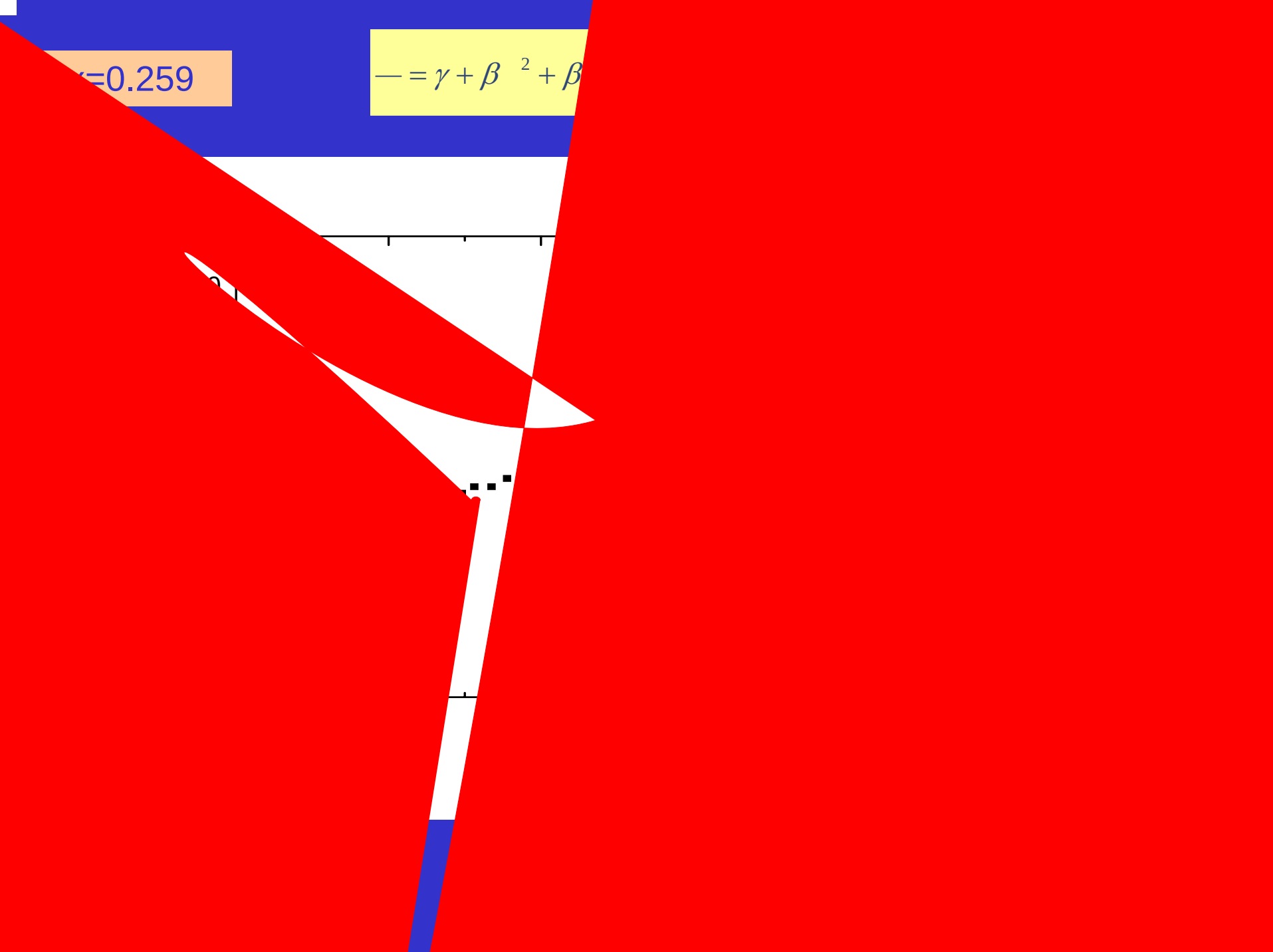
Samples: $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ single crystals From Tohoku University



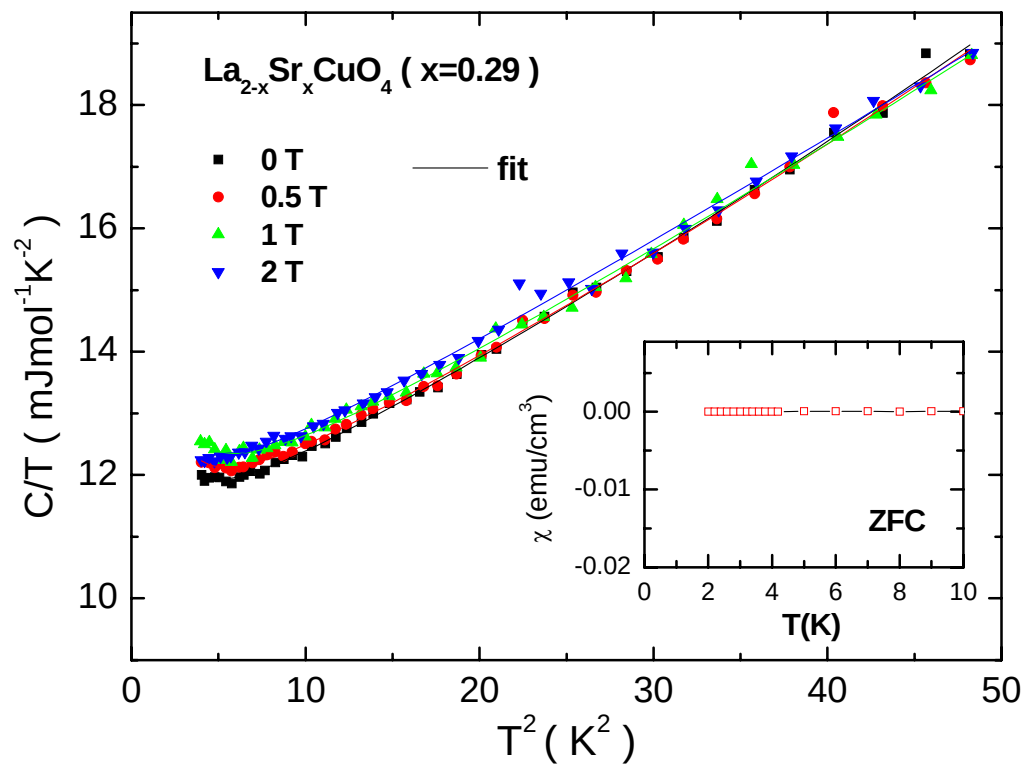
$$x=0.18\sim 0.5$$

16
14
12
10



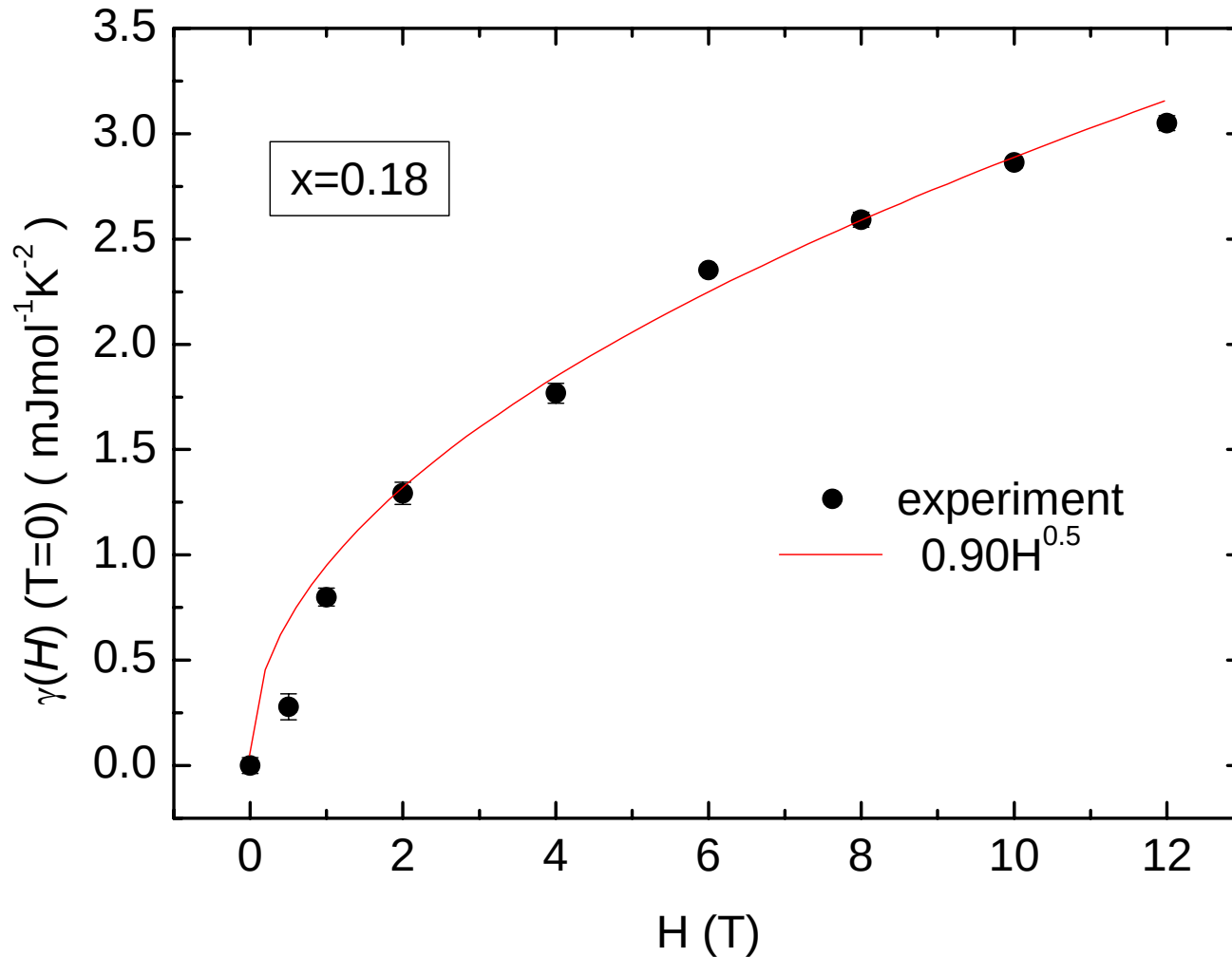


$x=0.29$



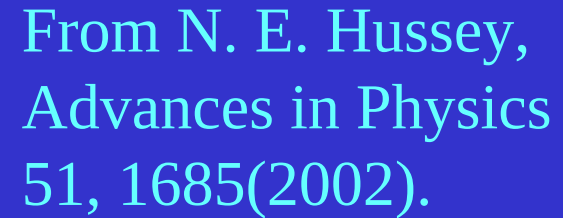
x	β	β_5	θ_D	γ_N	Notes
	(mJmol ⁻¹ K ⁻⁴)	(μJmol ⁻¹ K ⁻⁶)	(K)	(mJmol ⁻¹ K ⁻²)	
0.29	0.13	0.74	471	10.8	present work (2 K-12 K)
0.30	0.1	0.7	550	6.9	PRB 68,R100502(2003)(Nakama

The field-induced specific heat for $x=0.18$:



$$\frac{(\quad)}{(\quad)} = \gamma(\quad) = \sqrt{\quad}$$

Volovik's relation
G. E. Volovik, JETP Lett. 58, 469(1993).



$$(\hbar\omega) = \sqrt{\mathcal{E}^2 + \Delta^2} = \hbar\sqrt{\frac{2}{\parallel} + \frac{2}{\Delta \perp}}$$

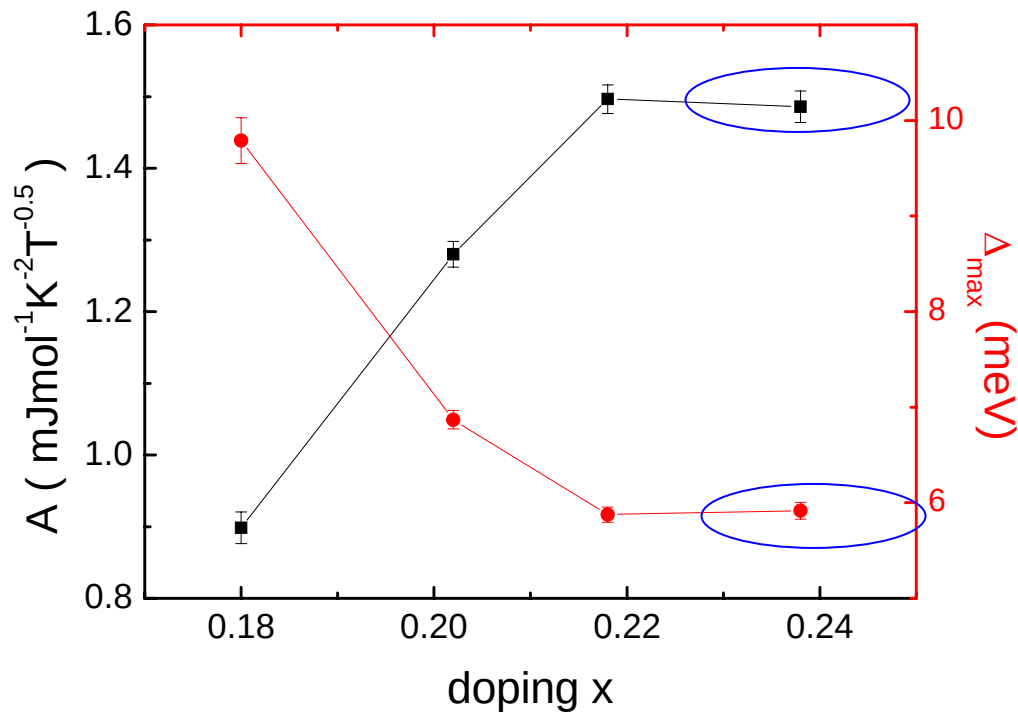
$$\langle \Delta \rangle = \frac{2}{\pi \hbar^2} \left(\frac{1}{\Delta} \right) \quad \Delta = \left[\frac{\Delta}{\phi} \right] / \hbar$$

$$= \frac{4}{3\hbar} \left(\frac{\pi}{\Phi_0} \right)^{1/2} \frac{1}{v_{\Delta}}$$

$$= \frac{4}{3\hbar} \left(\frac{\pi}{\Phi_0} \right)^{1/2} \frac{1}{\nu_{\Delta}}$$

$$\Delta_0 = \frac{1}{2} \hbar \nu_{\Delta}$$

$$= \frac{1}{\Delta_{\max}}$$

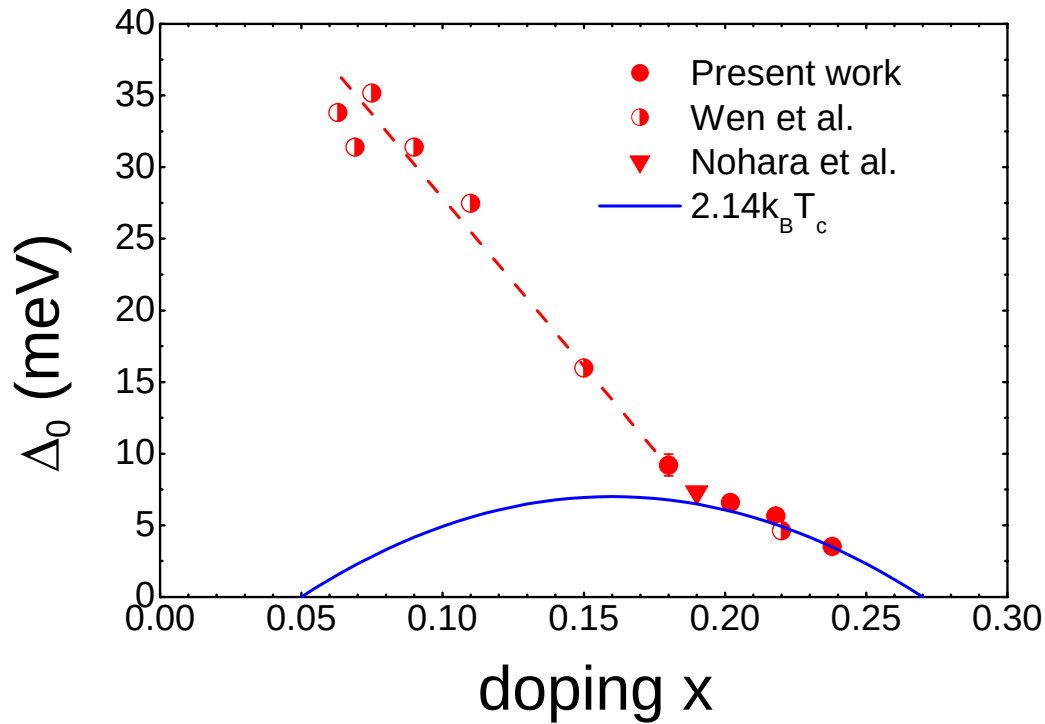


No longer increase in A: due to the decrease of the superconducting volume fraction

$$= \gamma / [\gamma - \gamma(0)]$$

Correction needed!

Weak coupling d-wave superconductivity in overdoped region!



$$= \frac{4}{3\hbar} \left(\frac{\pi}{\Phi_0} \right)^{1/2} \frac{1}{\nu_{\Delta}}$$

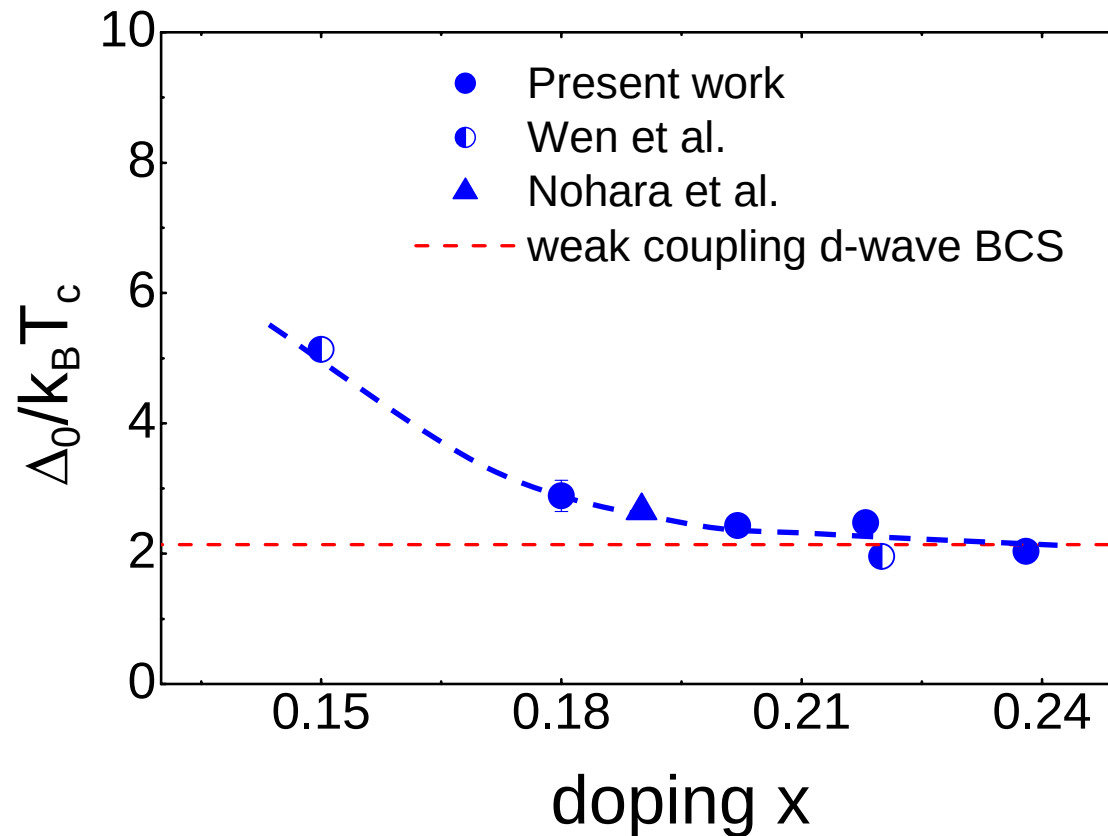
$$\Delta_0 = \frac{1}{2} \hbar \nu_{\Delta}$$

Underdoped LSCO:

H. H. Wen, et al., PRB72, 134507(2005).

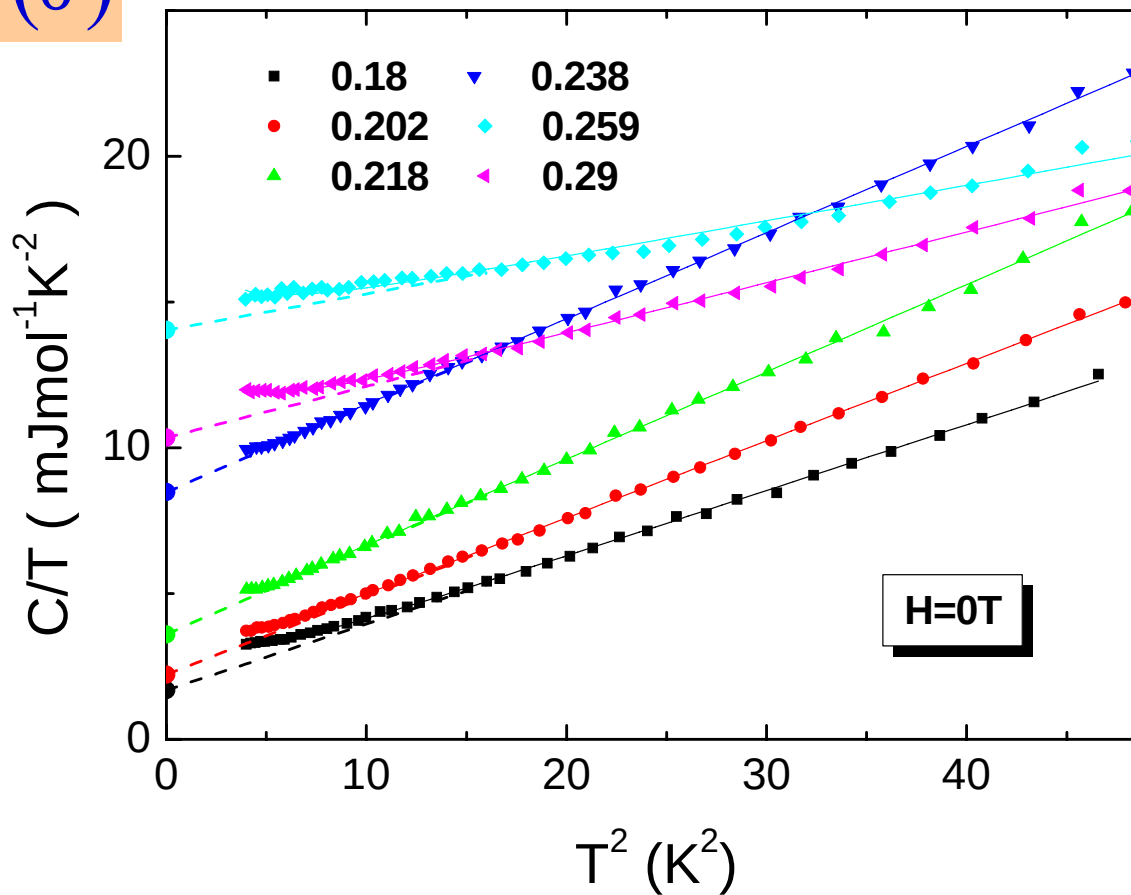
H. H. Wen, et al., PR B70, 214505(2004).

