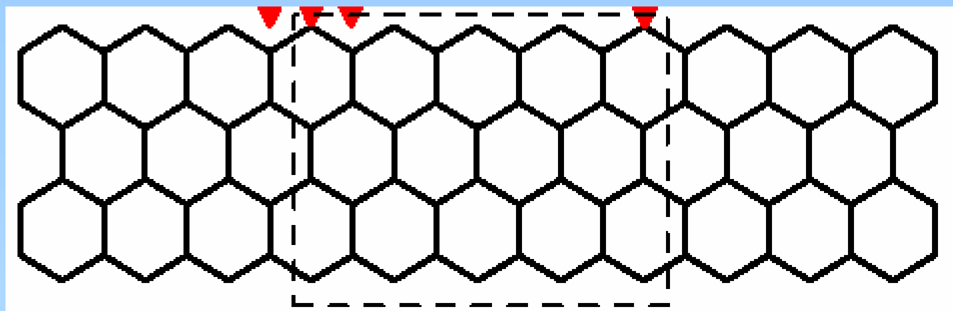
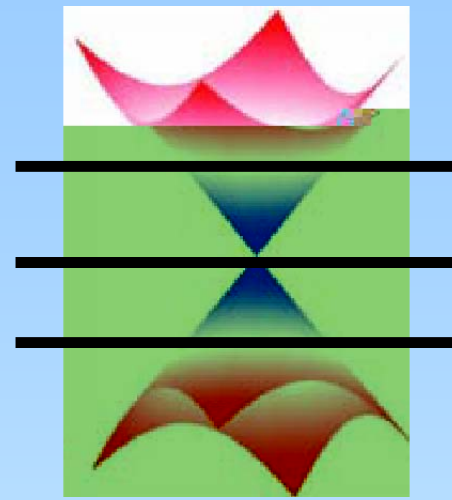
2008 10 30

p-n

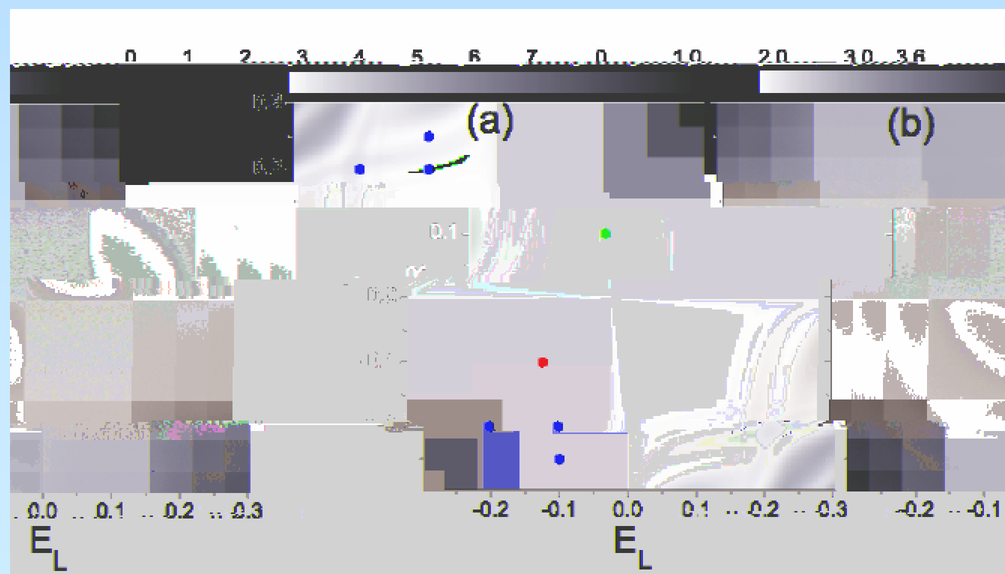
W. Long, Q.F. Sun, J. Wang, Phys. Rev. Lett. 101, 166806(2008)





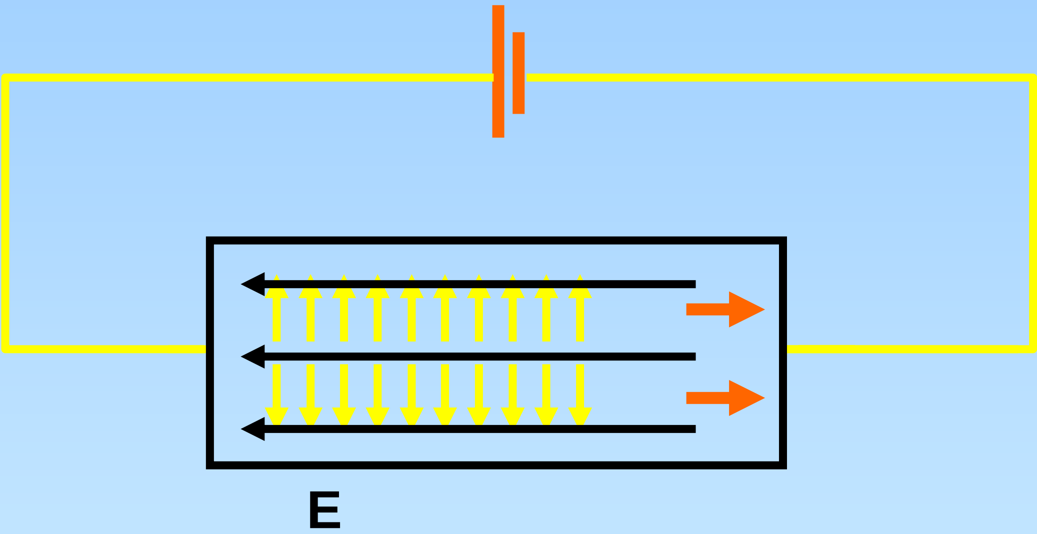
EF

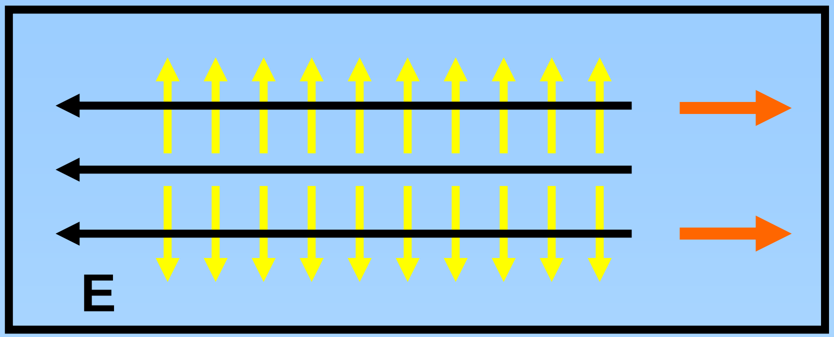
0 Dirac

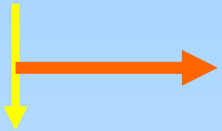


Hall

1. ,





	I_e	I_s
	$eI_\uparrow$	$\frac{}{2} I_\uparrow$
	$eI_\downarrow$	$-\frac{}{2} I_\downarrow$

: $I_e = e(I_\uparrow + I_\downarrow)$ $\neq 0$

: $I_s = \frac{}{2}(I_\uparrow - I_\downarrow)$ $= 0$

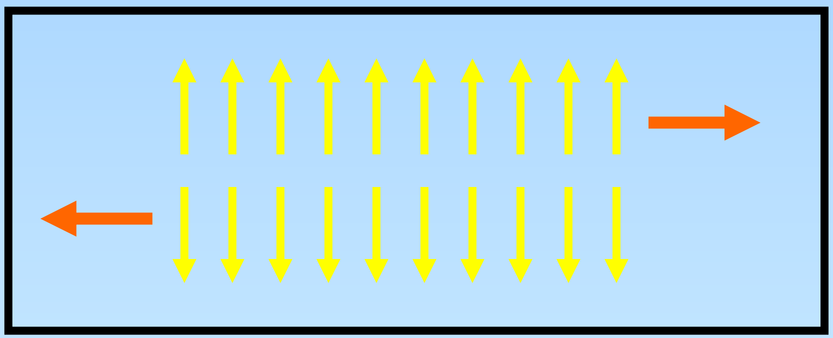
spintronics)

spintronics)