Weak coupling d-wave superconductivity in overdoped region!

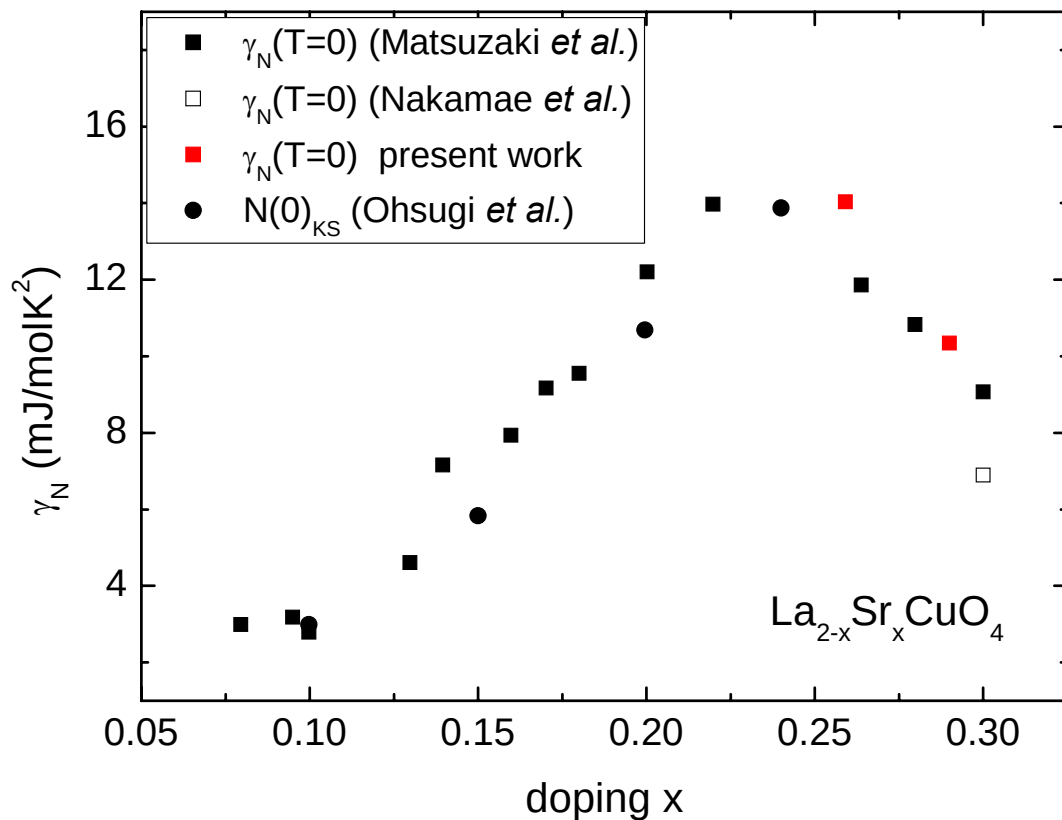


Yue Wang, et al., Phys. Rev. B, 2007, in press.

$\gamma(0)$

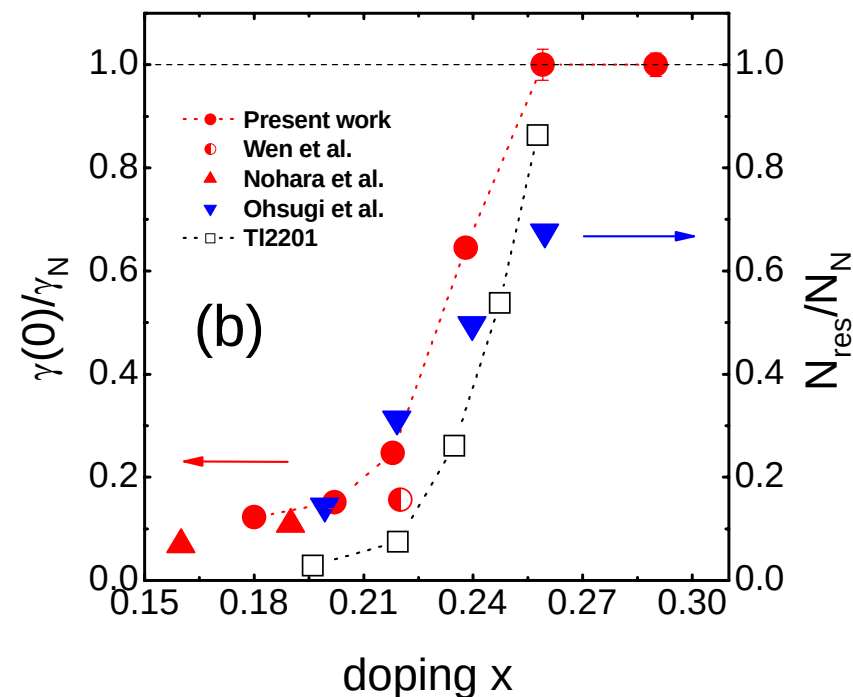
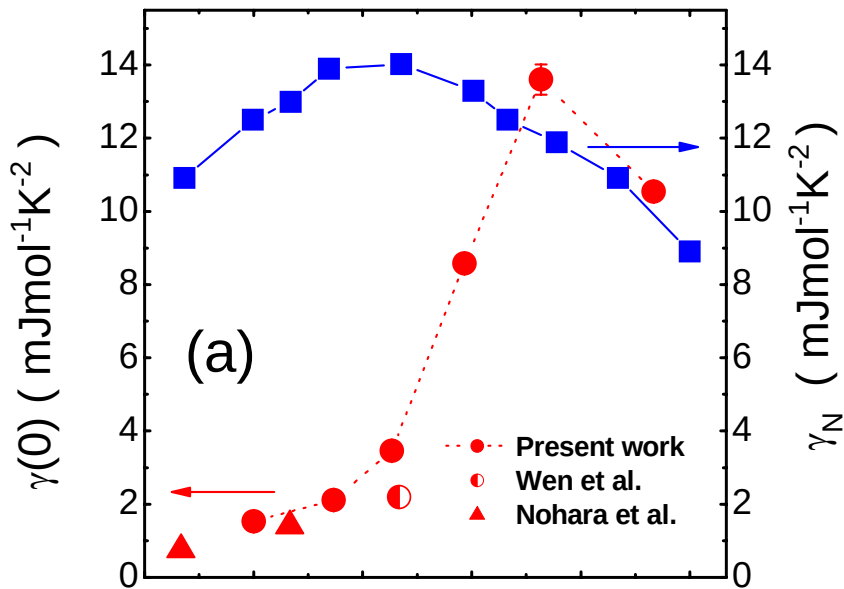


$\gamma(0)$ increases rapidly with increasing doping ($0.18 \leq x \leq 0.259$) and evolves into the normal state γ_N at $x = 0.29$



S. Nakamae *et al.*, PRB 68, R100205 (2003)

T. Matsuzaki *et al.*, J. Phys. Soc. Jpn. 73, 2232 (2004)



Crudely estimate the fraction of the residual unpaired carries:

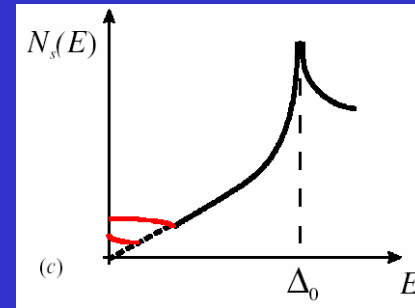
$\gamma(0)$ (mJmol ⁻¹ K ⁻²)	γ_N (%)	$\frac{\gamma(0)}{\gamma_N}$ (mJmol ⁻¹ K ⁻²)	$\frac{\gamma(0)}{\gamma_N}$ (mJmol ⁻¹ K ⁻²)
1.68	17.5	0.18	9.6
2.22	18.2	0.202	12.2
3.60	25.9	0.218	13.9
8.48	61.0	0.238	13.9
14.03	~100	0.259	~14.03
10.34	100	0.29	10.34

Growing of the normal state regions in the sample towards mo

Impurity scattering:

$$\frac{\gamma}{\gamma_0} = \frac{\Delta_0}{8\gamma_0} \left(\frac{\gamma}{\Delta_0} \right)^2 \ln \left[\frac{\pi}{2} \left(\frac{\Delta_0}{\gamma} \right)^2 \right]$$

$$\frac{\gamma}{\gamma_0} = \frac{2\gamma_0}{\pi\Delta_0} \ln \left(\frac{\Delta_0}{\gamma_0} \right)$$

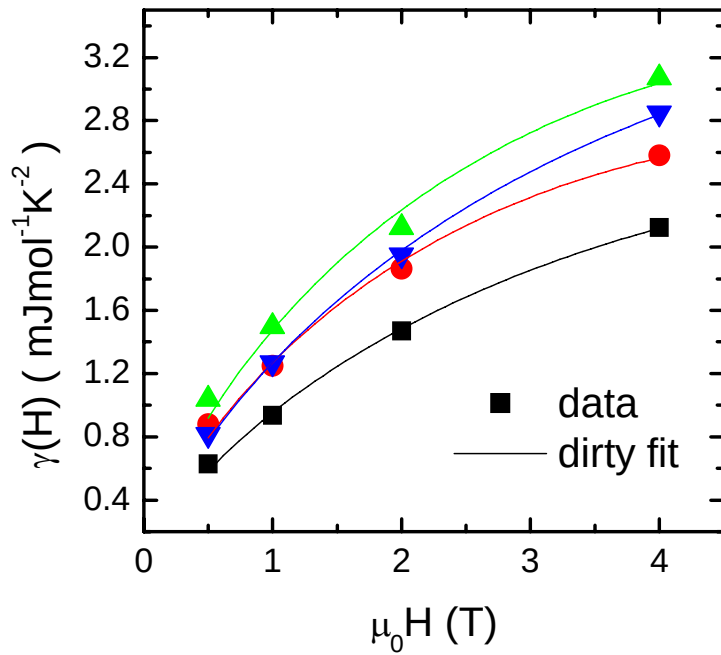


$$\gamma_0 \cong 0.61 \sqrt{\Gamma \Delta_0} \quad \text{pair-breaking parameter}$$

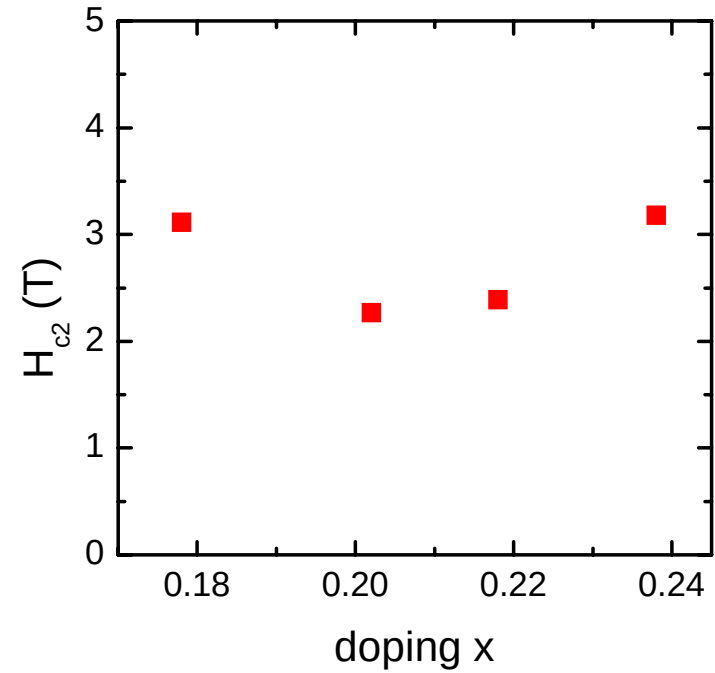
$$\Gamma = \overline{\hbar \pi} \quad (0) \quad \text{quasi-particle scattering rate}$$

C. K. Bert et al., Solid State Commun. 105, 459(1998)

G. Preosti et al., PRB **50**, 1259 (1994)



dirty fit

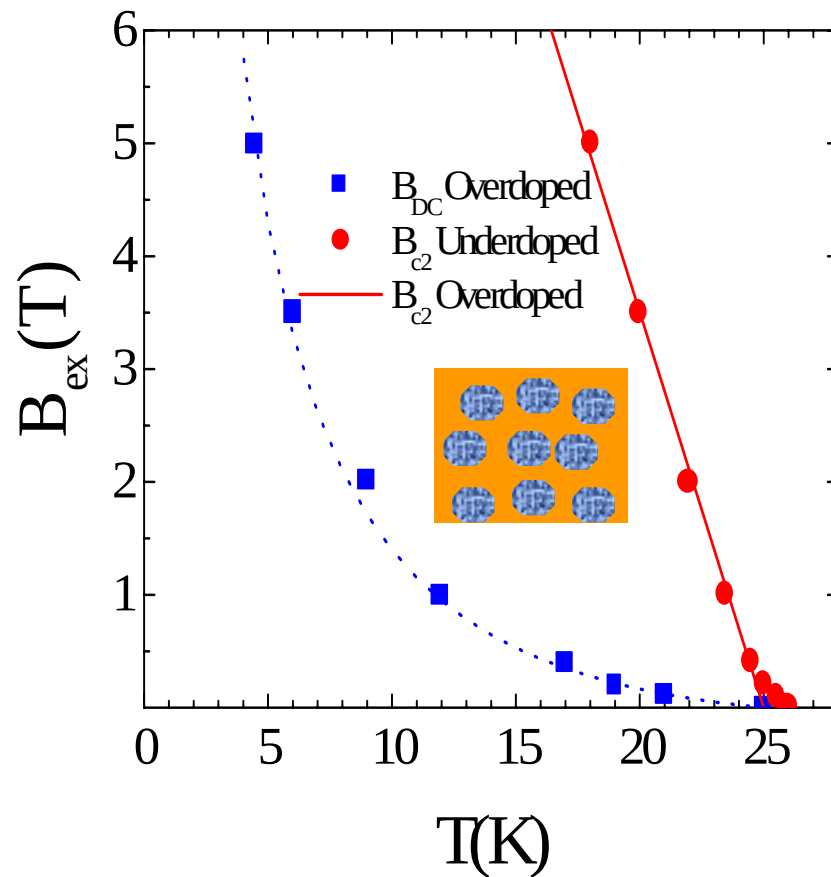
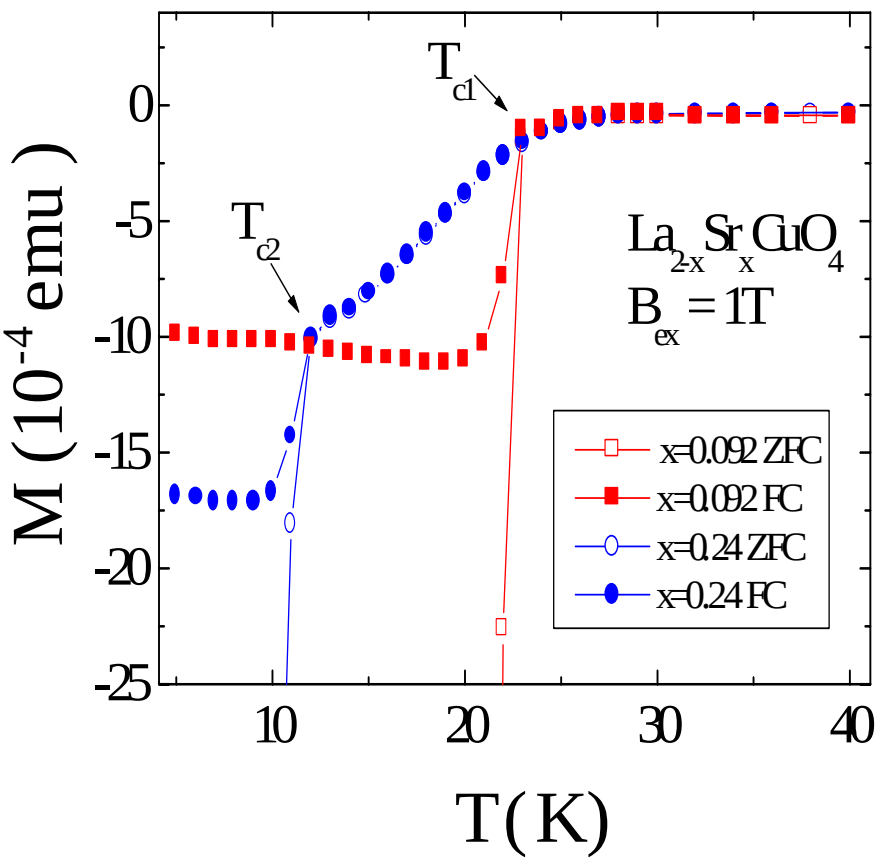


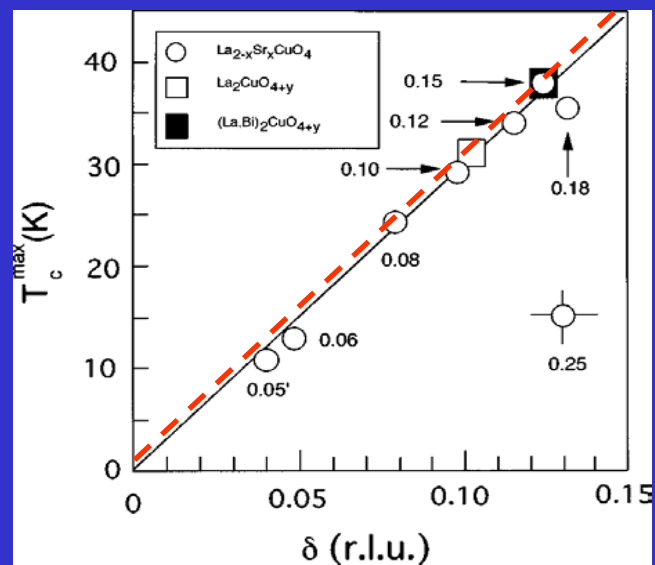
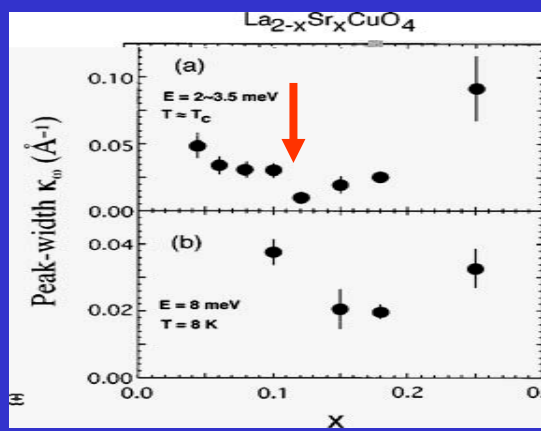
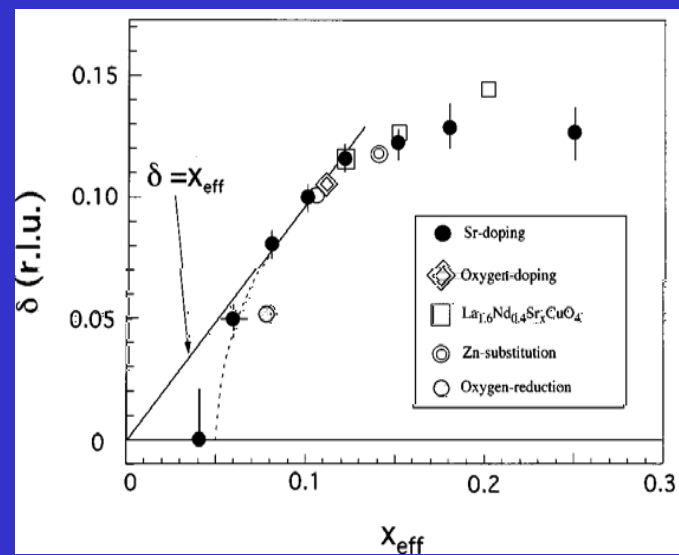
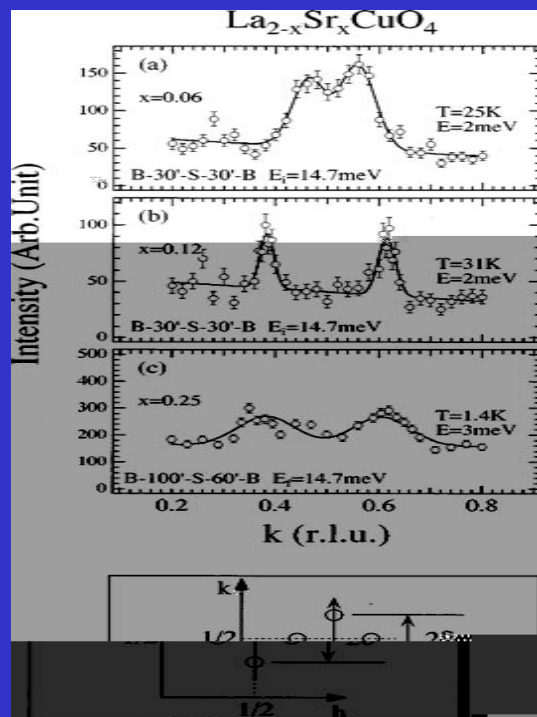
derived H_{c2} extremely small,
unreasonable!

$$\frac{\gamma(H)}{\gamma_0} = \frac{\Delta_0}{8\gamma_0} \left(\frac{H}{H_0} \right)^2 \log \left[\frac{\pi}{2} \left(\frac{H}{H_0} \right)^2 \right]$$

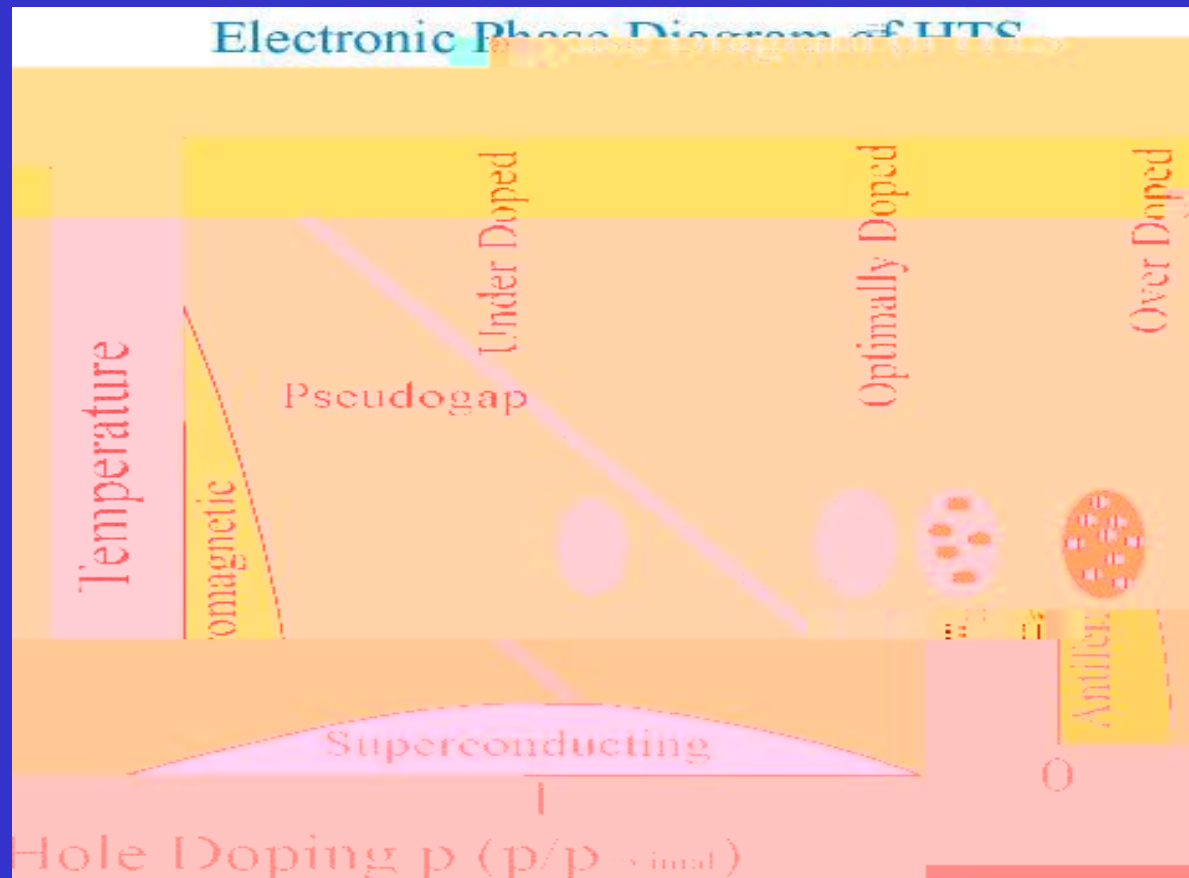
Failure of the fitting !

Data from $\text{La}_{2-x}\text{Sr}_x\text{CuO}_6$ single crystals





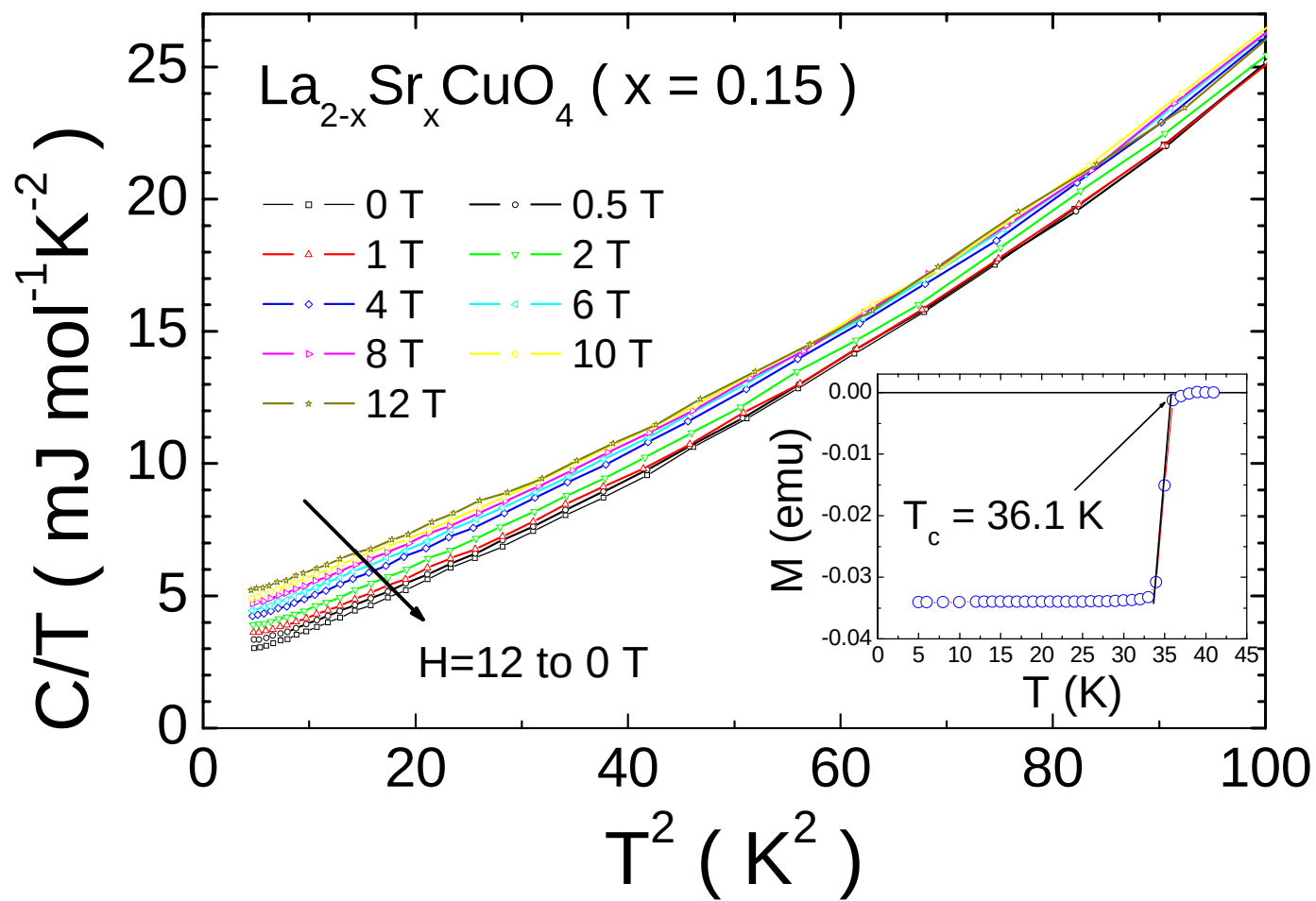
*Proposed Picture for
Electronic Phase Diagram (By Hai-Hu Wen 2000)*



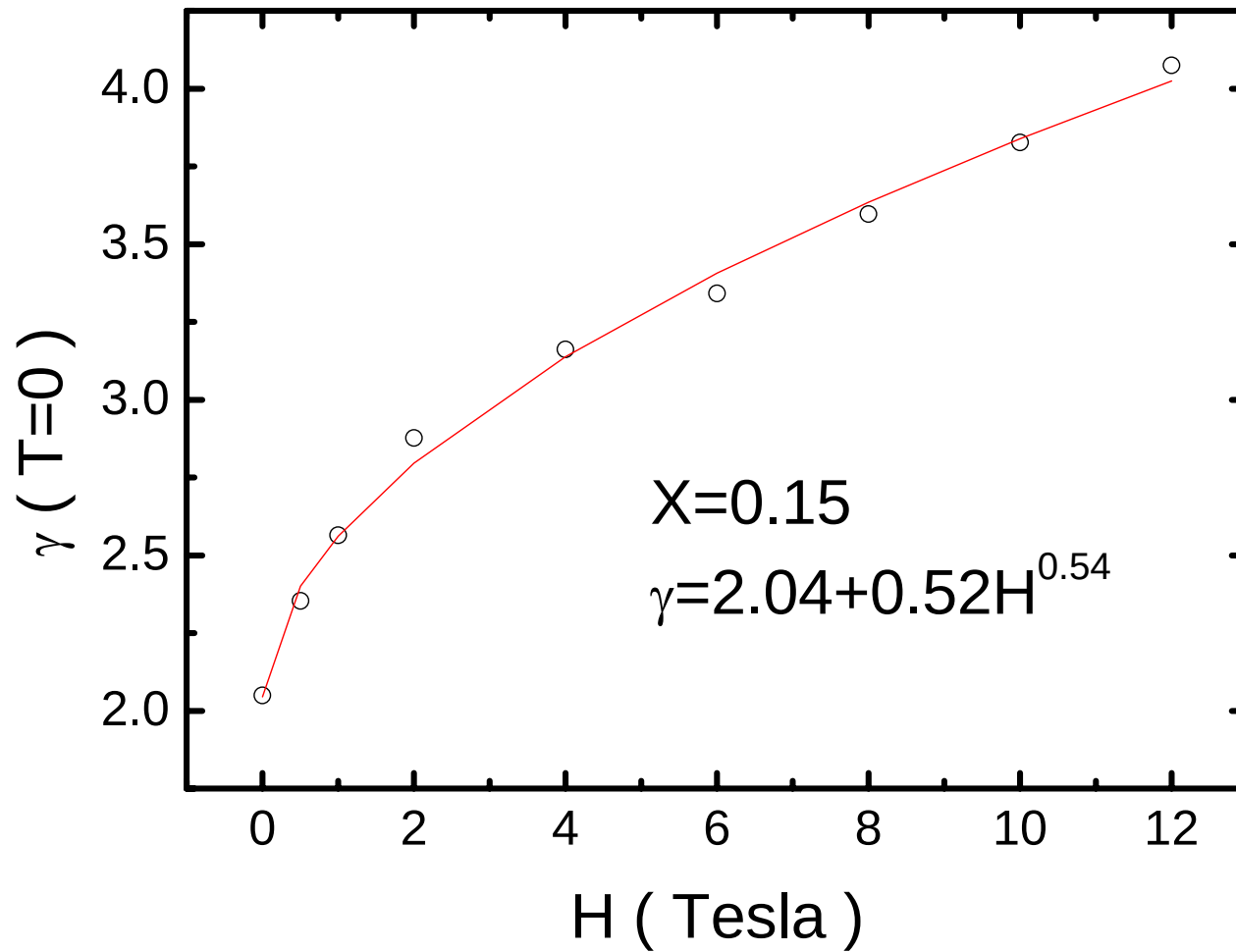
Similar conclusion: S. Wakimoto, et al., Phys. Rev. Lett. 98, 247003(2007)

Underdoped region:

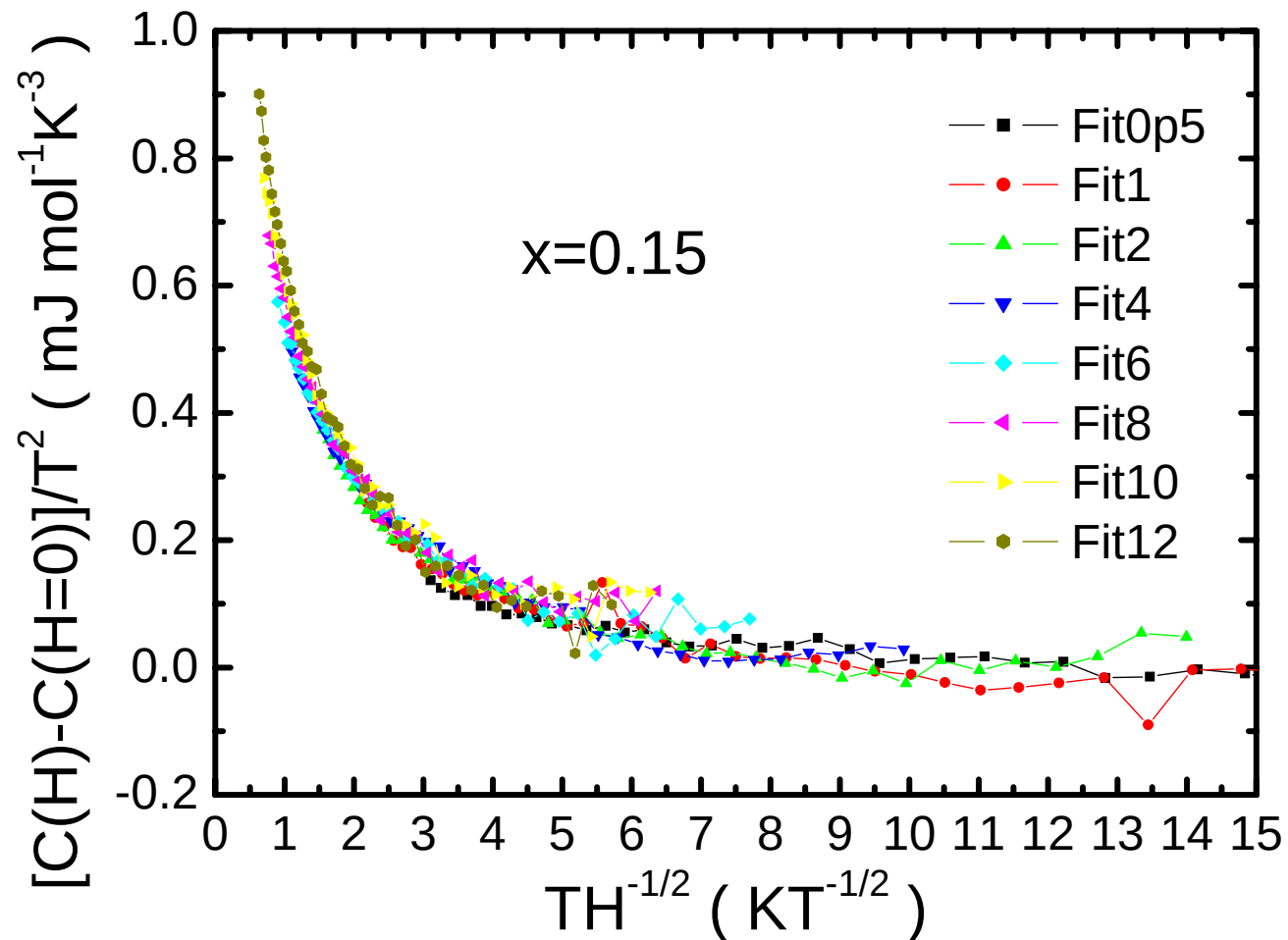
**Correlation between pseudogap
and superconductivity**



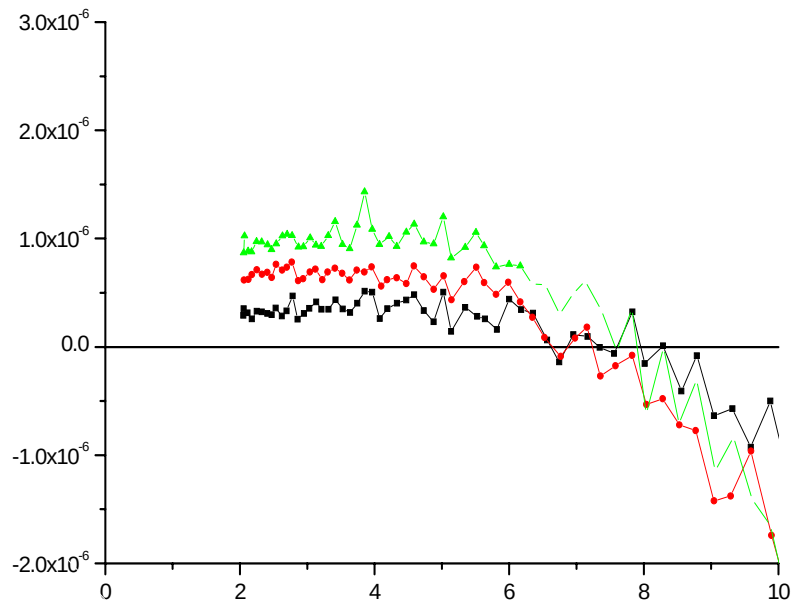
$x=0.15$



Pure d-wave scaling $x=0.15$

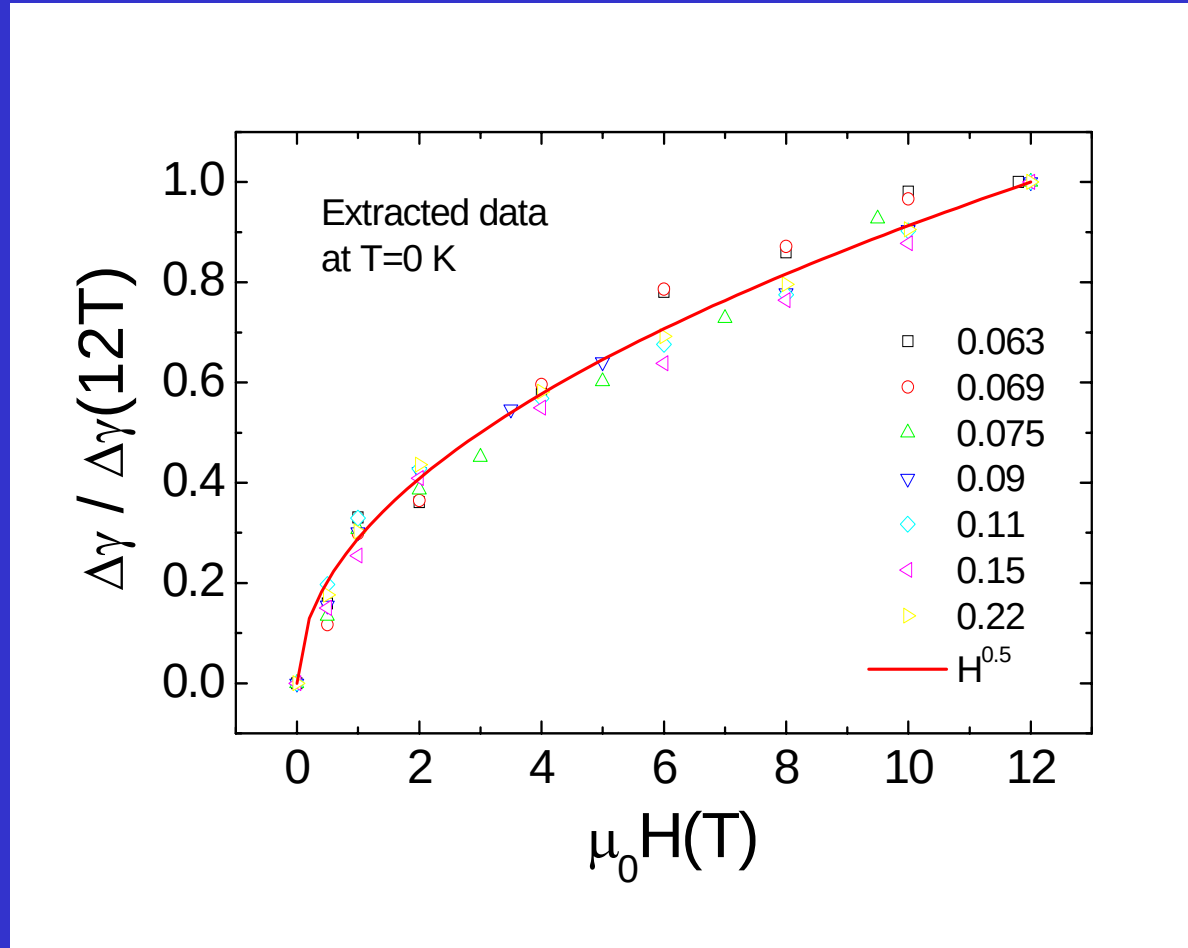


LaSrCuO, $p=0.063$



Good scaling to Volovik's relation at $T=0$ for all doping concentrations:

$$\gamma = \sqrt{\mu_0 H(T)}$$

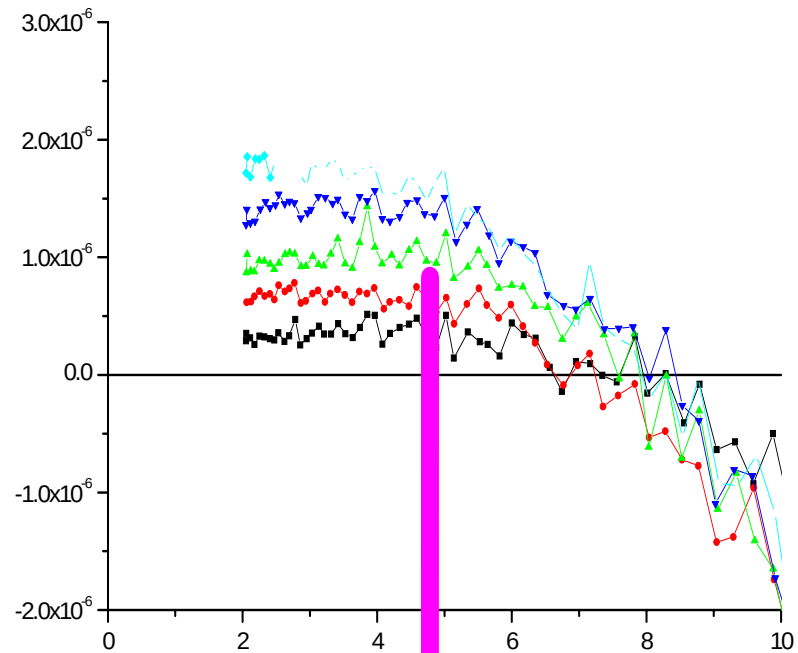


H. H. Wen, et al., Phys. Rev. B70, 214505(2004).

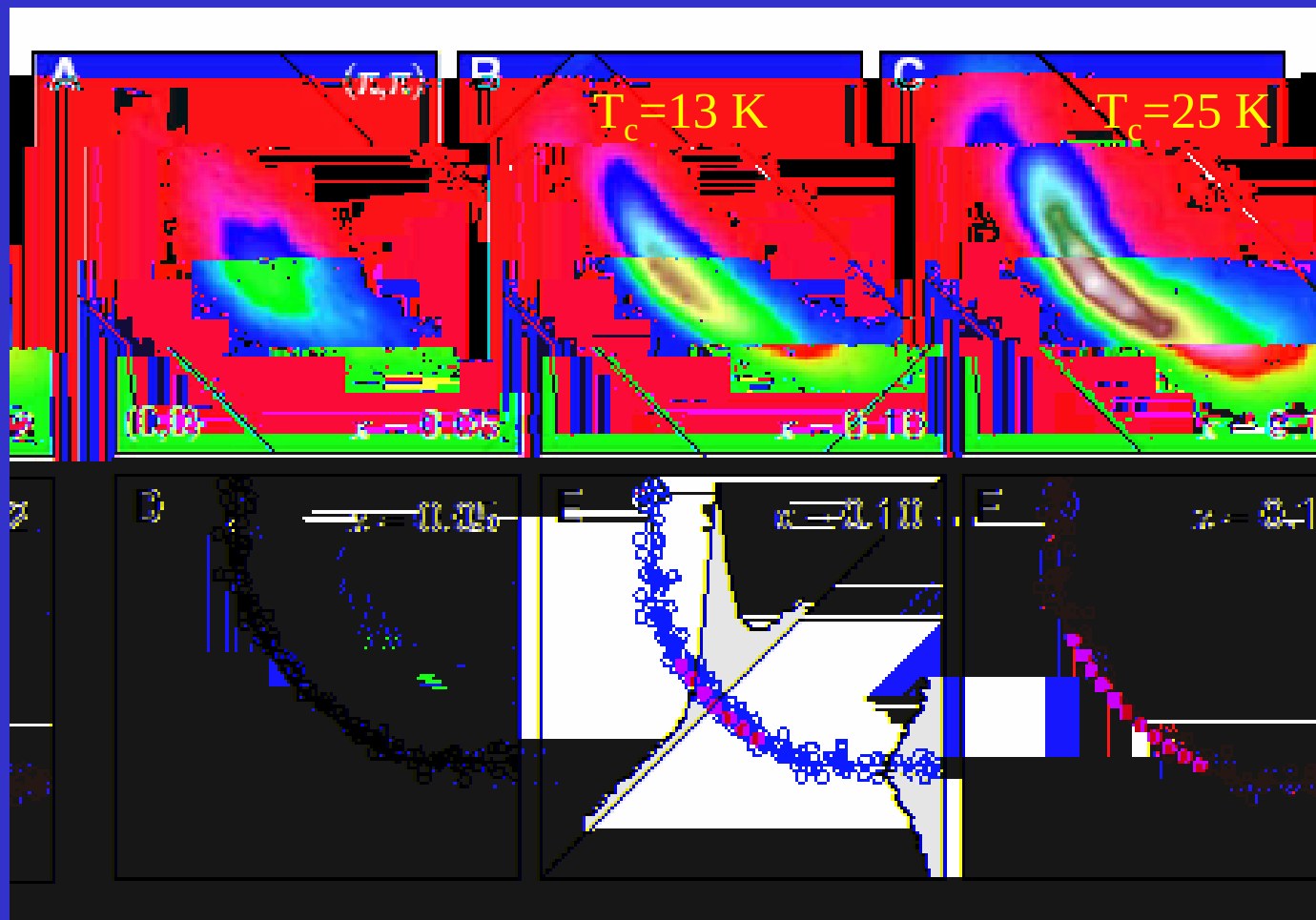


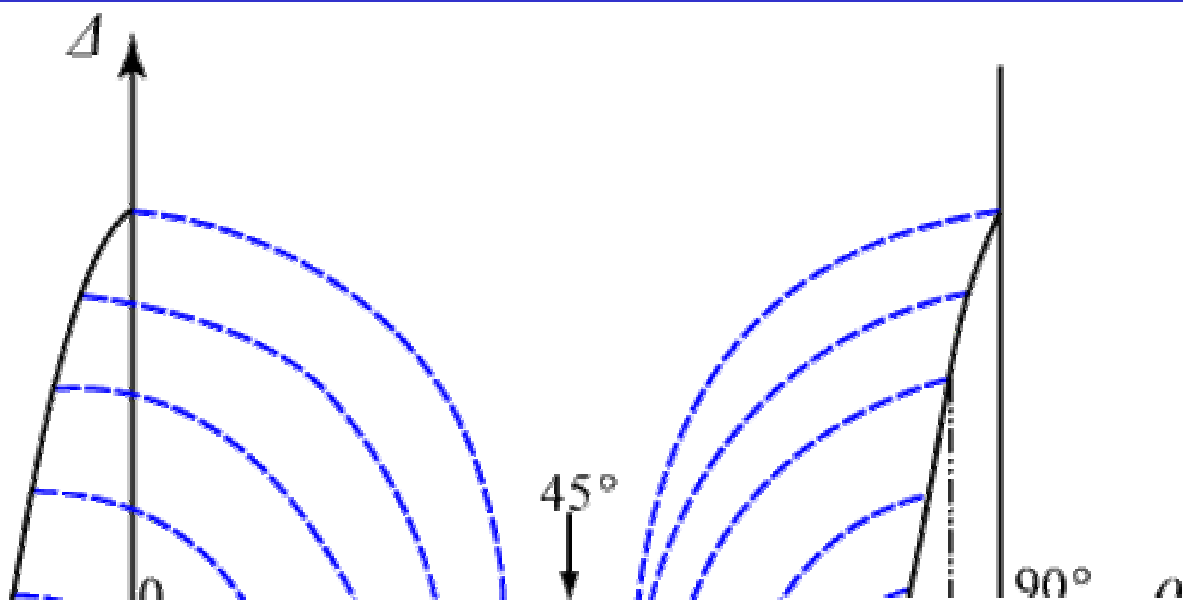
LaSrCuO, $p=0.063$

$p=0.08$

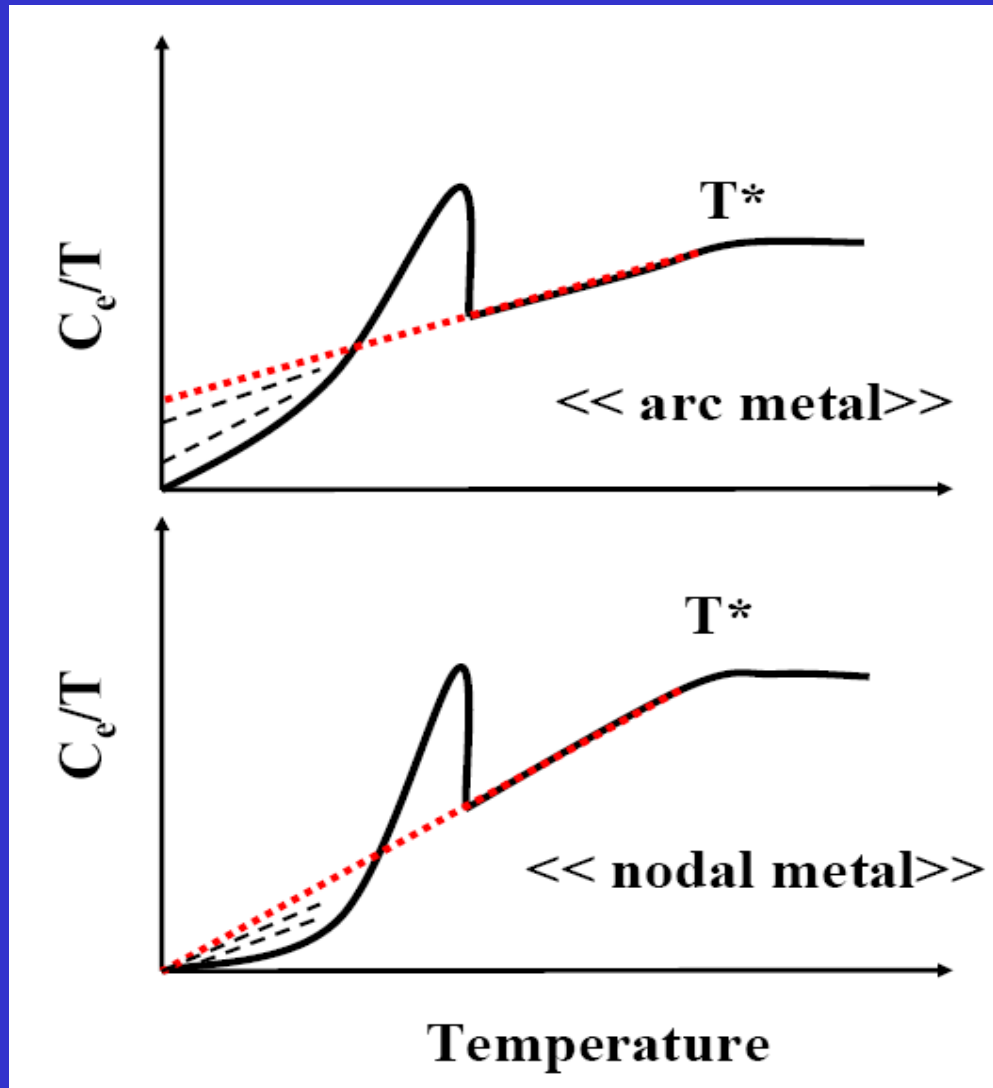


Na-CaCuOCl Single crystals





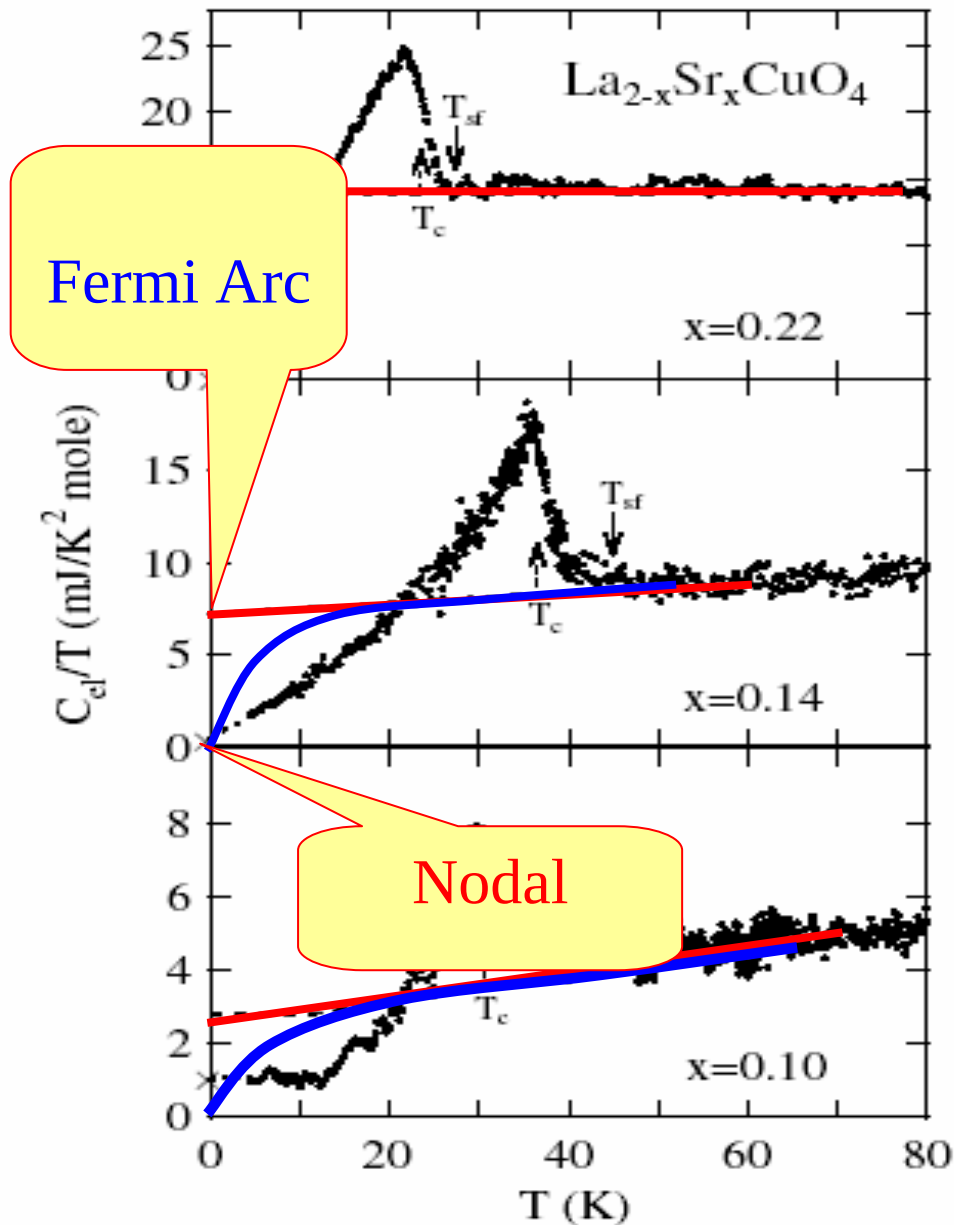
There should be Fermi arcs in the pseudogap state if the superconductivity would be suppressed completely!



$$\gamma \approx (\gamma_0 + \gamma_{\text{arc}})$$

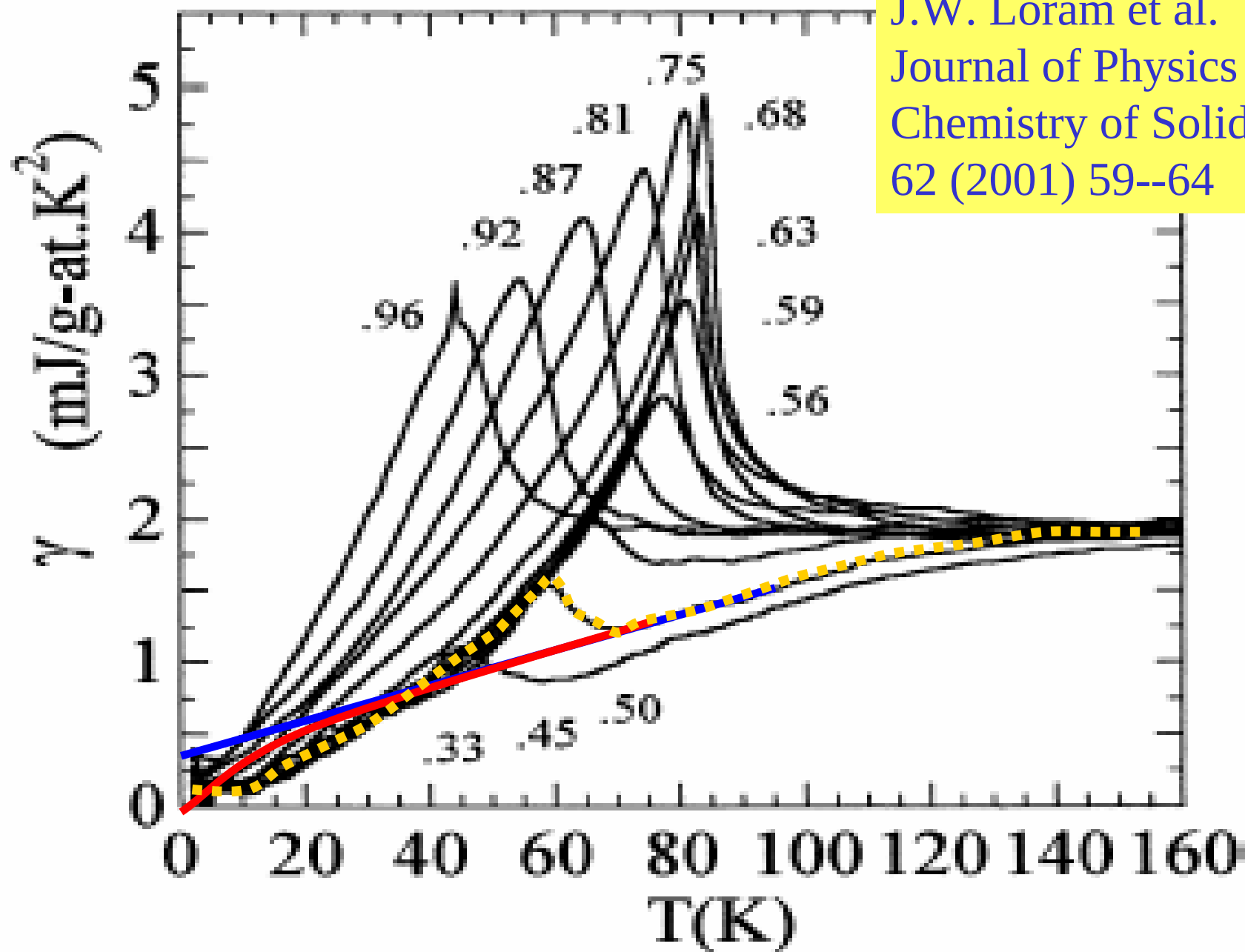
$$\gamma(\theta) = \gamma(0) \int \frac{\theta}{2\pi} \text{Re} \frac{1}{\sqrt{1 - \Delta_0^2 \cos^2(2\theta)}}$$