-
-

2

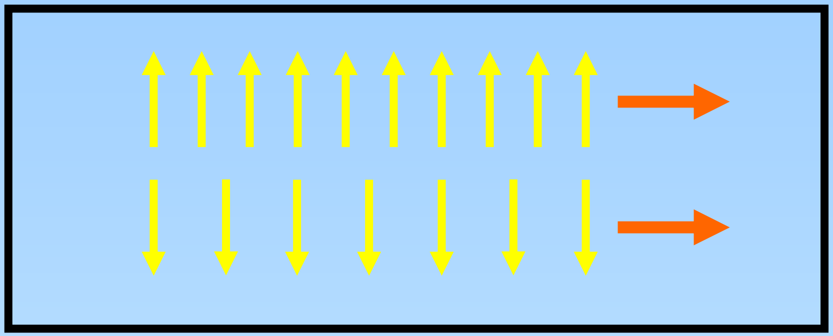
( )



: $I_e = e(I_\uparrow + I_\downarrow) = 0$

: $I_s = \frac{}{2}(I_\uparrow - I_\downarrow) \neq 0$

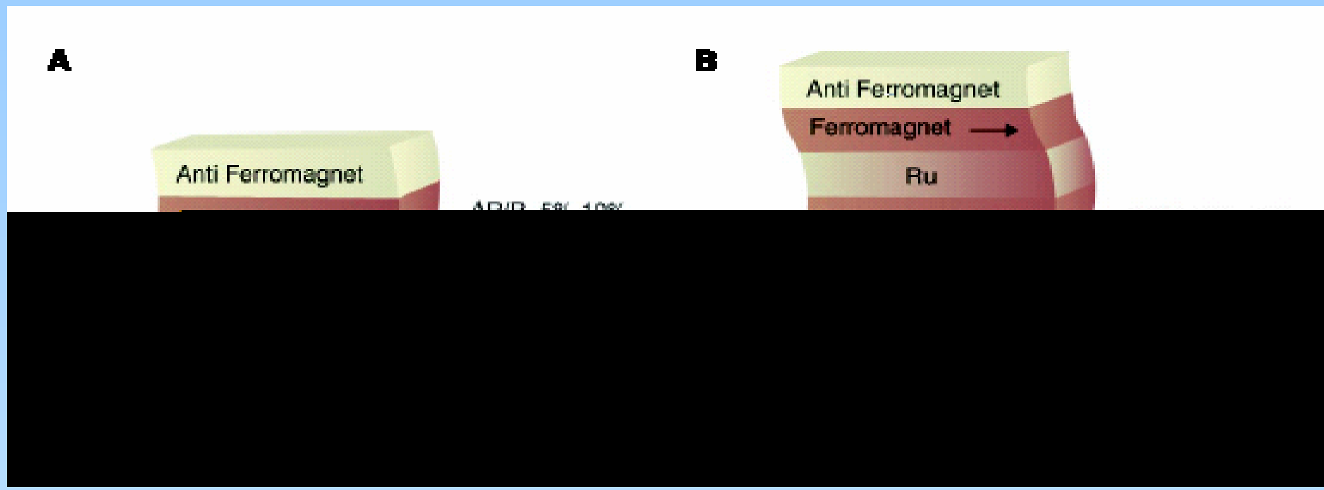
3



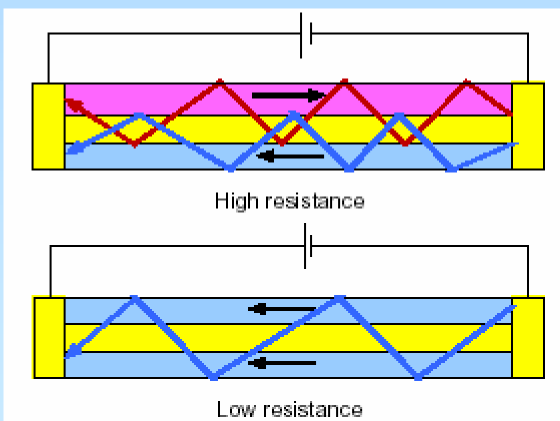
: $I_e = e(I_\uparrow + I_\downarrow) \neq 0$

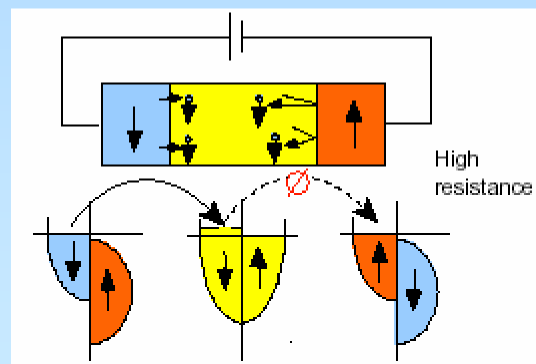
: $I_S = \frac{}{2}(I_\uparrow - I_\downarrow) \neq 0$

GMR TMR



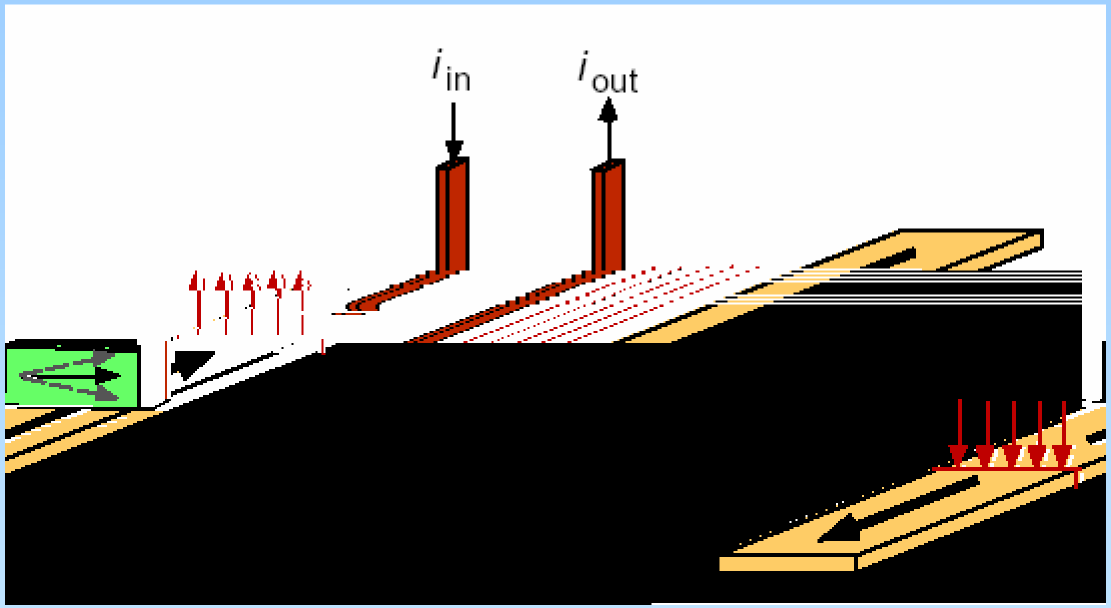
[redacted]



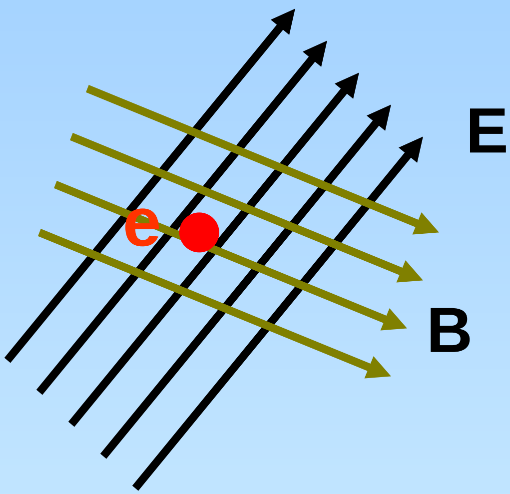


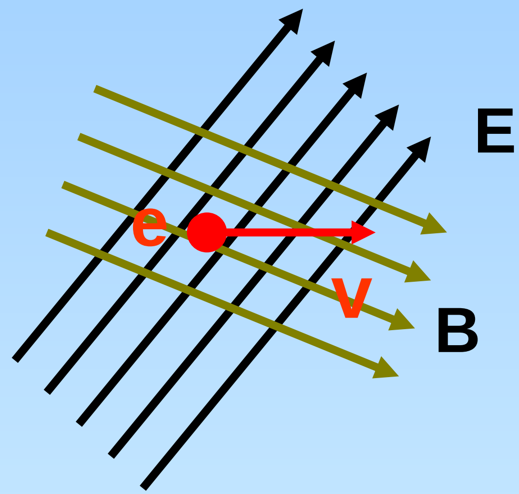
1. G.A. Prinz, science 282, 1660 (1998);
2. S.A. Wolf, et. al. science 294, 1488 (2001).

GMR



1



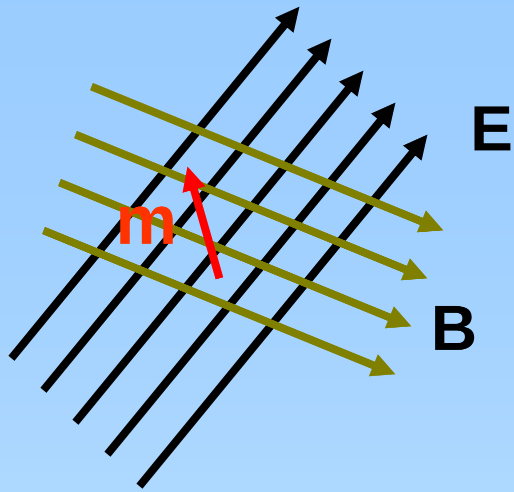


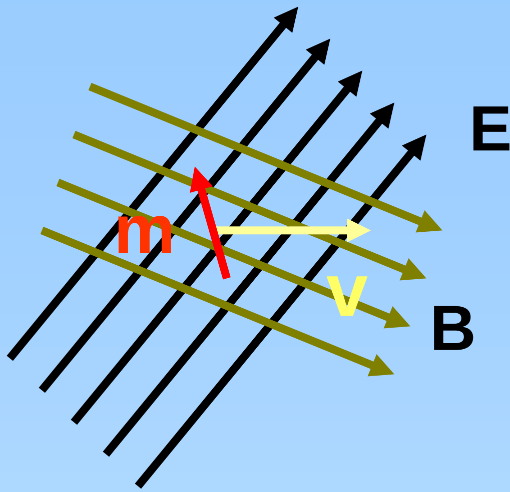
$$\mathbf{F} = e\mathbf{E}$$

$$\mathbf{F} = e\mathbf{E} + e\mathbf{v} \times \mathbf{B}$$

B

B





$$-m \bullet B$$

$$m \bullet (v \times E)/c^2$$
$$= g\mu_B \sigma \bullet (v \times E)/c^2$$

-
-
-

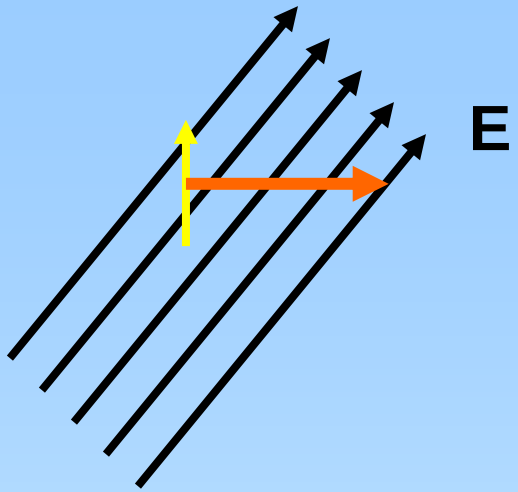
*

* (   )

*

* (   )

$$m \bullet (v \times E)/c^2 \quad = \quad g\mu_B s \bullet (v \times E)/c^2$$



$$\alpha s \bullet (v \times E)$$

ii.

$$\frac{\alpha}{2}\left[\sigma \bullet (p \times \nabla V(r)) - \sigma \bullet (\nabla V(r) \times p)\right]$$

$$= \frac{\alpha}{2}\left[(\sigma \times p) \bullet \nabla V(r) + \nabla V(r) \bullet (\sigma \times p)\right]$$

(v/c)^2

Dirac

$$\frac{\alpha}{2}\left[(\sigma \times \mathbf{p}) \bullet \nabla V(r) + \nabla V(r) \bullet (\sigma \times \mathbf{p})\right]$$

(i) V(r)

$$\nabla V(\mathbf{r}) = \frac{\mathbf{r}}{r}\frac{d}{dr}V(r)$$

$$\nabla V(\mathbf{r}) \bullet (\sigma \times \mathbf{p}) = \frac{\mathbf{r}}{r}\frac{d}{dr}V(r) \bullet (\sigma \times \mathbf{p})$$

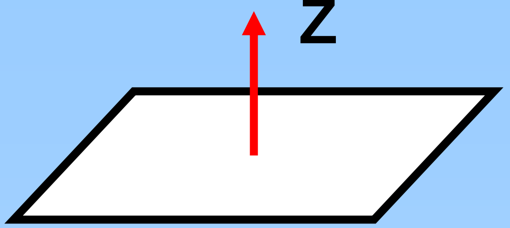
$$= -\frac{1}{r}\frac{d}{dr}V(r)\sigma \bullet (\mathbf{r} \times \mathbf{p})$$

$$= -\frac{1}{r}\frac{d}{dr}V(r)\sigma \bullet \hat{l}$$

Thomas

$$\alpha(\mathbf{r})\hat{s} \bullet \hat{l}$$

(ii) **V(r) = V(z) + V(x,y)**

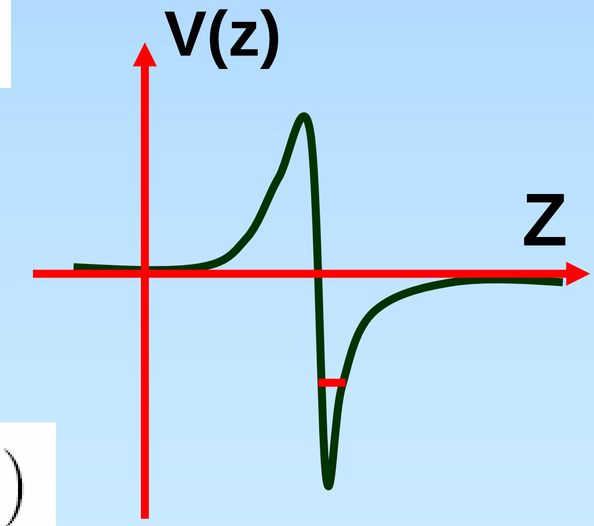


$\nabla V(z)$ $\qquad$ $\nabla V(x,y)$

$$\frac{\alpha}{2}\left[(\sigma \times \mathbf{p}) \bullet \nabla V(r) + \nabla V(r) \bullet (\sigma \times \mathbf{p})\right]$$

$$= \frac{\alpha}{2}\left[(\sigma \times \mathbf{p}) \bullet \hat{z}\frac{d}{dz}V(z) + \hat{z}\frac{d}{dz}V(z) \bullet (\sigma \times \mathbf{p})\right]$$

$$= \alpha\frac{d}{dz}V(z)\hat{z} \bullet (\sigma \times \mathbf{p})$$



V(z)

Rashba : $\alpha\hat{z} \bullet (\sigma \times \mathbf{p})$

2

⊛

H = H0 $\frac{\alpha}{2}\left[(\sigma\times\mathbf{p})\bullet\nabla V(r)+\nabla V(r)\bullet(\sigma\times\mathbf{p})\right]$

$$H_0 = \mathrm{p}^2/2m + V(\mathrm{r})$$

$T = -i\sigma_y K$ T K

$$Kp_x = -p_x K$$

$$Tp_x = -p_x T$$

$$T\sigma_x = -\sigma_x T$$

⊛ $[T, H] = 0$

Kramers

φ **H**

$T\varphi$ $T\varphi$ φ

$$HT\varphi = TH\varphi = \varepsilon_n T\varphi$$

$$<\varphi|T\varphi> = <T^2\varphi|T\varphi> = -<\varphi|T\varphi>$$

$$\therefore \quad <\varphi|T\varphi> = 0$$

$<f|\varphi> = <T\varphi|Tf>$

$$\varphi = \begin{pmatrix} \varphi_\uparrow(\mathrm{r}) \\ \varphi_\downarrow(\mathrm{r}) \end{pmatrix} \qquad T\varphi = -i\sigma_y K\varphi = \begin{pmatrix} -\varphi_\downarrow^*(\mathrm{r}) \\ \varphi_\uparrow^*(\mathrm{r}) \end{pmatrix}$$