$$\gamma = \gamma_{\text{arc}}$$



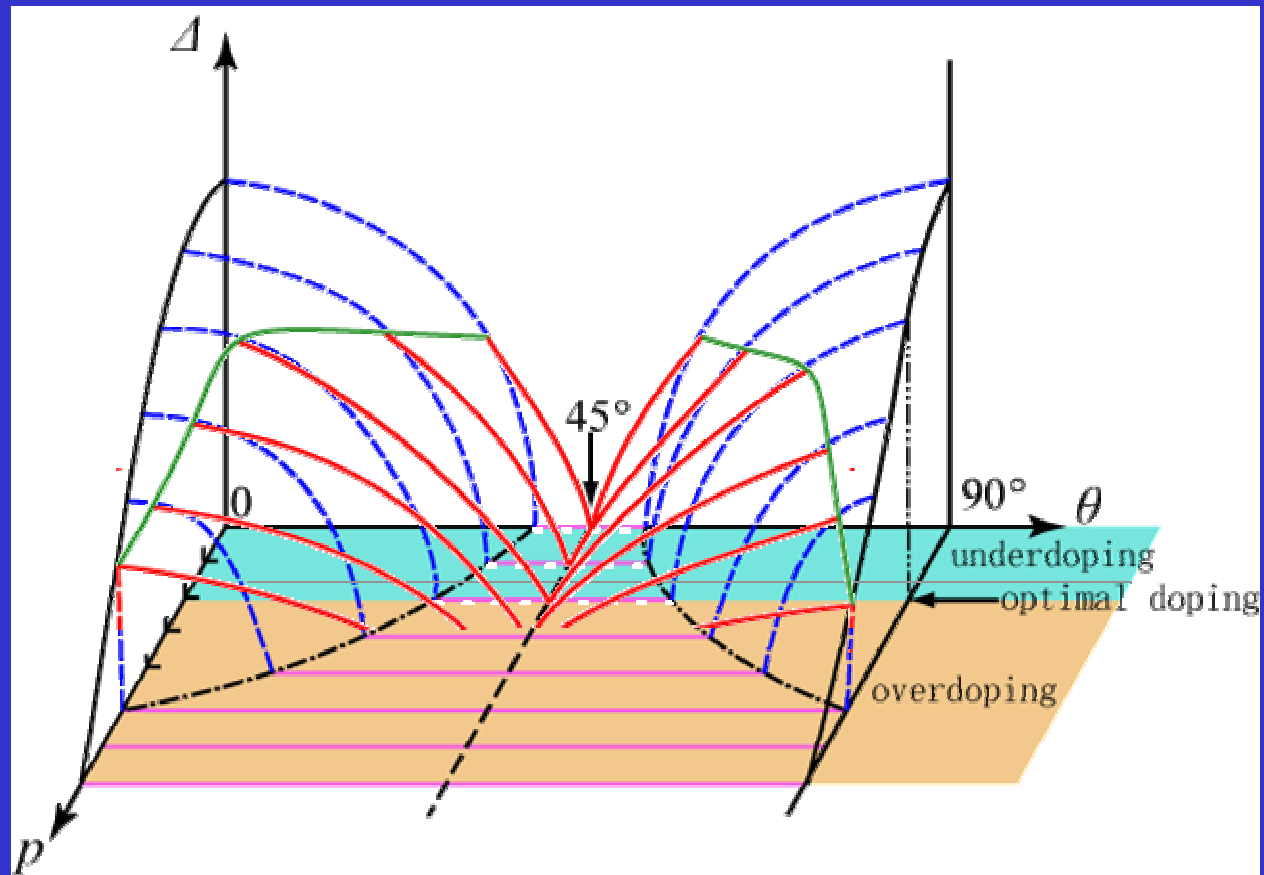
T. Matsuzaki, N. Momono, M. Oda, M. Ido, J. Phys. Soc. Japn. 73, 2232(2004).

From the entropy point of view, the nodal metal picture for the PG phase is not reasonable.



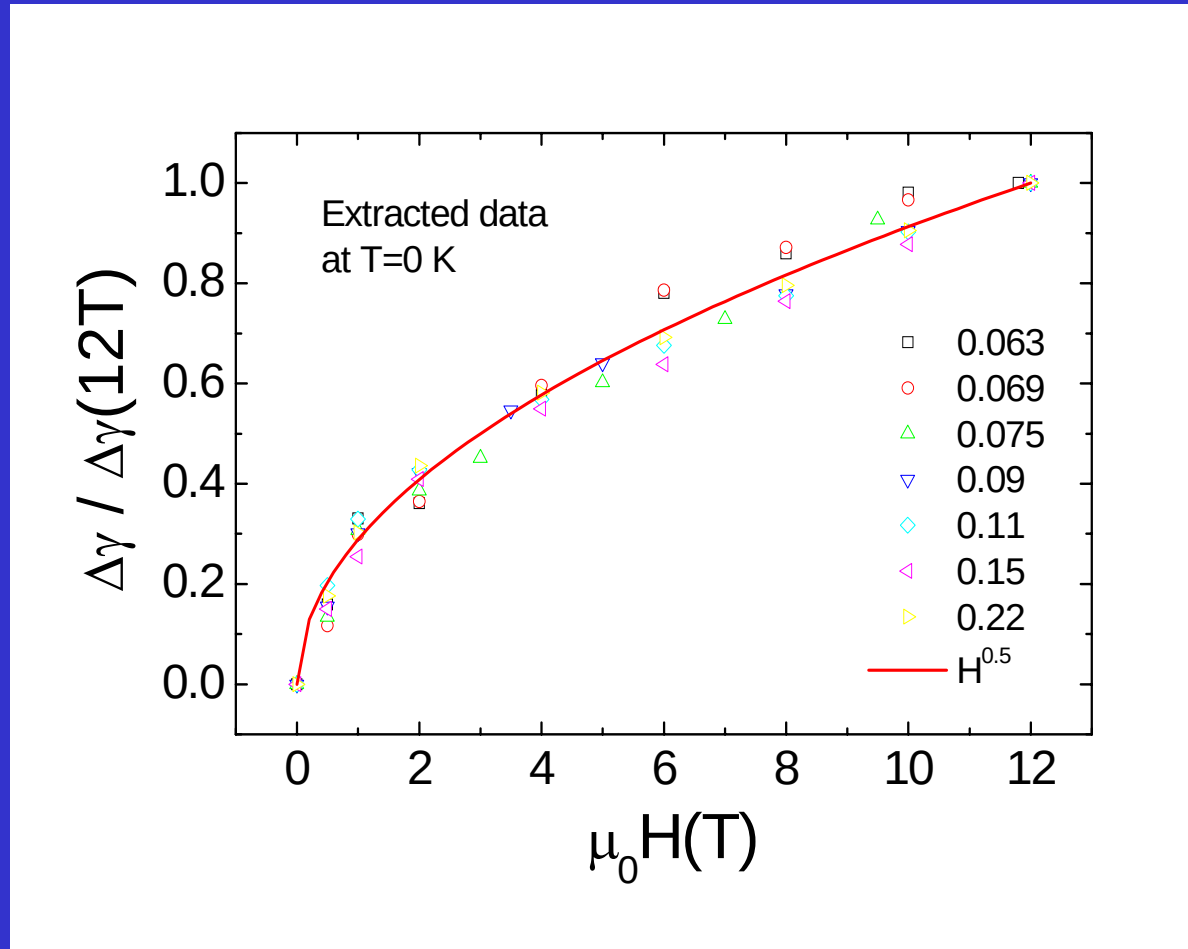
J.W. Loram et al.
Journal of Physics and
Chemistry of Solids
62 (2001) 59--64

What governs the
superconducting
transition temperature?

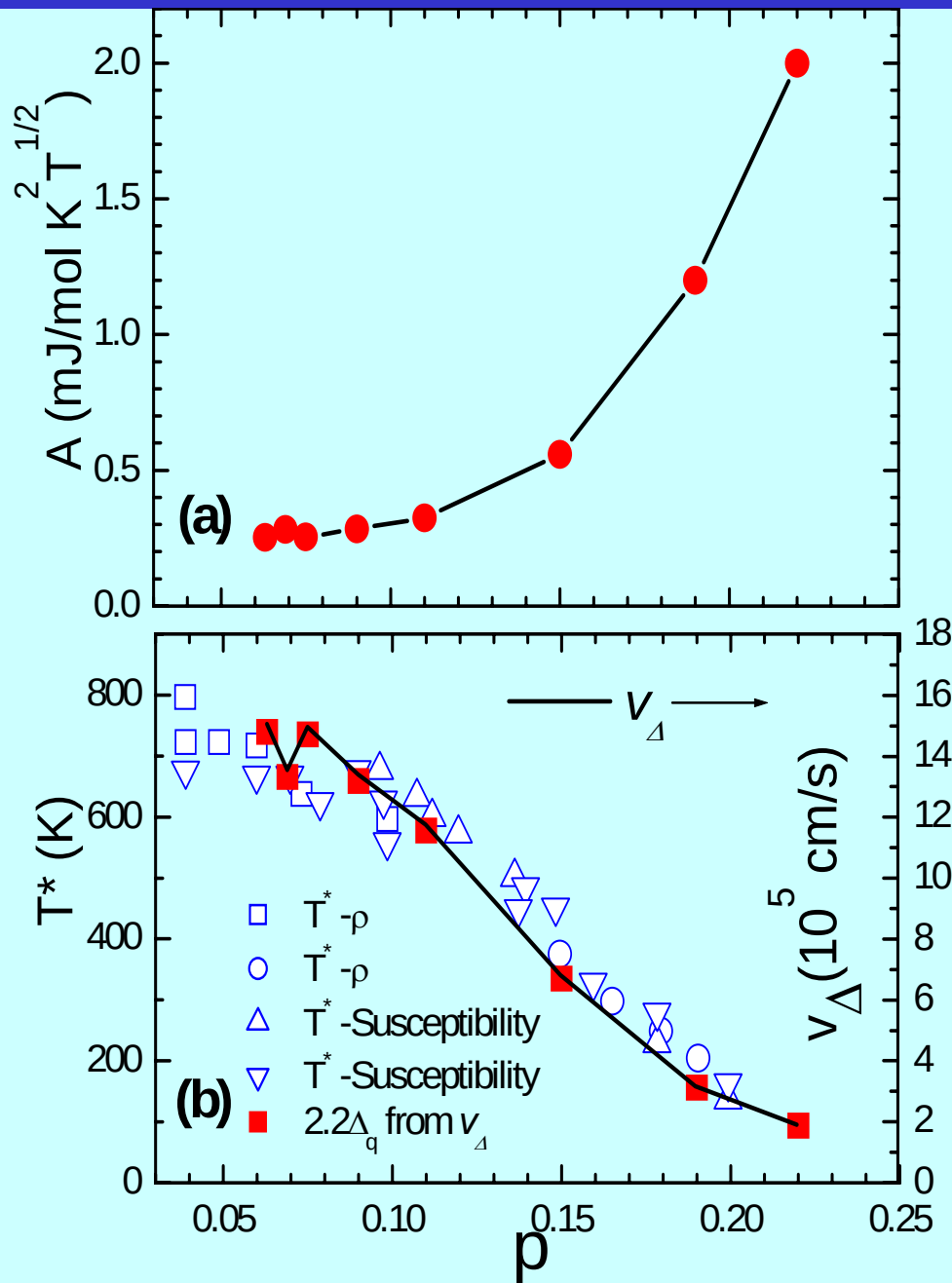


Good scaling to Volovik's relation at $T=0$ for all doping concentrations:

$$\gamma = \sqrt{\mu_0 H(T)}$$



H. H. Wen, et al., Phys. Rev. B70, 214505(2004).



$$\gamma = \sqrt{\frac{2}{\pi}}$$

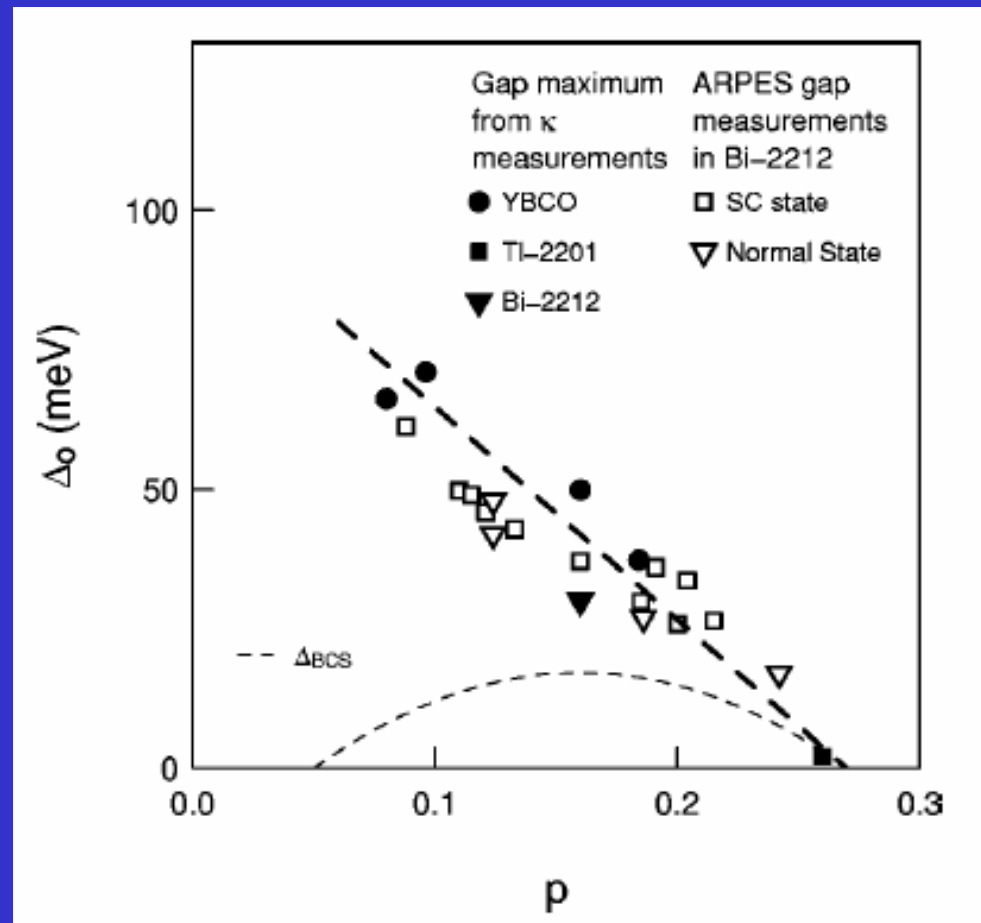
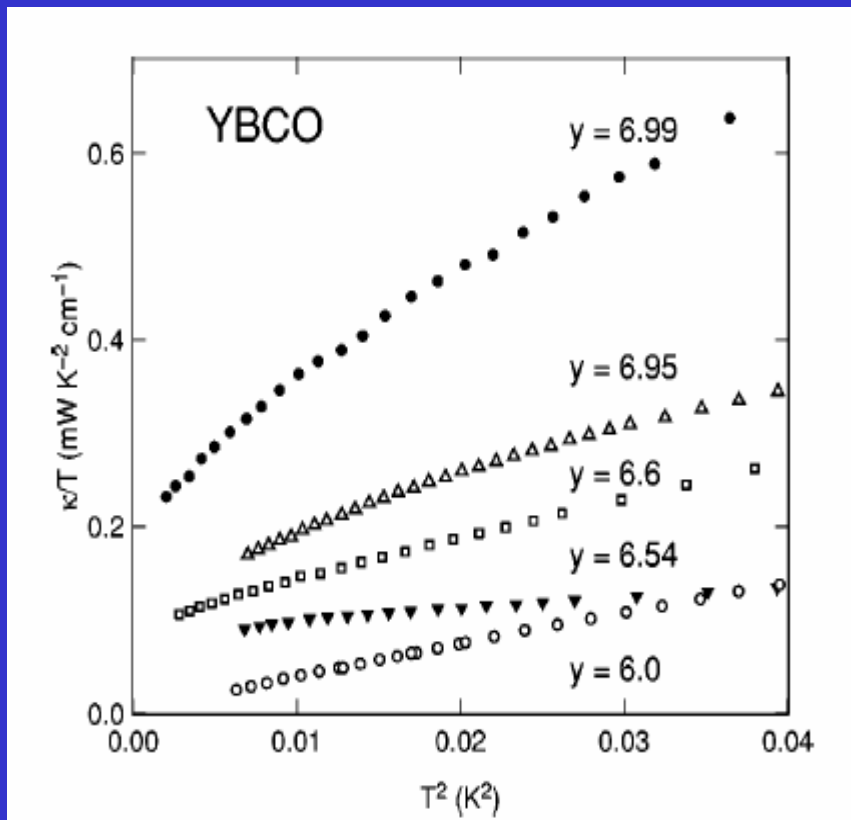
$$= \alpha \frac{4}{3\hbar} \sqrt{\frac{\pi}{\Phi_0}} \frac{1}{\Delta}$$

$$\Delta = \left[\frac{\Delta}{\phi} \right] / \hbar$$

$$\Delta = \Delta \hbar / 2$$

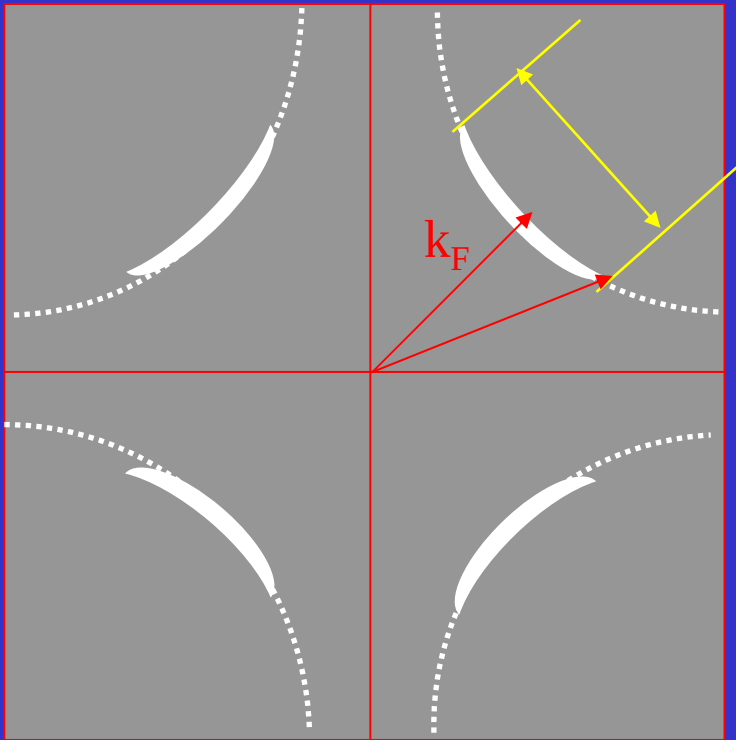
$$2.2\Delta = *$$

Open symbols: T. Timusk, B. Statt, Rep. Prog. Phys. 62, 61(1999).



$$\frac{\kappa_0}{3\hbar} = \frac{2}{\Delta} \left(\frac{1}{\Delta} + \frac{\Delta}{\Delta} \right)$$