(1993)

$$
\begin{aligned}
<\sigma_z> &= f(\varepsilon_n)[<\varphi|\sigma_z|\varphi> + <T\varphi|\sigma_z|T\varphi>] \\
&= f(\varepsilon_n)[<\varphi|\sigma_z|\varphi> + <T\varphi|-T\sigma_z\varphi>] \\
&= f(\varepsilon_n)[<\varphi|\sigma_z|\varphi> - <\sigma_z\varphi|\varphi>] \\
&= f(\varepsilon_n)[<\varphi|\sigma_z|\varphi> - <\varphi|\sigma_z|\varphi>] \\
&= 0
\end{aligned}
$$

$$H = p^2/2m + V(r) + g\hat{\sigma} \bullet B$$

$$[T, H] \neq 0$$

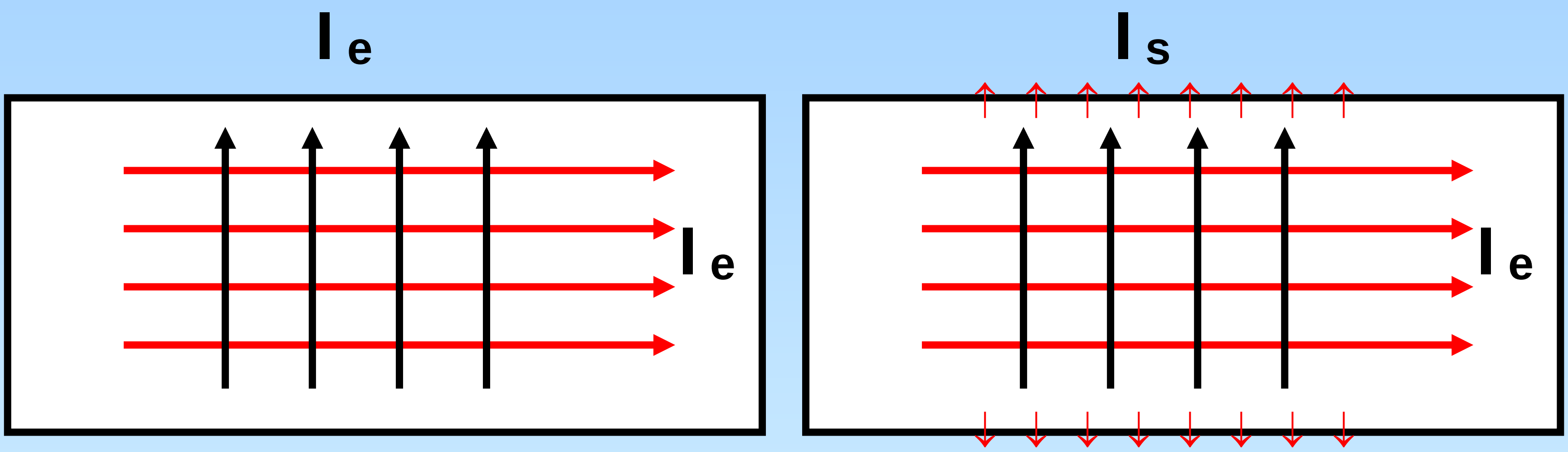
$$[T, H] = 0$$

Kramers

3

i. :

Spin Hall effect:



Spin Hall effect

S. Murakami, et.al. Science 301,1348 (2003);

S. Murakami, et.al. Phys.Rev.B 69, 235206 (2004);

J. Sinova et.al. Phys.Rev.Lett. 92, 126603 (2004);

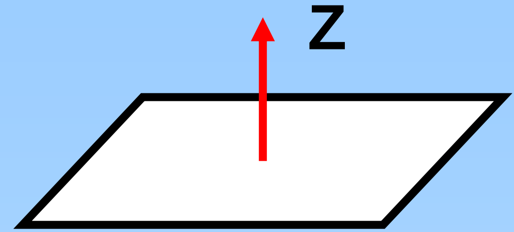
S.-Q. Shen, et.al. Phys.Rev.Lett. 92, 256603 (2004);

etc.

Y. K. Kato, R. C. Myers, A. C. Gossard, and D. D.
Awschalom, Science 306, 1910 (2004); V. Sih, R. C. Myers,
, Y. K. Kato, W. H. Lau, A. C. Gossard, and D. D. ers
schalom, Nature Phys. 1, 31 (2005); V. Sih, W. H. Lau, Aw
C. Myers, V. R. Horowitz, A. C. Gossard, and D. D.
Awschalom, Phys. Rev. Lett. 97, 096605 (2006).
J. Wunderlich, B. Kaestner, J. Sinova, and T. Jungwirth,
Phys. Rev. Lett. 94, 047204 (2005).
a and M. Tinkham, Nature (London) 442, S. O. Valenzuela
176 (2006).

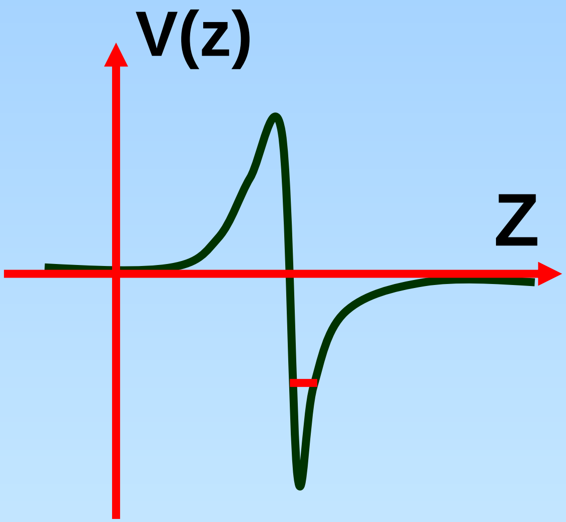
ii

( )

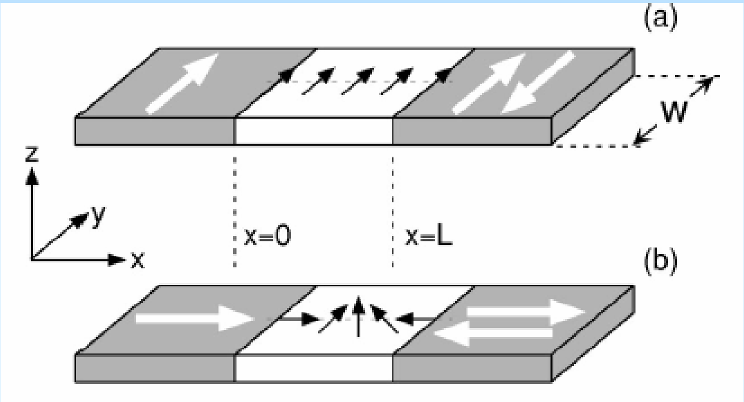


$$\alpha$$

$$\alpha \hat{z} \bullet (\sigma \times \mathbf{p})$$



iii



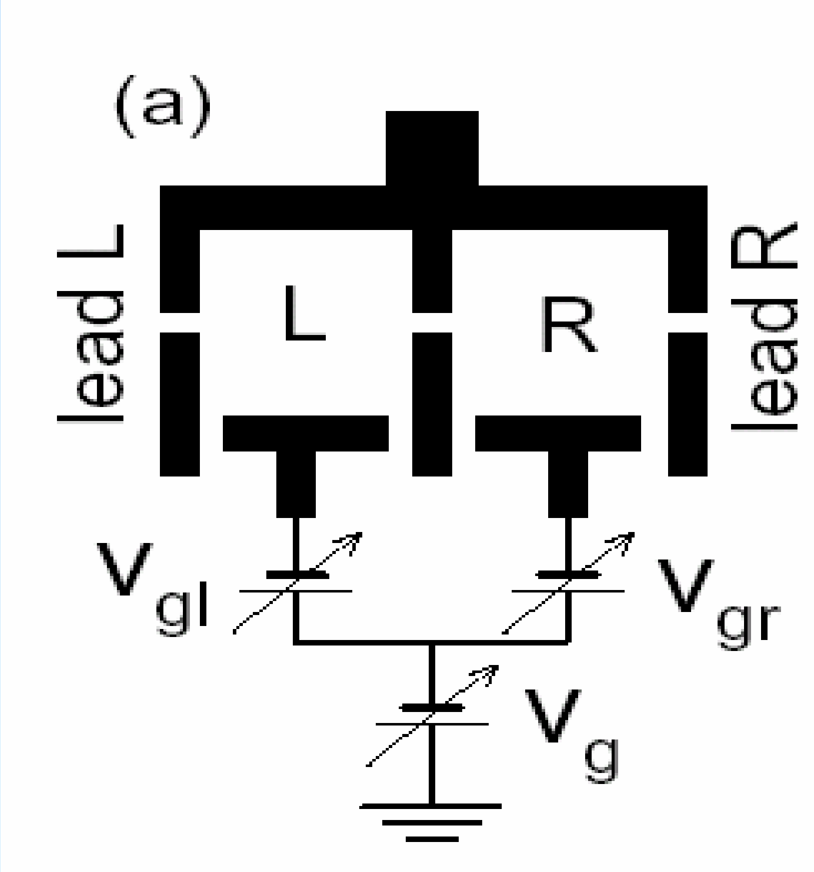
S. Datta and B. Das, Appl. Phys. Lett. 56, 665(1990)

Q.-f. Sun, H. Guo, J. Wang, Phys. Rev. Lett. 90, 258301 (2003).

1

2

3



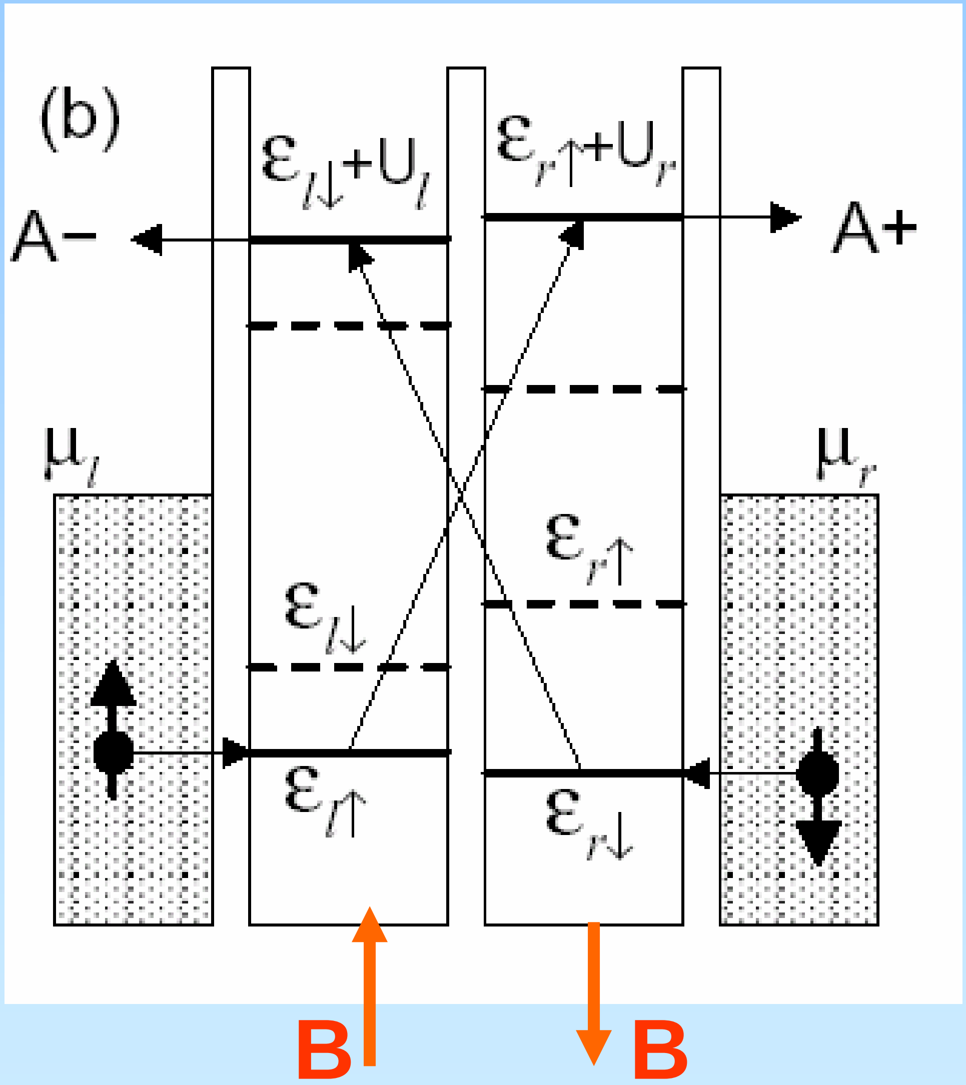
1 Zeeman

2

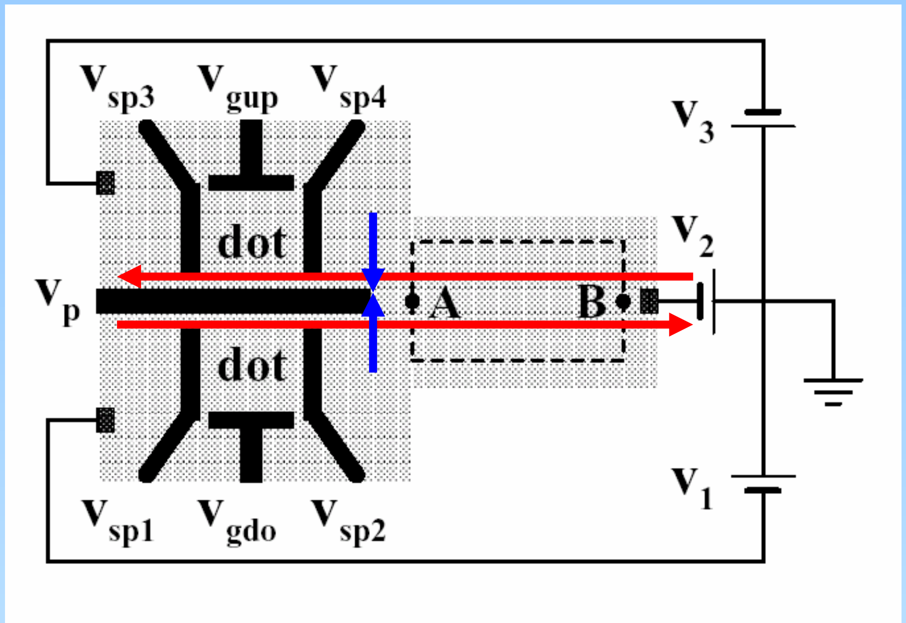
3 A_

A+ ;

, .