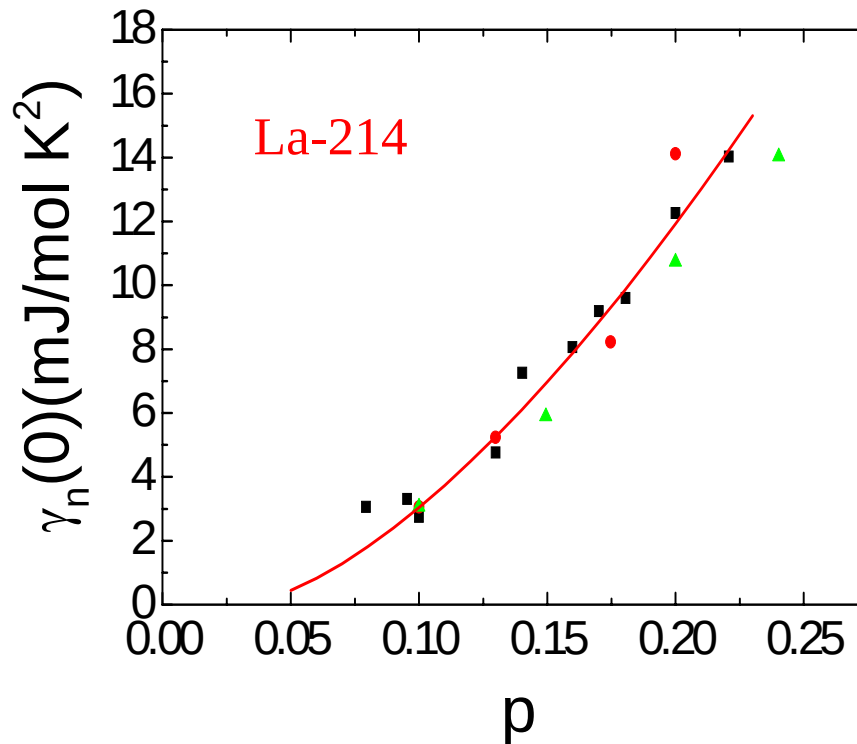
$$\Delta = \frac{1}{2} \Delta \hbar$$



$$\gamma(0) = 2 \quad / 3\hbar$$

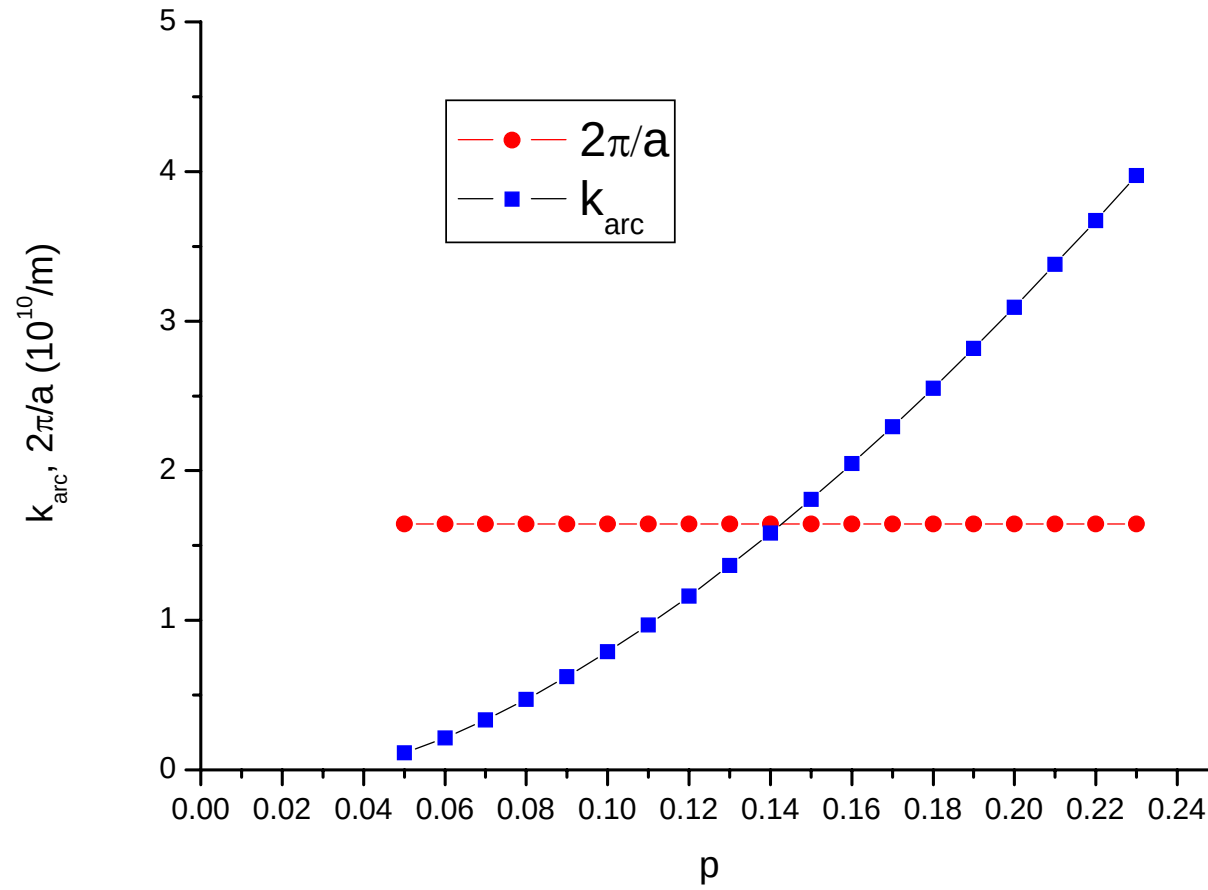
$$= \alpha^{-1} \frac{3\hbar^2}{4} \frac{\gamma(0)}{3} \Delta = \beta \gamma(0) \Delta$$

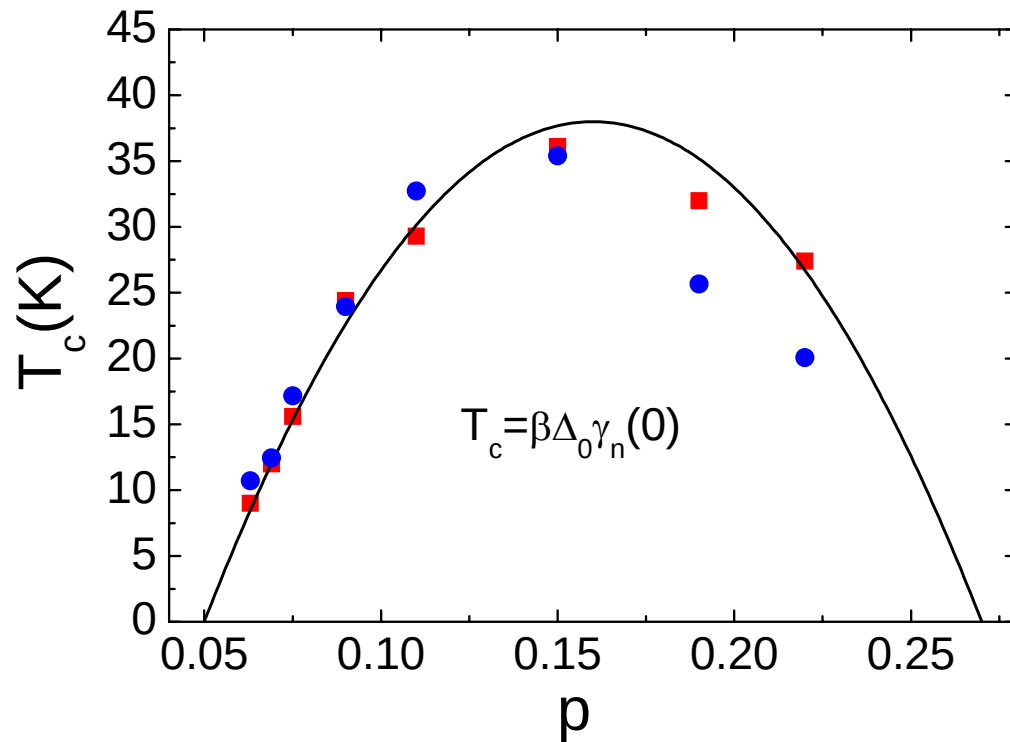
$$\gamma_n(0) = 182.6(-0.03)^{1.54}$$



T. Matsuzaki, N. Momono, M. Oda, M. Ido, J. Phys. Soc. Japn. 73, 2232(2004).

Comparison between k_{arc} and $2\pi/a$





$$\beta = 0.7445 \quad \text{3} \quad /$$

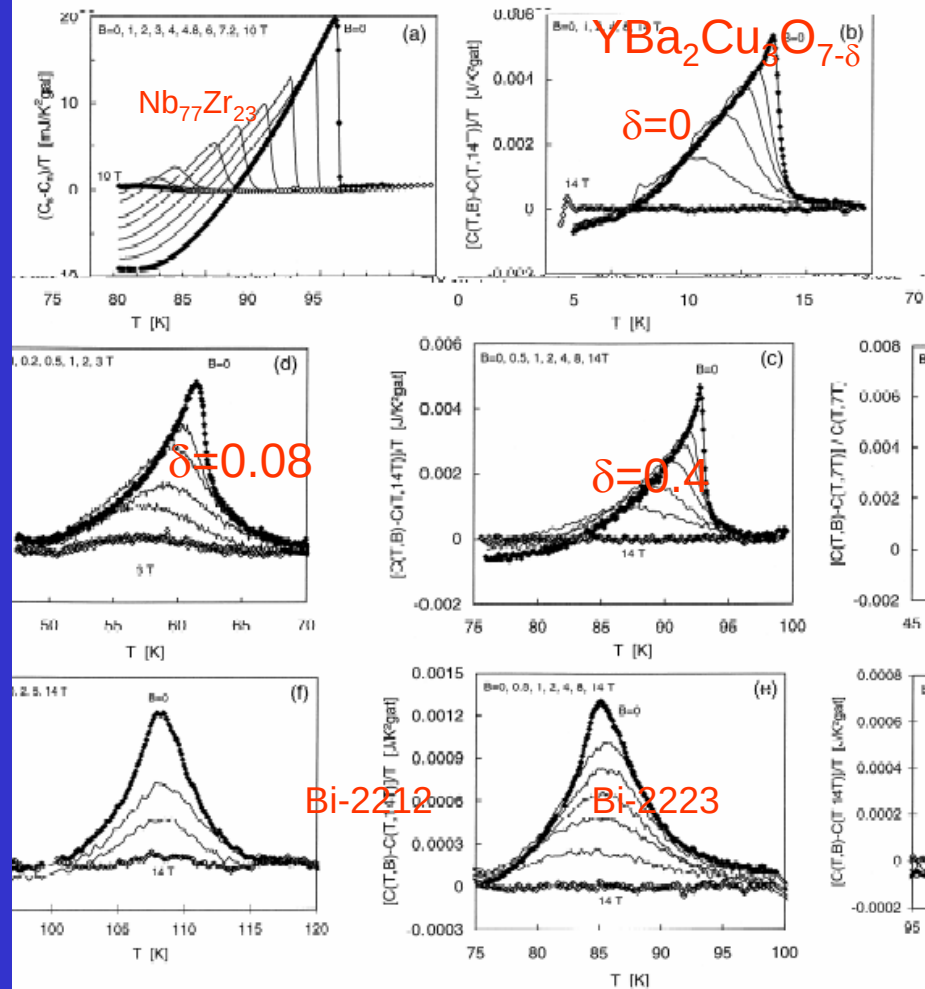
$$= 2.73 \times 10^7 \quad /$$

$$\alpha = 13.8$$

$$= \alpha^{-1} \frac{3\hbar^2}{4} \frac{\gamma(0)}{3} \Delta = \beta \gamma(0) \Delta$$

H. H. Wen, et al., Phys. Rev. B 72, 134507(2005).



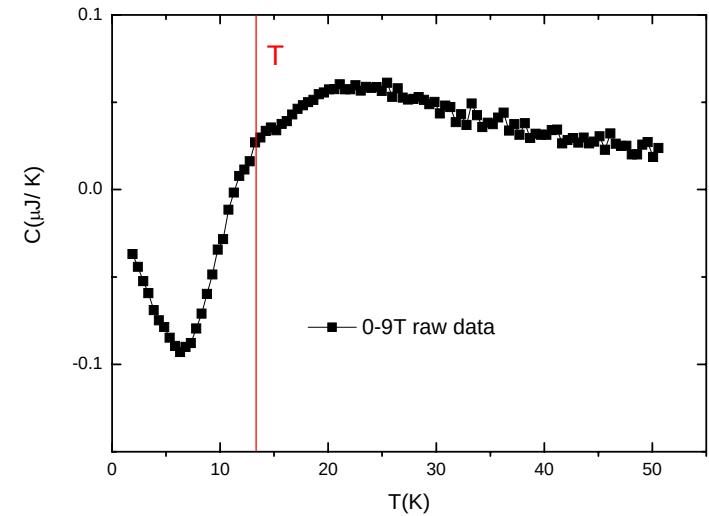
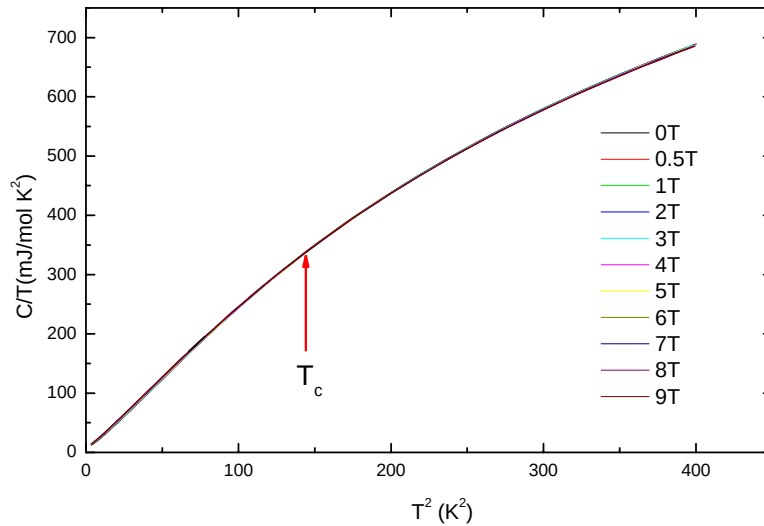


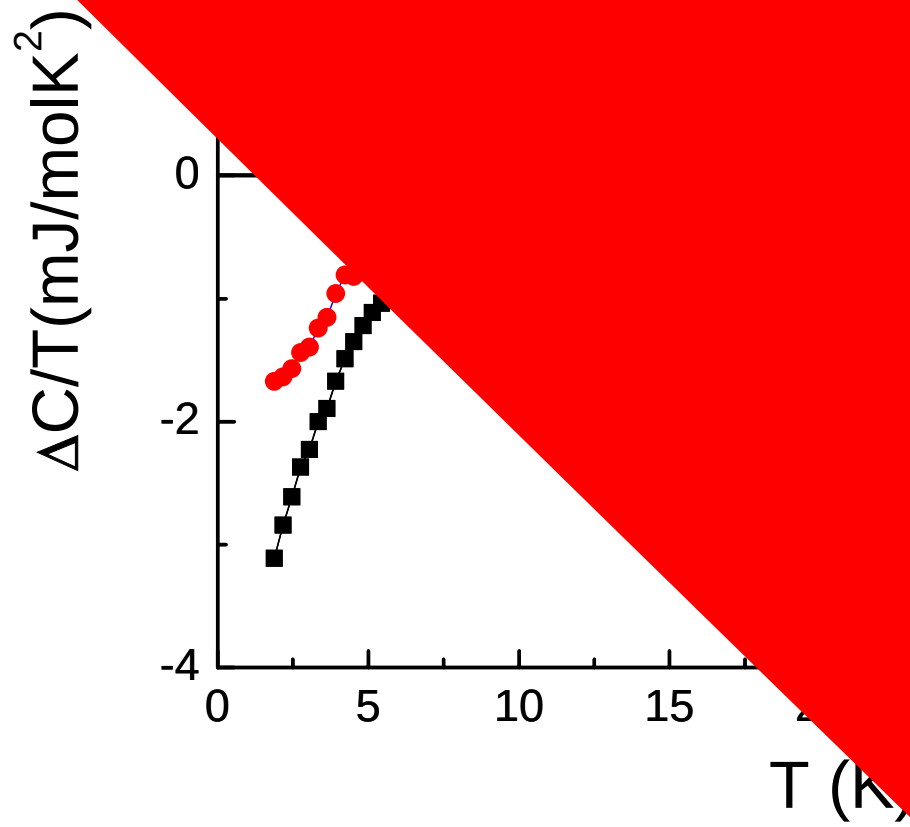
A. Junod, A. Erb, and C. Renner,
Physica C 317-318, 333 (1999).

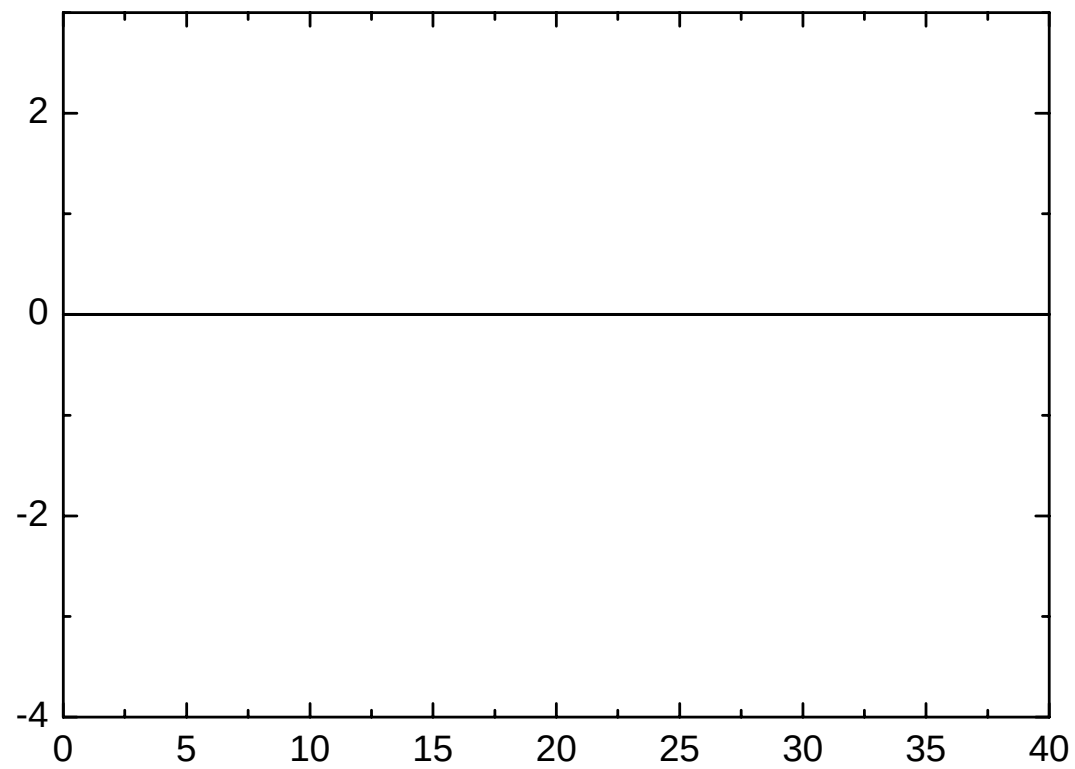


To get useful message from “nothing”

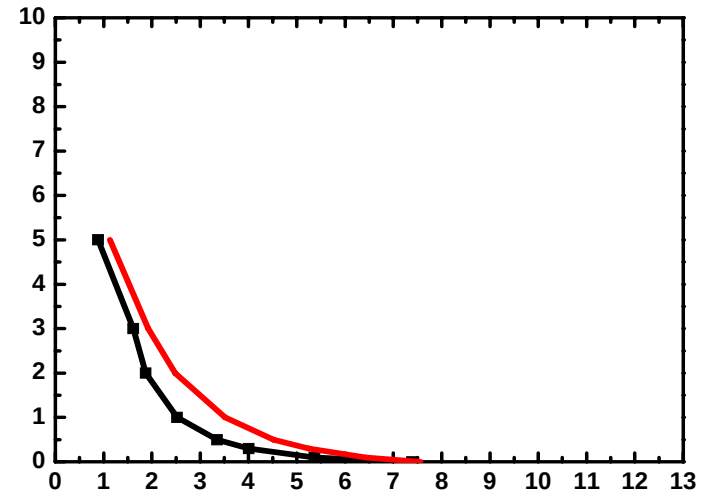
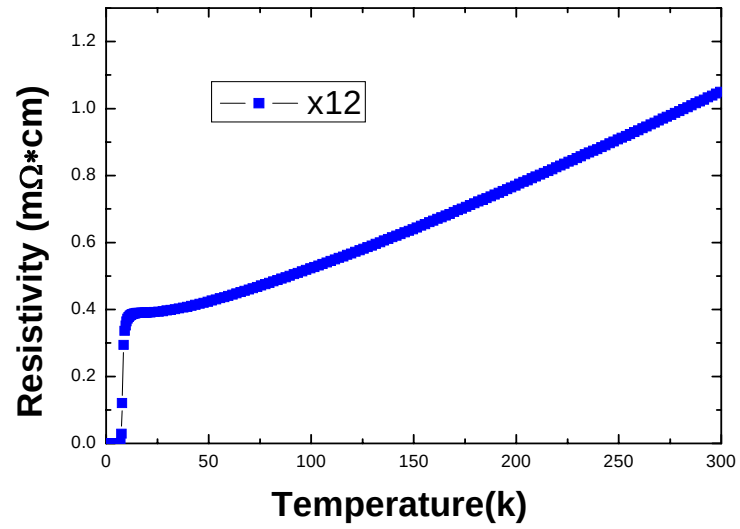
Improved Low-temperature specific heat measurements based on PPMS