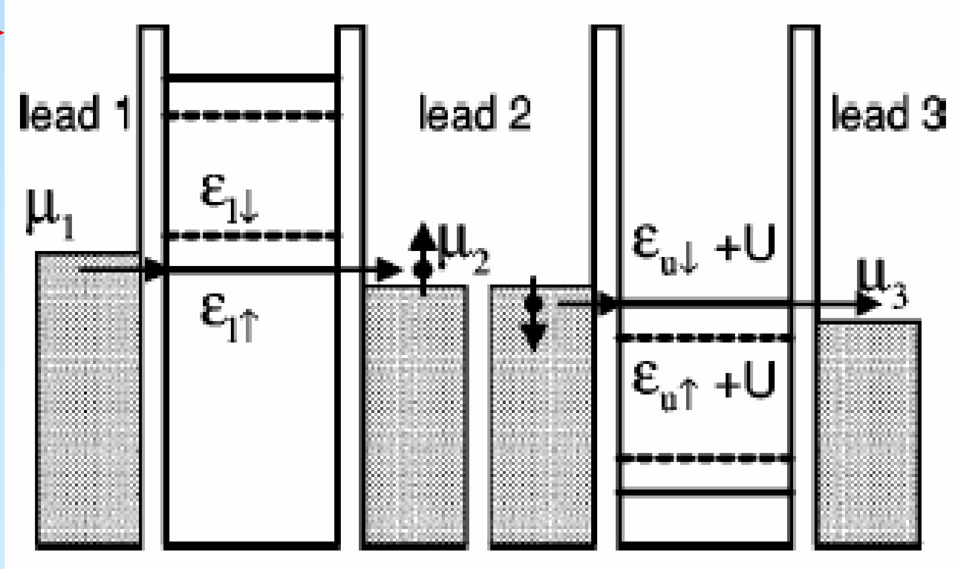


W. Long, Q.F. Sun, H. Guo, J. Wang, Appl.Phys.Lett. 83, 1397 (2003).



(

) : 1 3

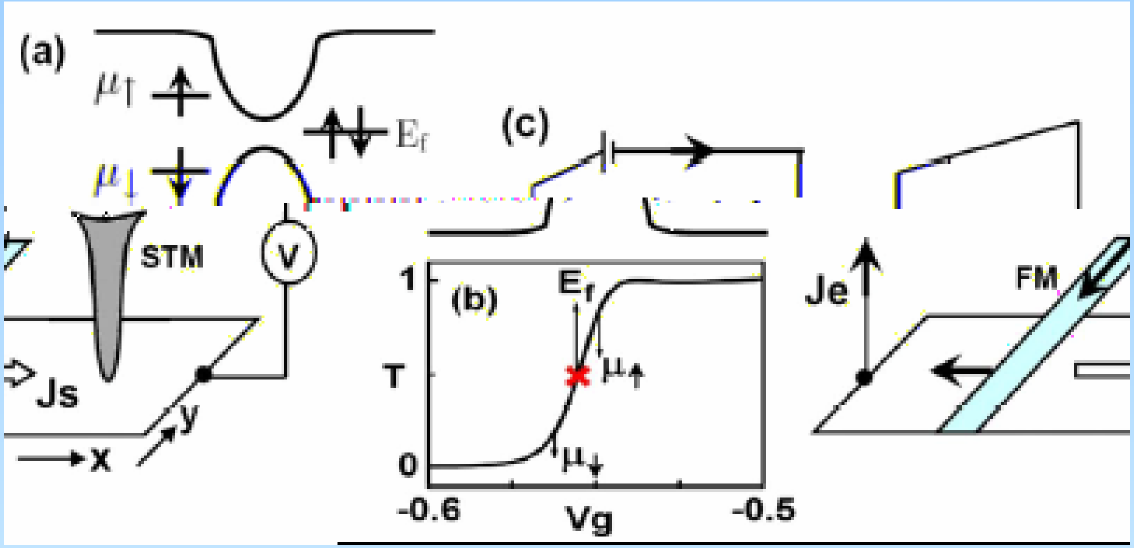


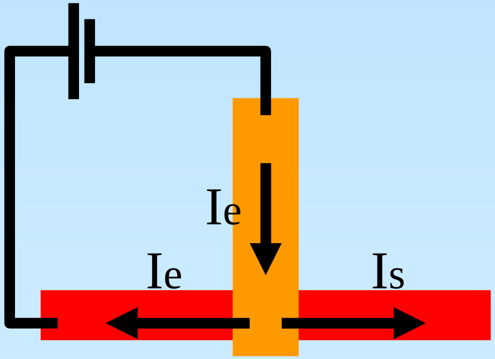
0 1

1 2

A, B

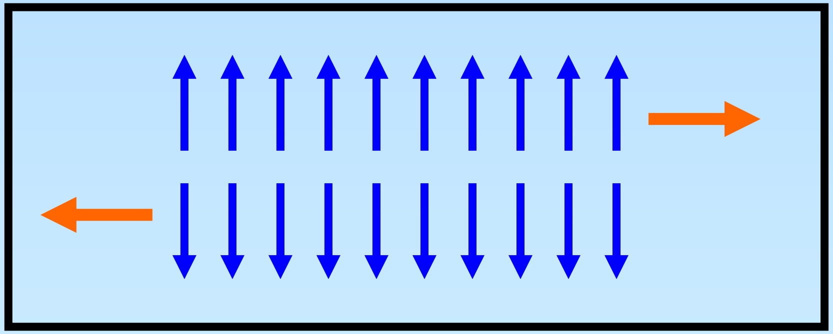
Q.F. Sun, Y. Xing S.Q. Shen, Phys. Rev. B 77, 195313(2008).
Y. Xing, Q.F. Sun, J. Wang, Appl.Phys.Lett. 93, 142107(2008).





Q.F. Sun, H. Guo, J. Wang, Phys. Rev. B 69, 054409(2004);
Q.F. Sun and X.C. Xie, Phys. Rev. B 72, 245305(2005).

( ) 毕奥－萨伐尔定
律



- $$\vec{E}_s = \frac{-\mu_0 g \mu_B}{4\pi} \nabla \times \int \mathbf{j}_s \, dV \times \frac{\mathbf{r}}{r^3},$$
- $$\vec{E}_\omega = \frac{-\mu_0 g \mu_B}{4\pi} \int \vec{j}_\omega \, dV \times \frac{\mathbf{r}}{r^3},$$

*

*

*

*

*

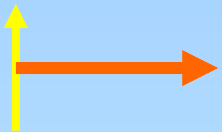
* ( )

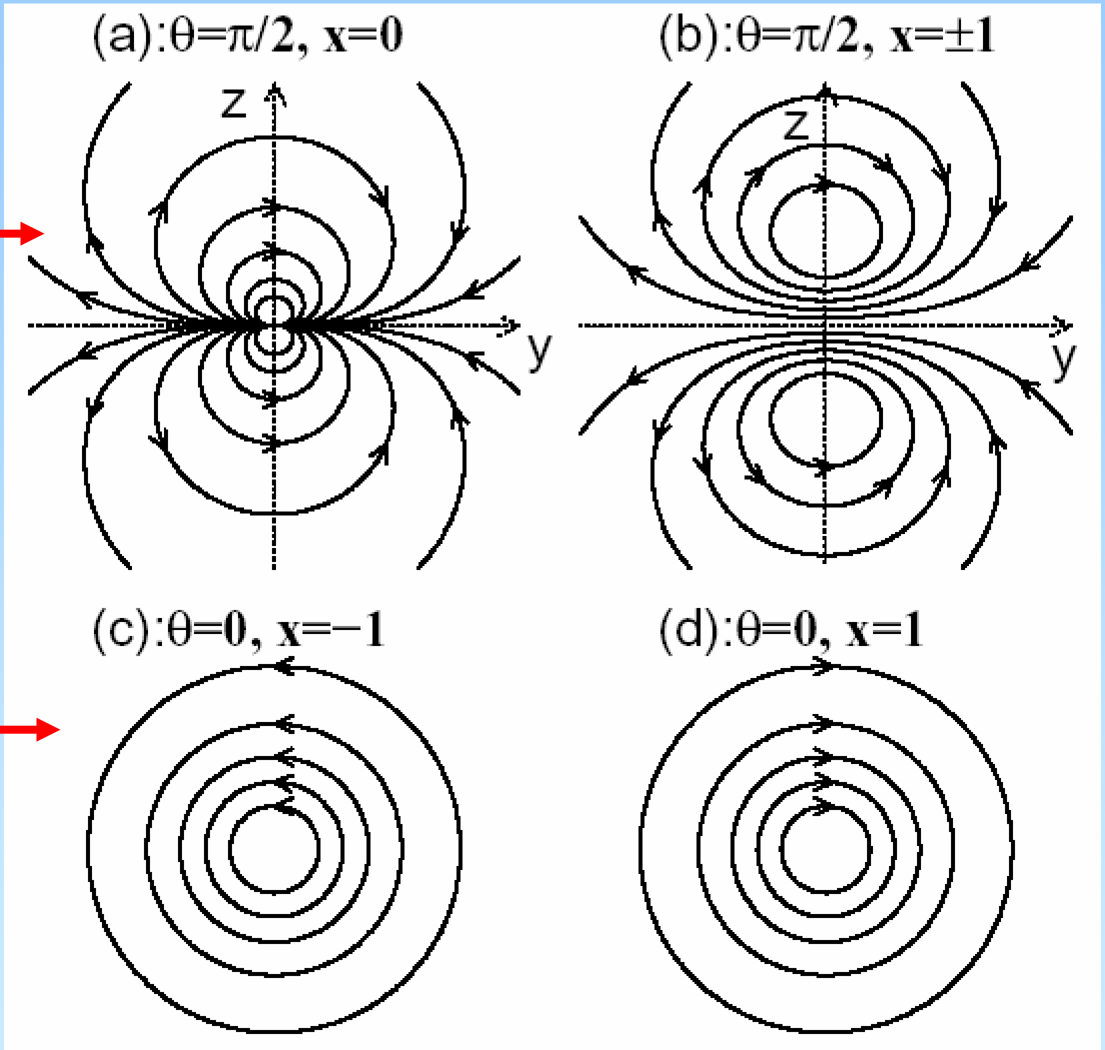
*

*

x

z





x

Maxwell

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} - \mu_0 \mathbf{J}_{mc},$$

$$\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}_{ec},$$

$$\nabla \bullet \mathbf{E} = \rho_{ec}/\epsilon_0,$$

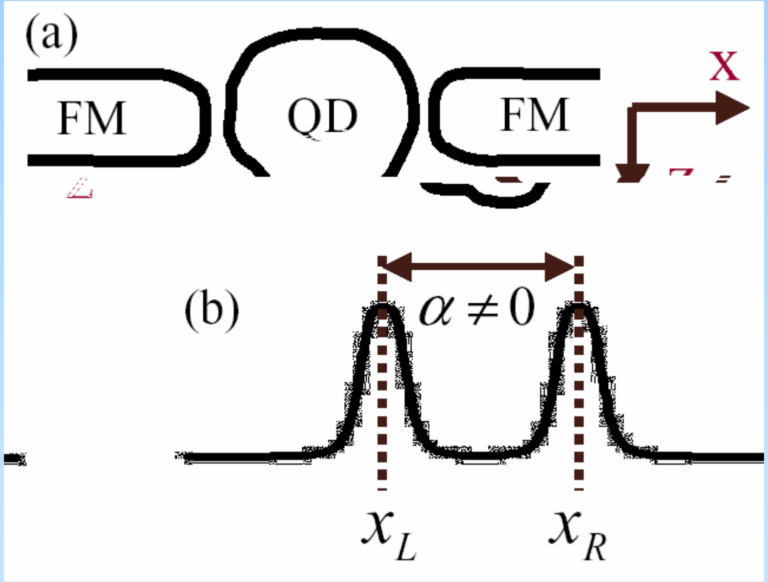
$$\nabla \bullet \mathbf{B} = \mu_o \rho_{mc},$$

Q.F. Sun, J. Wang, H. Guo, Phys.Rev.B 71,165310(2005)

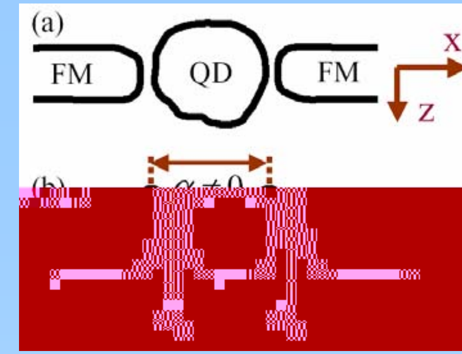
i

ii -

iii



(a)
FM
QD
FM
X
(b)
$\alpha \neq 0$
x_L
x_R



$$H(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) = \sum_i H_s(\mathbf{r_i}) + \sum_{i,j(i\neq j)} H_I(\mathbf{r}_i, \mathbf{r}_j)$$

$$H_I(\mathbf{r}_1, \mathbf{r}_2) = \frac{e^2}{2|\mathbf{r}_1 - \mathbf{r}_2|^2}$$

$$H_s(\mathbf{r}) = \frac{\mathbf{p}^2}{2m^*} + V(\mathbf{r}) + \hat{\sigma}\cdot\mathbf{M}(\mathbf{r}) + \hat{\sigma}\cdot\mathbf{B}(\mathbf{r}) + H_{SO}(\mathbf{r})$$

Rashba

$$H_{SO} = \frac{\hat{y}}{2\hbar}\bullet[\alpha(\hat{\sigma}\times\mathbf{p}) + (\hat{\sigma}\times\mathbf{p})\alpha]$$

$$H = H_{QD} + \sum_{\beta=L,R} H_\beta + H_T$$

$$H_{QD} = \sum_{n,s} (\epsilon_n + sB_z) d^\dagger_{ns} d_{ns} + \sum_{ns,ms'(ns\neq ms')} U_{ns,ms'} \hat{n}_{ns} \hat{n}_{ms'}$$

$$+ \sum_{m,n} \left[t^{so}_{mn} d^\dagger_{m\downarrow} d_{n\uparrow} + B_x t^B_{mn} d^\dagger_{m\downarrow} d_{n\uparrow} + H.c. \right]$$

$$H_\beta = \sum_{k,s} (\epsilon_{k\beta} + sM_\beta) a^\dagger_{k\beta s} a_{k\beta s}$$

$$H_T = \sum_{k,n,s,\beta} \left[t_{k\beta n} \left(\cos\frac{\theta_\beta}{2} a^\dagger_{k\beta s} - s \sin\frac{\theta_\beta}{2} a^\dagger_{k\beta \bar{s}} \right) \times e^{is\phi_\beta/2} e^{-isk_R x_\beta} d_{ns} + H.c \right]$$

where $t^{so}_{mn} = -t^{so}_{nm}$ and $t^B_{mn} = t^B_{nm}$.

$e^{-is\phi_{so}}$

AB $e^{i\phi}$

$e^{-is\phi_{so}}$

$e^{i\phi}$

.

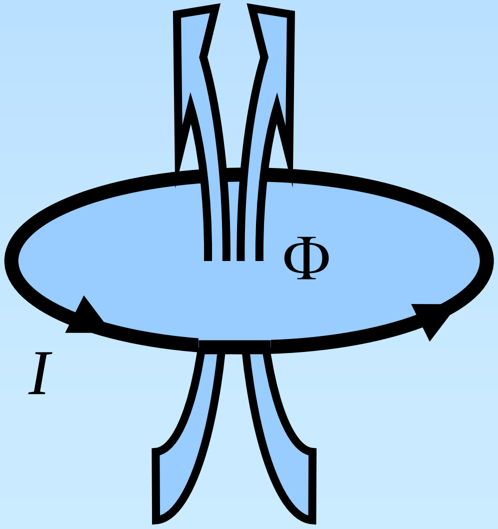
$$H_{R2} = \sum_{m,n} t^{so}_{mn} d^{\dagger}_{m\downarrow} d_{n\uparrow} + H.c.$$

$$t^{so}_{mn} = -t^{so}_{nm}.$$

Q.F. Sun, X.C. Xie, J. Wang, Phys. Rev. Lett. 98,196801(2007)

Q.F. Sun, X.C. Xie, J. Wang, Phys. Rev. B 77,035327(2008)

20



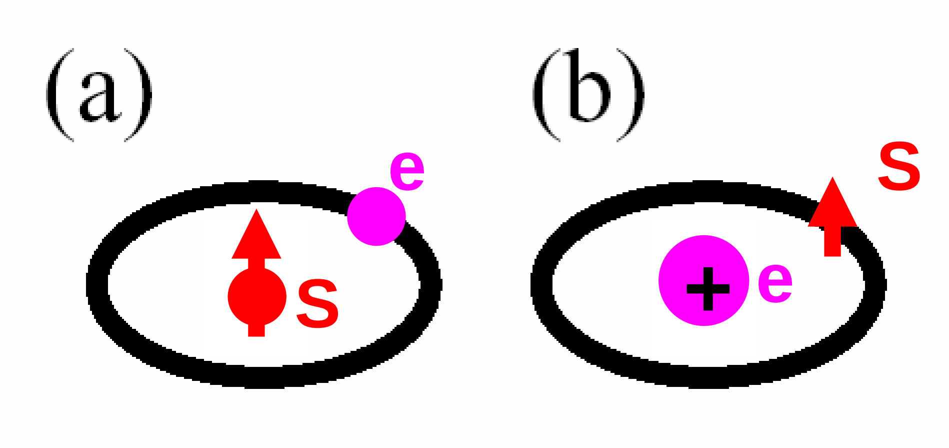
M. Buttiker, et al. Phys.Lett. 96A, 365(1983);

H.F. Cheung, et al. Phys.Rev.Lett. 62,587(1989);

L.P. Levy, et al. Phys.Rev.Lett. 64,2074(1990);