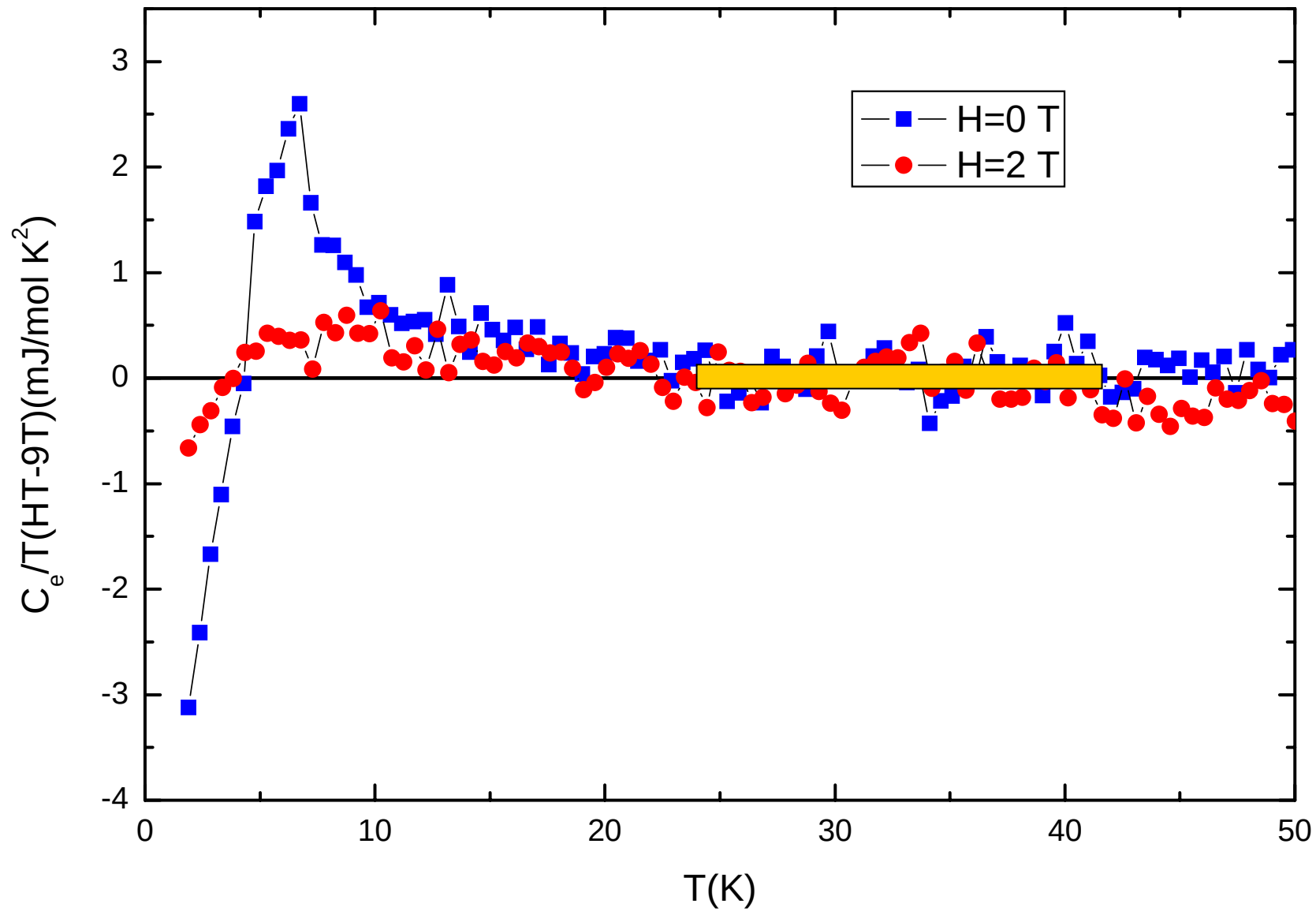


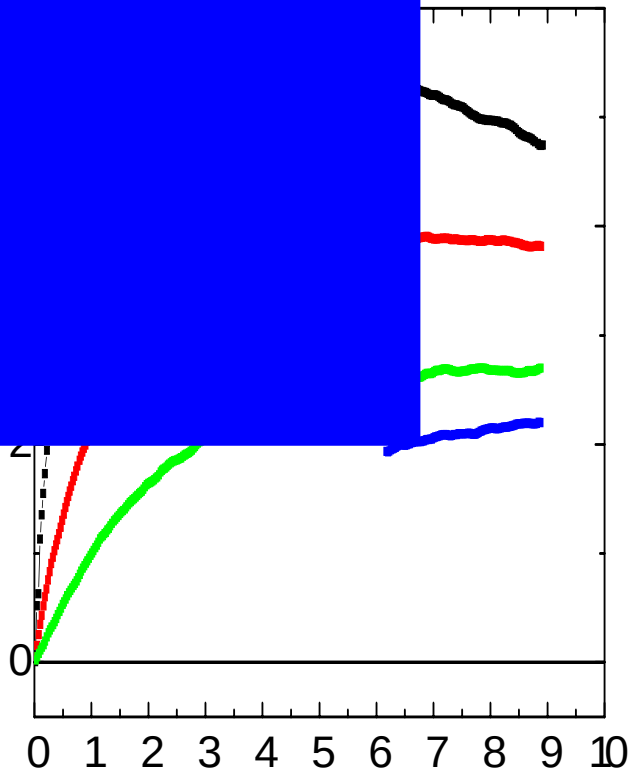




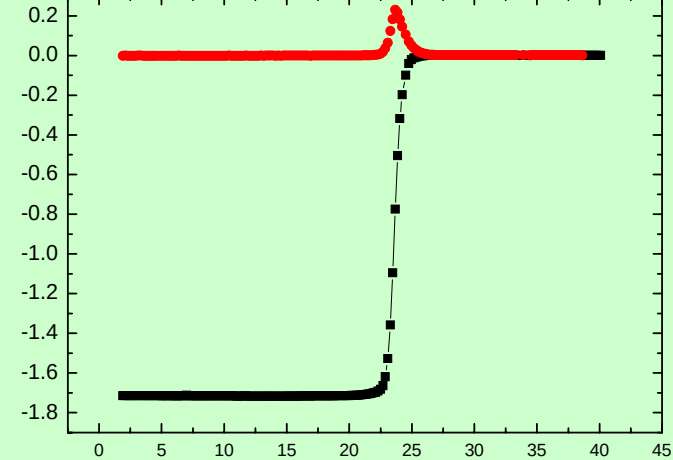
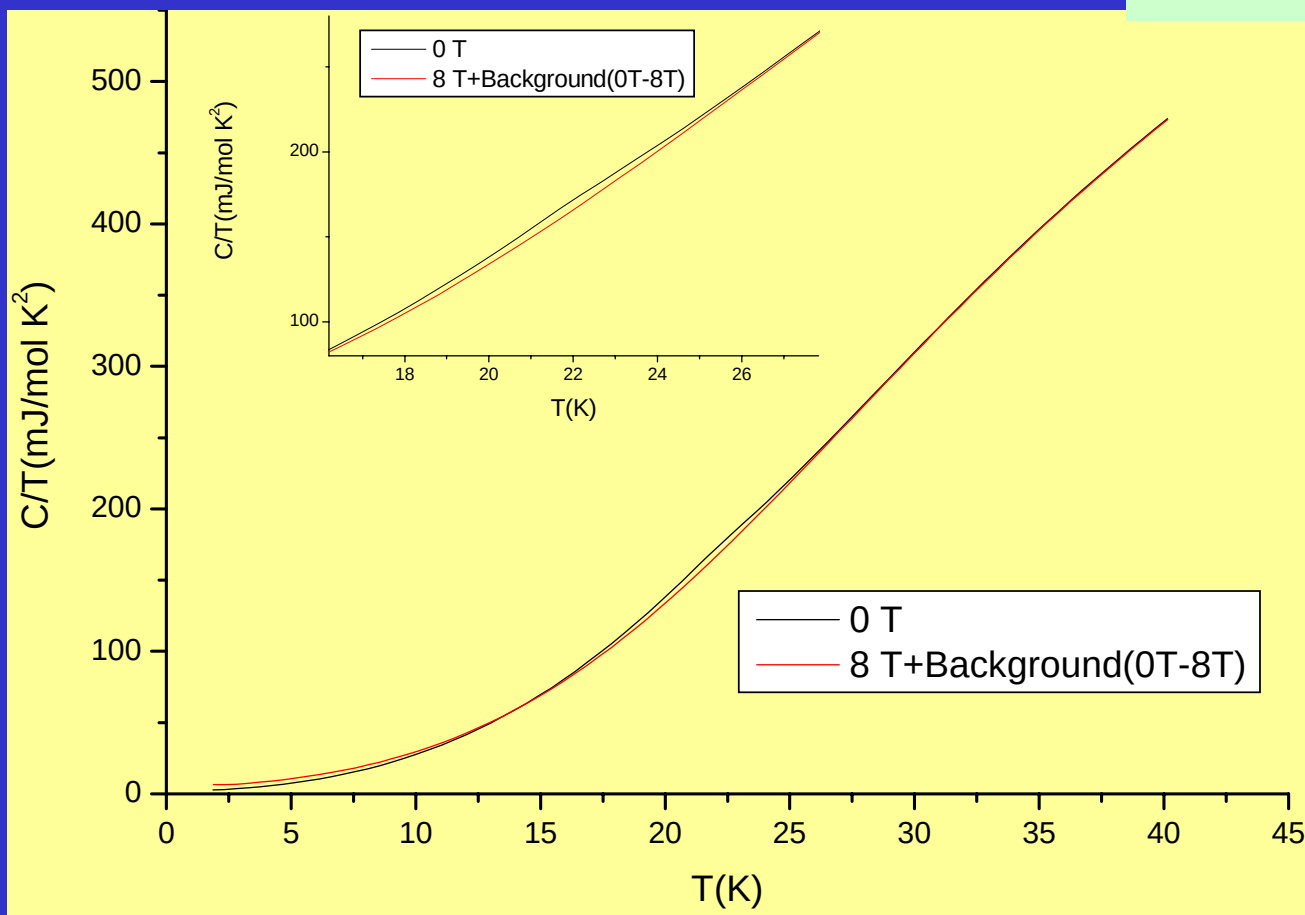
$\text{Bi}_{2+x}\text{Sr}_{2-x}\text{CuO}_6$ single crystals

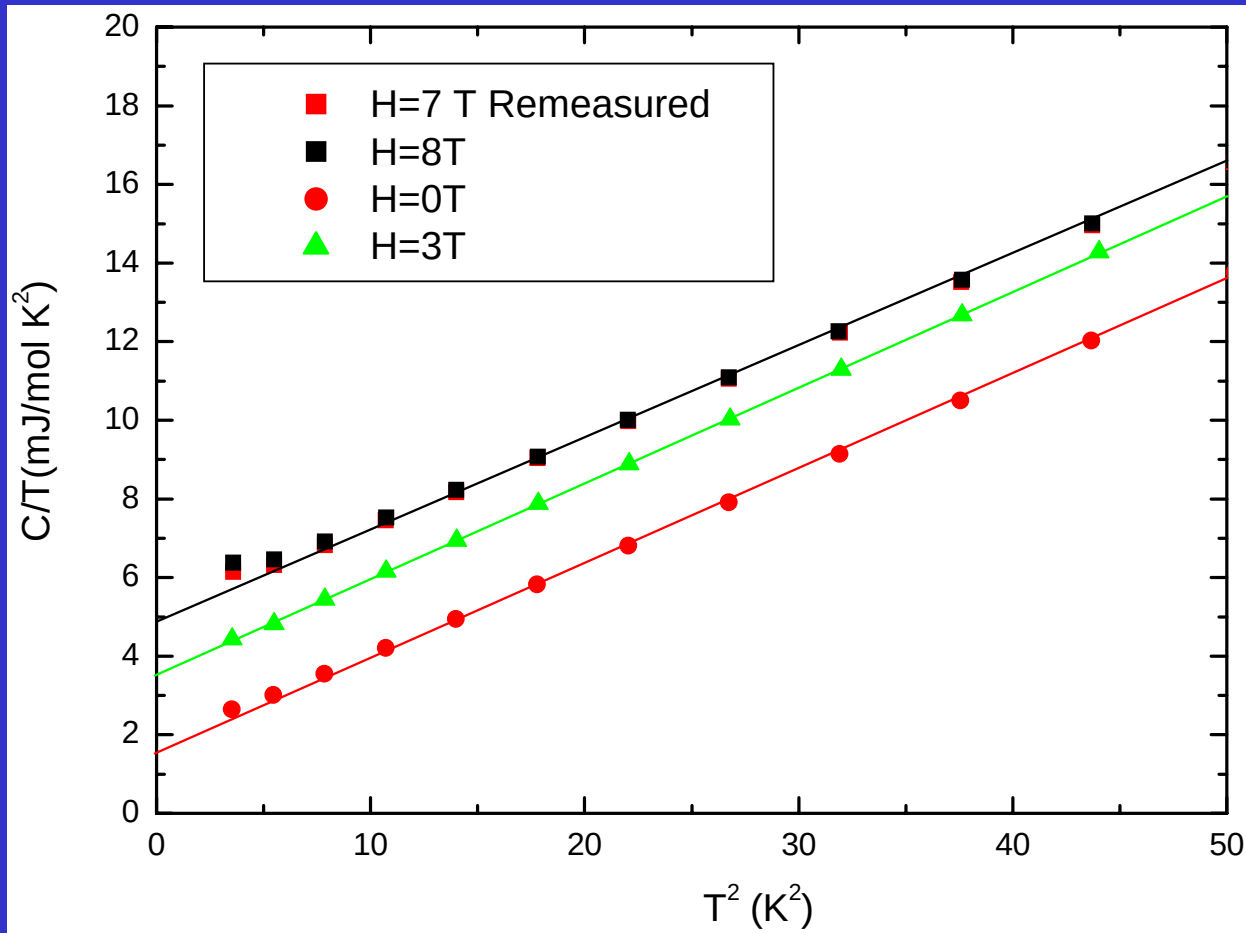












$$C/T = \gamma + \beta T^2$$

$$\gamma_n = 4.88 \text{ mJ/mol K}^2$$

$$\gamma_0 = 1.60 \text{ mJ/mol K}^2 \text{ (perhaps due to the impurity scattering? Nodal gap?)}$$

$$\beta = 0.2337 \text{ mJ/mol K}^4$$

Phonon Mediated Pairing?

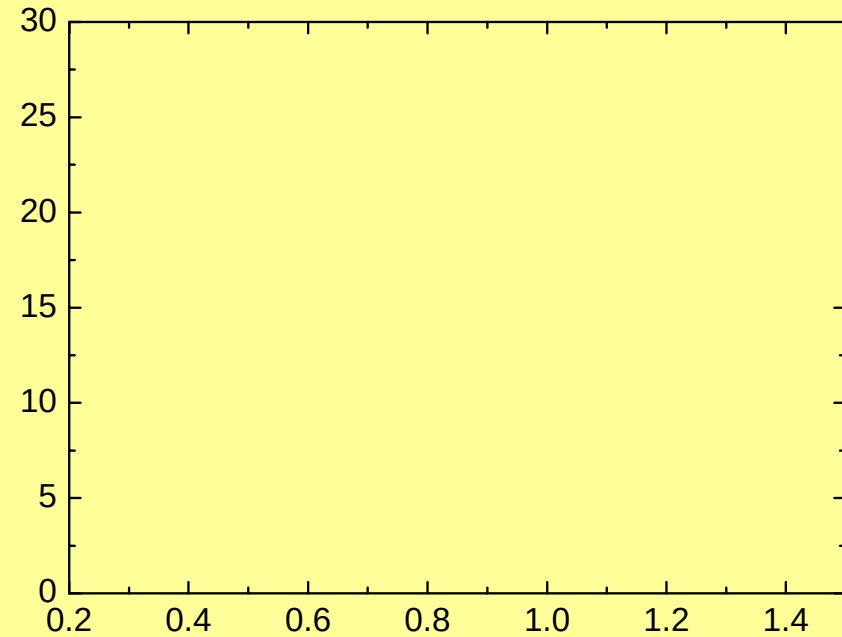
$$\Theta = (12\pi^4 / 5\beta)^{1/3}$$

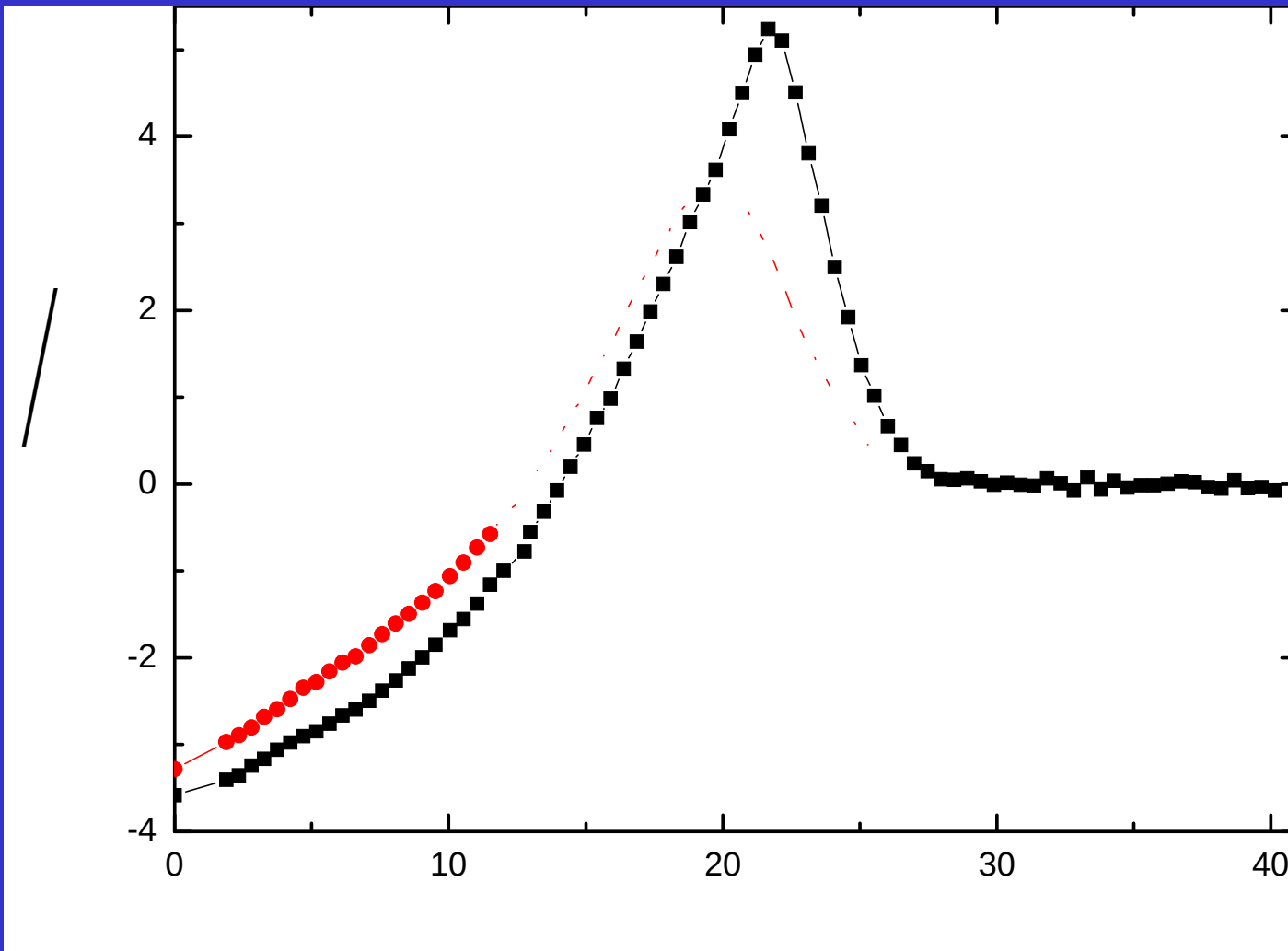
$$\beta = 0.233 / 4$$

$$= 7$$

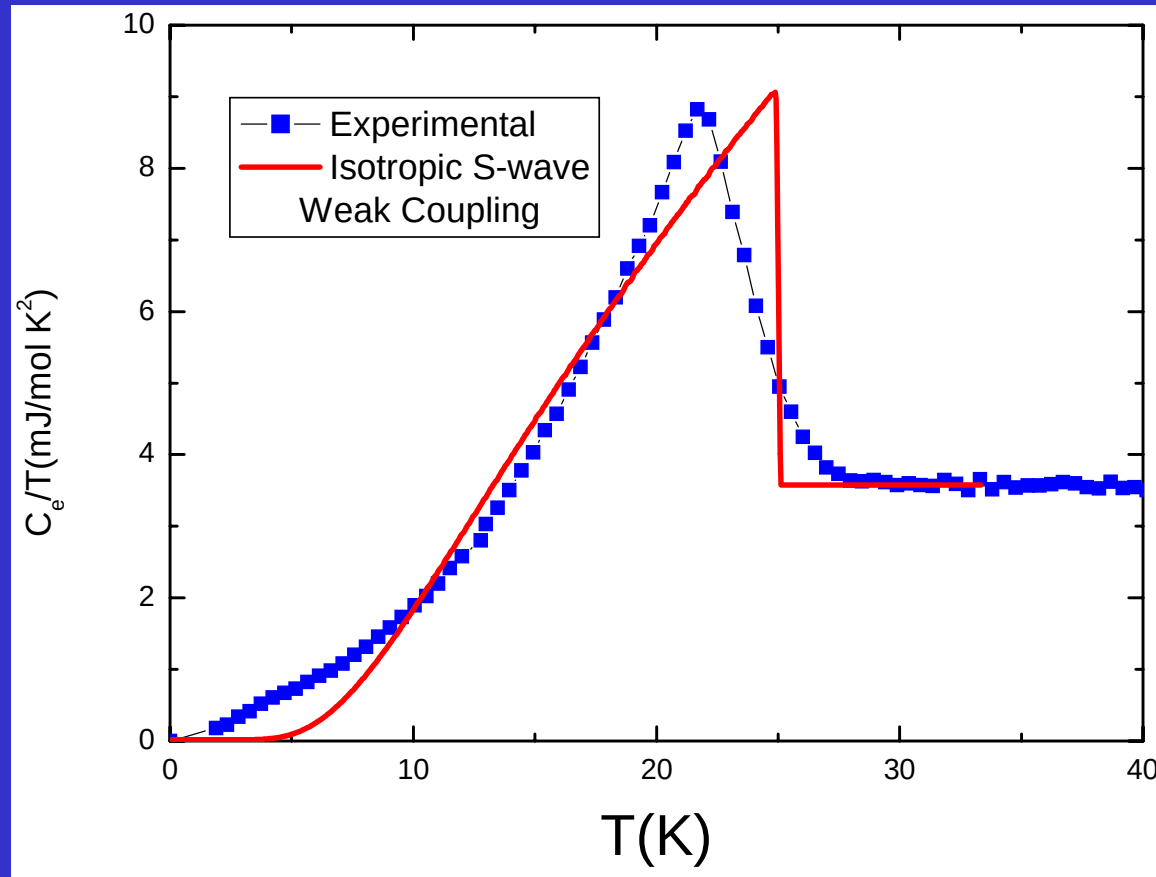
$$\Theta = 387$$

$$= \frac{\Theta}{1.45} \exp \left[- \frac{1.04(1+\lambda)}{\lambda - \mu^* (1 + 0.62\lambda)} \right]$$



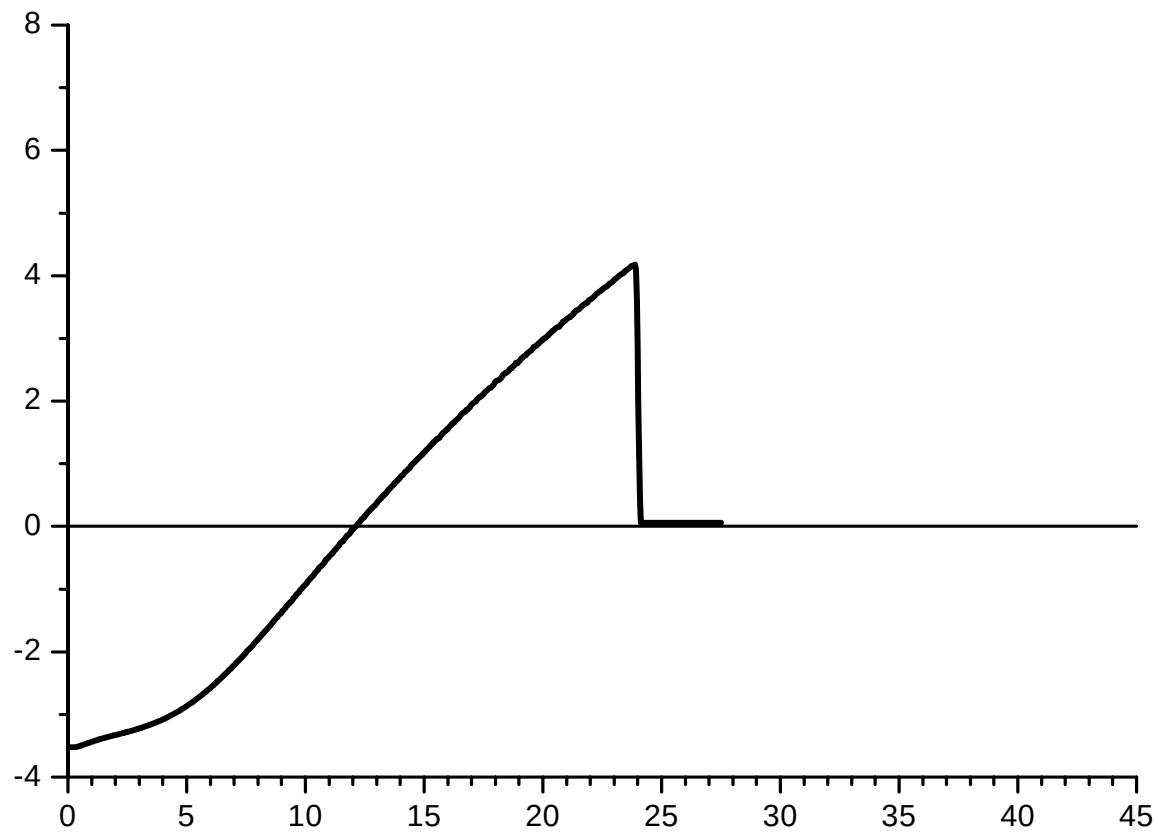


Failure to an isotropic s-wave gap



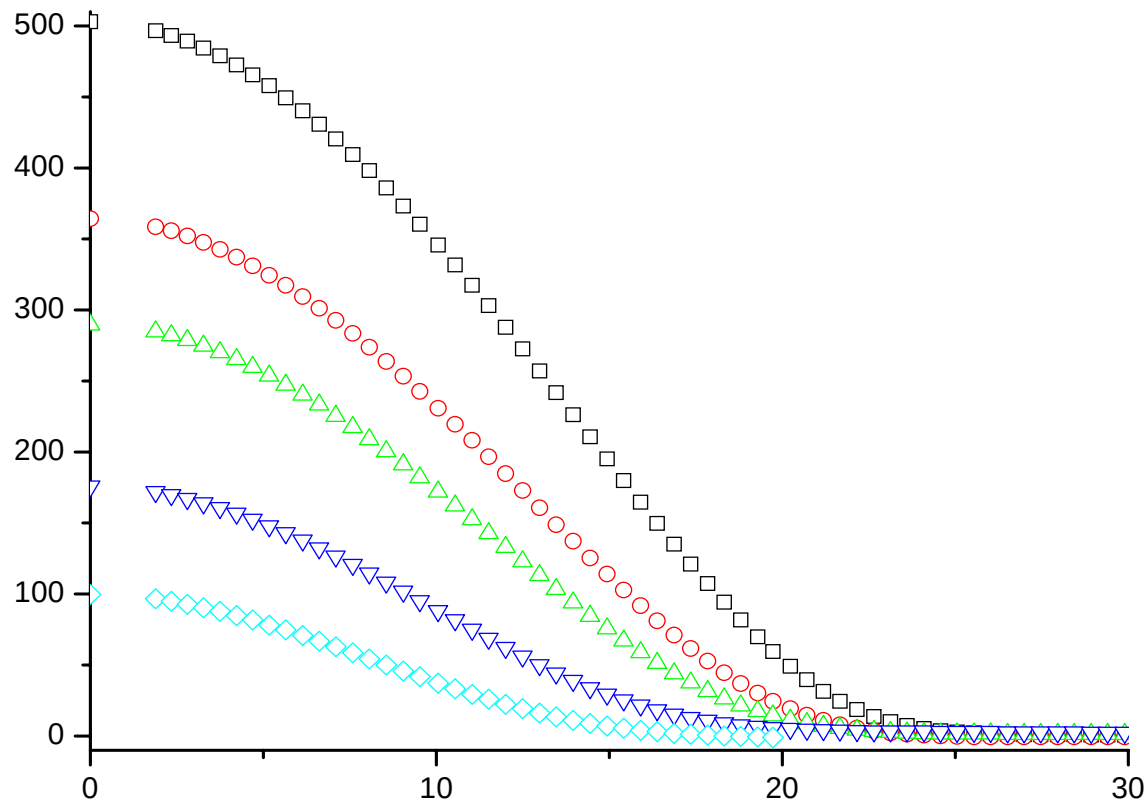
$$\gamma = \frac{4}{3} \int_0^{\hbar\omega} \varepsilon \left[\varepsilon^2 + \Delta^2 - \frac{\Delta^2}{2} \right] \frac{\zeta'}{(1 + \zeta')^2} d\varepsilon$$

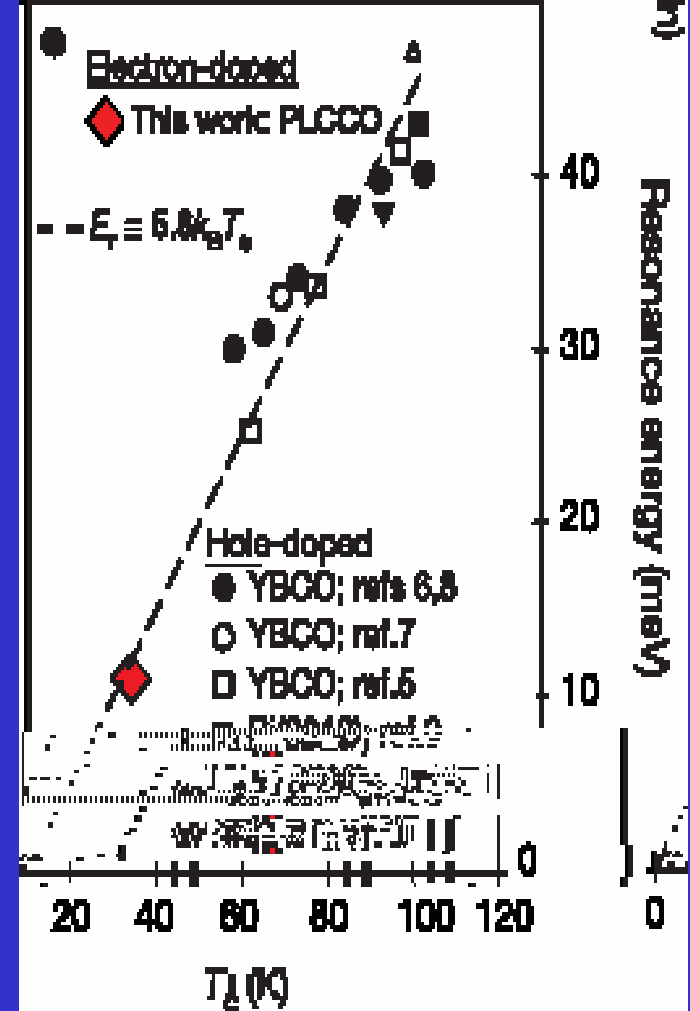
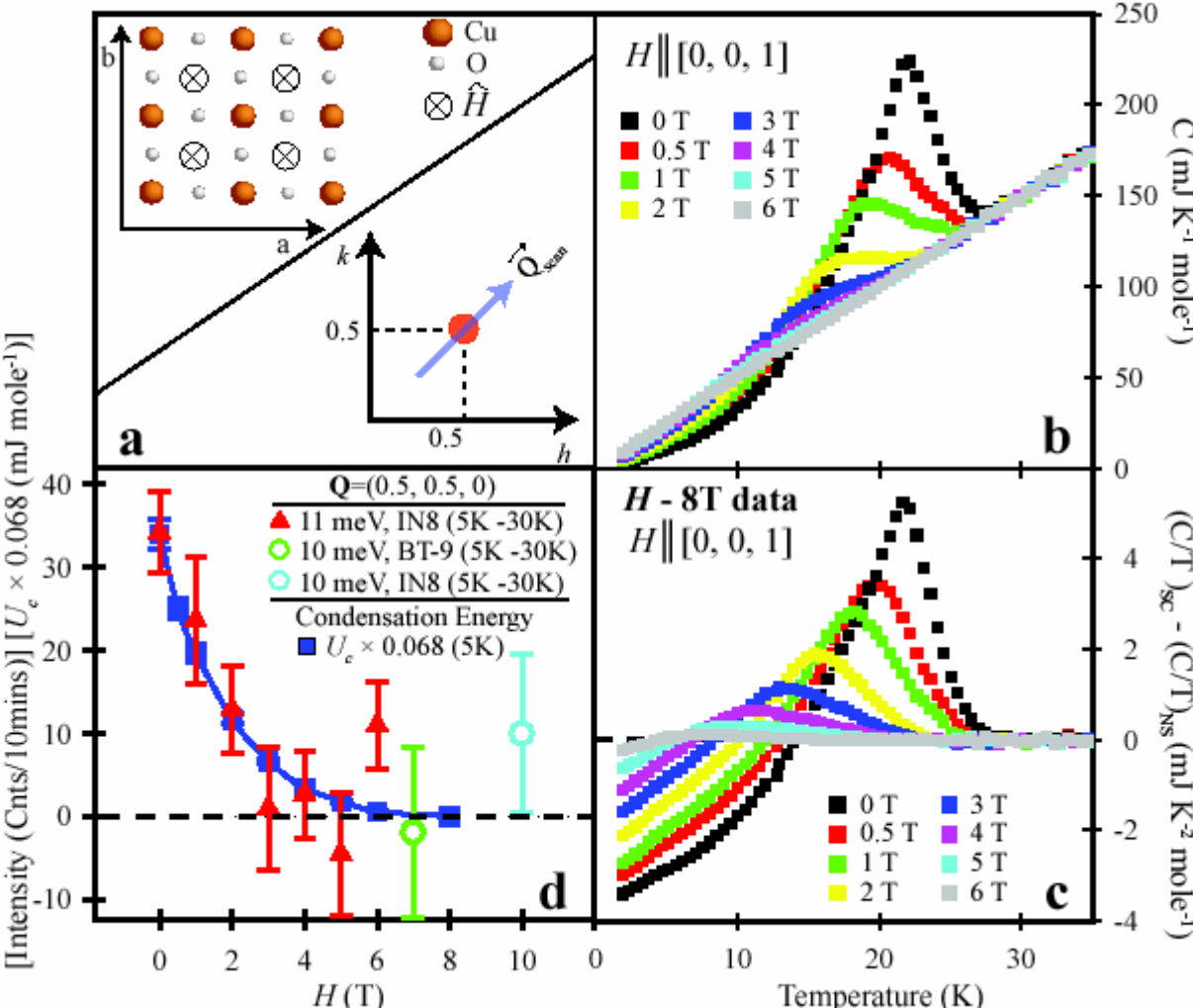
$$\zeta = \sqrt{\varepsilon^2 + \Delta^2}$$



$$= \frac{\sigma^2}{8\pi}$$

$$(\sigma) = \sigma(0) \left[1 - \left(\sigma / \sigma_0 \right)^2 \right]$$

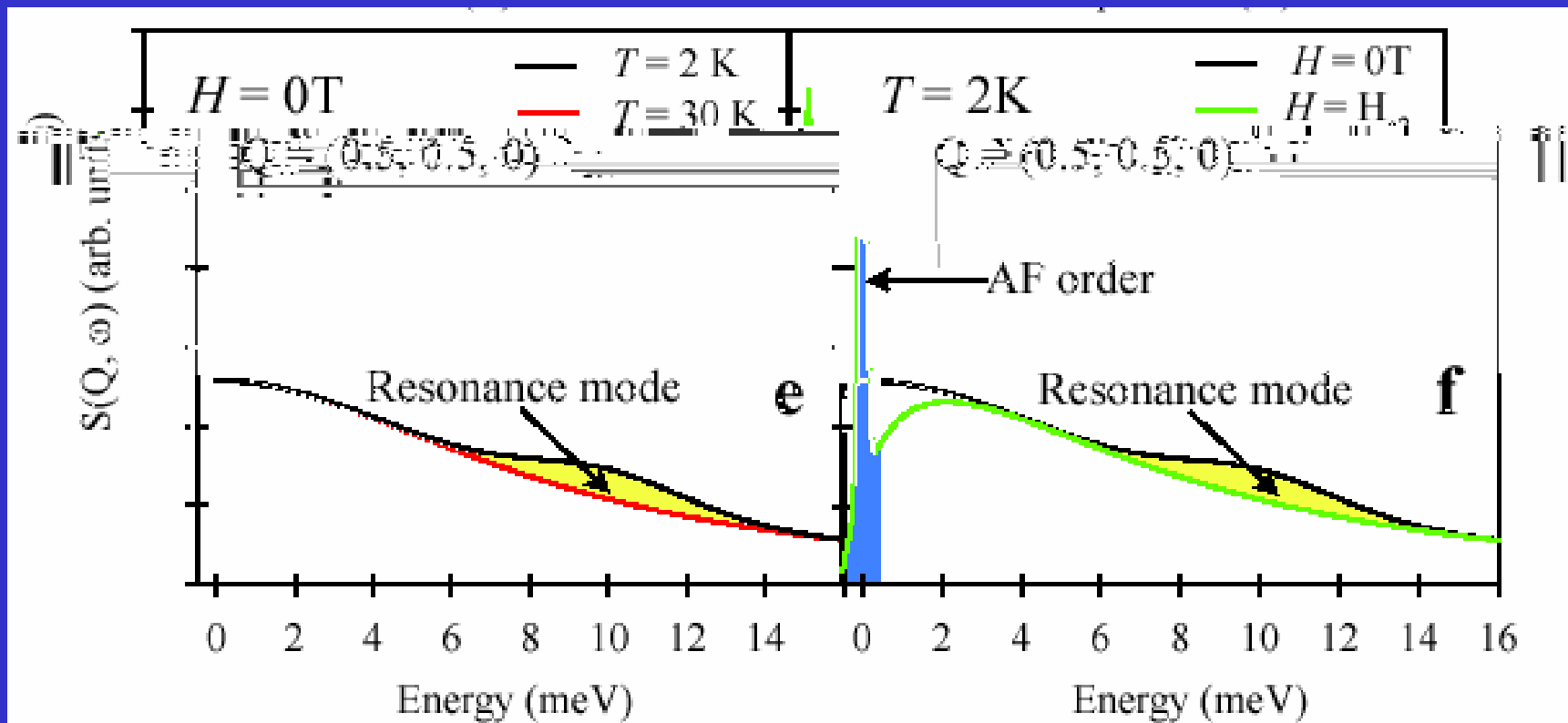




Proc. National Acad. Sci.104 15259 (2007).

Collaboration between P. C. Dai and H. H. Wen's group

S. Wilson, et al.,
 Nature 442, 59 (2006).



Proc. National Acad. Sci. 104 15259 (2007).

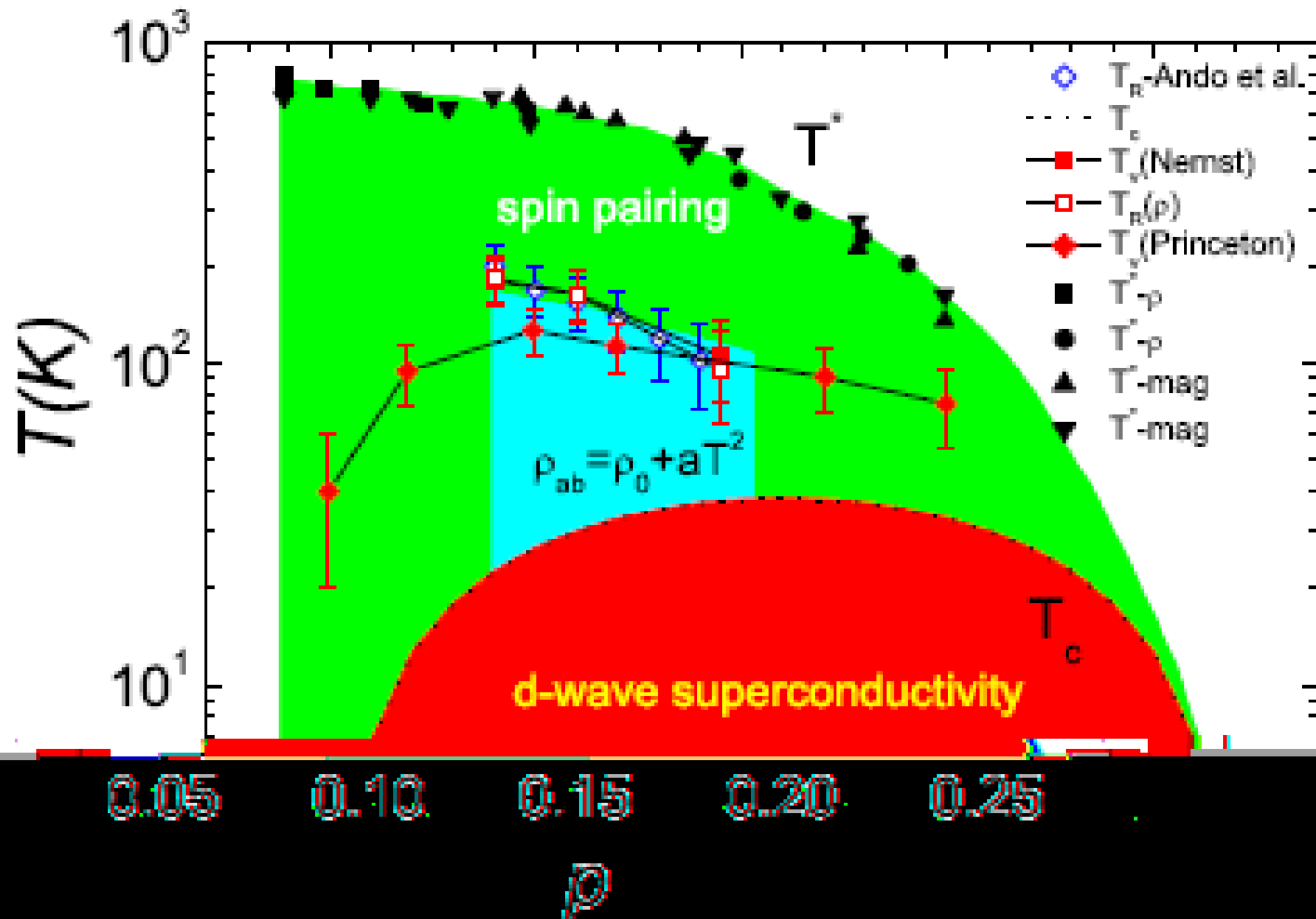
Collaboration between P. C. Dai and H. H. Wen's group

Summary from experiment

- In all doped regions, d-wave pairing symmetry holds;
- It seems to be BCS like in overdoped region although with some specialties: possible phase separation;
- The general quasiparticle gap becomes smaller towards more doping, which resembles the doping dependence of the AF correlation, or the RVB singlet pairing gap;
- Pseudogap phase has small Fermi surfaces, the superconductivity may be formed by the a new “gapping” on these small FS;
- There is a close relationship between the pseudogap and superconductivity.
- The resonance peak on the magnetic fluctuation spectrum has a close connection between the superconductivity condensation.

Phase diagram of cuprate superconductors

Based on the data of La-214



Possible pictures for HTS

1. Spin fluctuation mediated pairing (glue may or may not need)
2. RVB based picture: Mobil electrons swim in the Natural spin singlet-pairing background (glue does not need)
3. Electron-phonon pairing (glue need)