V. Ambegaokar and U. Eckern, Phys.Rev.Lett. 65,381(1990).

i



a

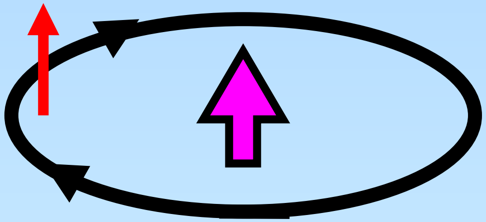
b

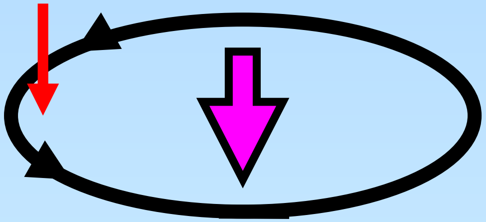
ii Berry

$$\alpha\hat{z} \bullet (\sigma \times \mathbf{p})$$

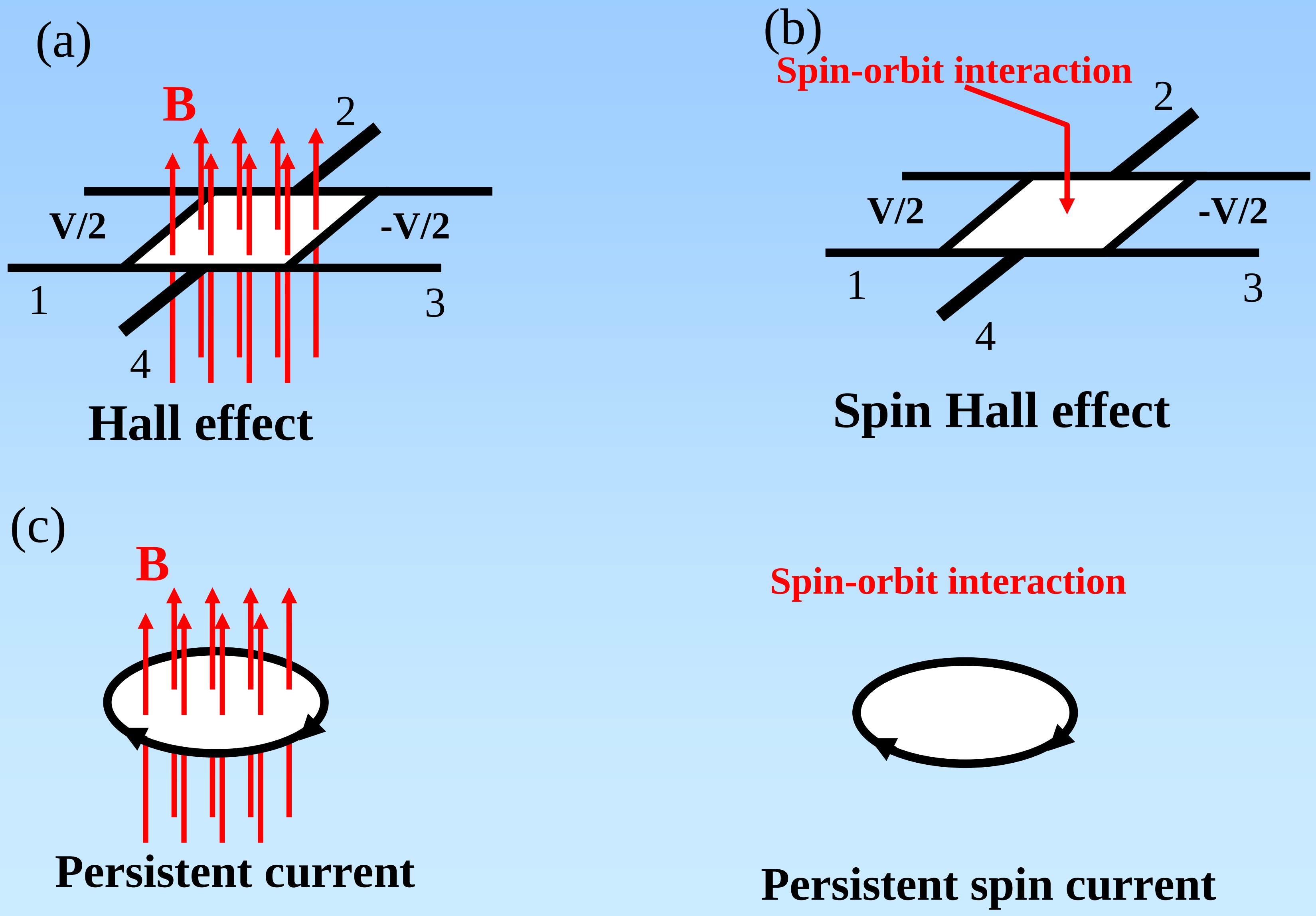
$e^{-is\phi_{so}}$

$e^{i\phi}$





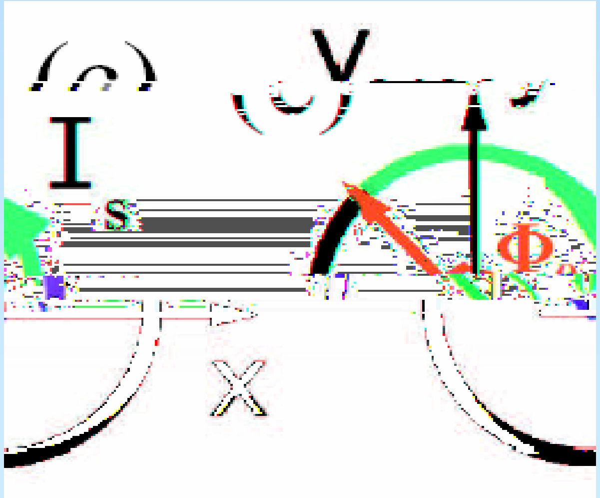
* Berry

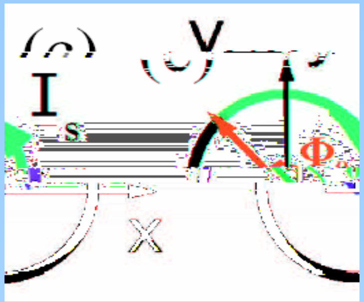

(a)
B
2
V/2
-V/2
1
3
4
Hall effect
(b)
Spin-orbit interaction
2
V/2
-V/2
1
3
4
Spin Hall effect
(c)
B
Persistent current
Spin-orbit interaction
Persistent spin current

Berry

-Rashba

Rashba





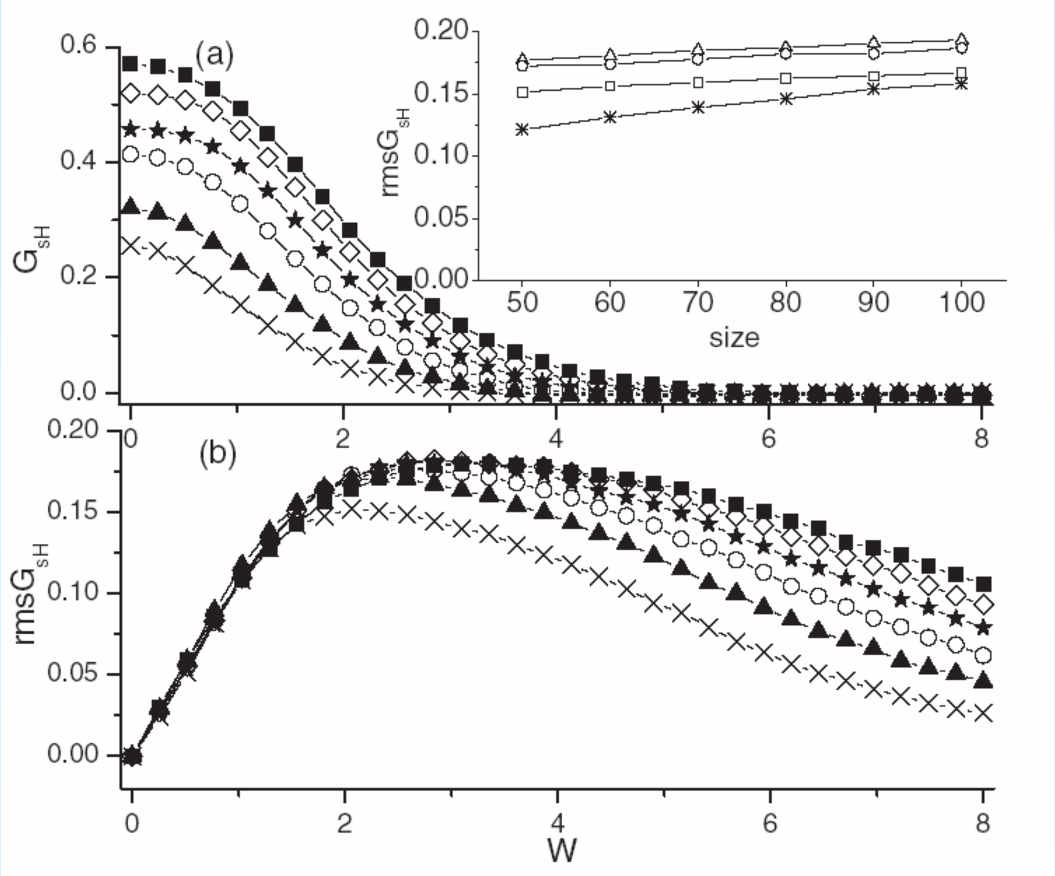
$$H = -E_a \frac{\partial^2}{\partial \varphi^2} - \frac{i\sigma_r}{2a}\left[\alpha_R(\varphi)\frac{\partial}{\partial \varphi} + \frac{\partial}{\partial \varphi}\alpha_R(\varphi)\right] - i\frac{\alpha_R(\varphi)}{2a}\sigma_\varphi \qquad (1)$$

(1)

$$H\Psi = E\Psi$$

(2)

W. Ren, Z. Qian, J. Wang, Q.F. Sun, H. Guo, Phys. Rev. Lett. 97,066603(2006)

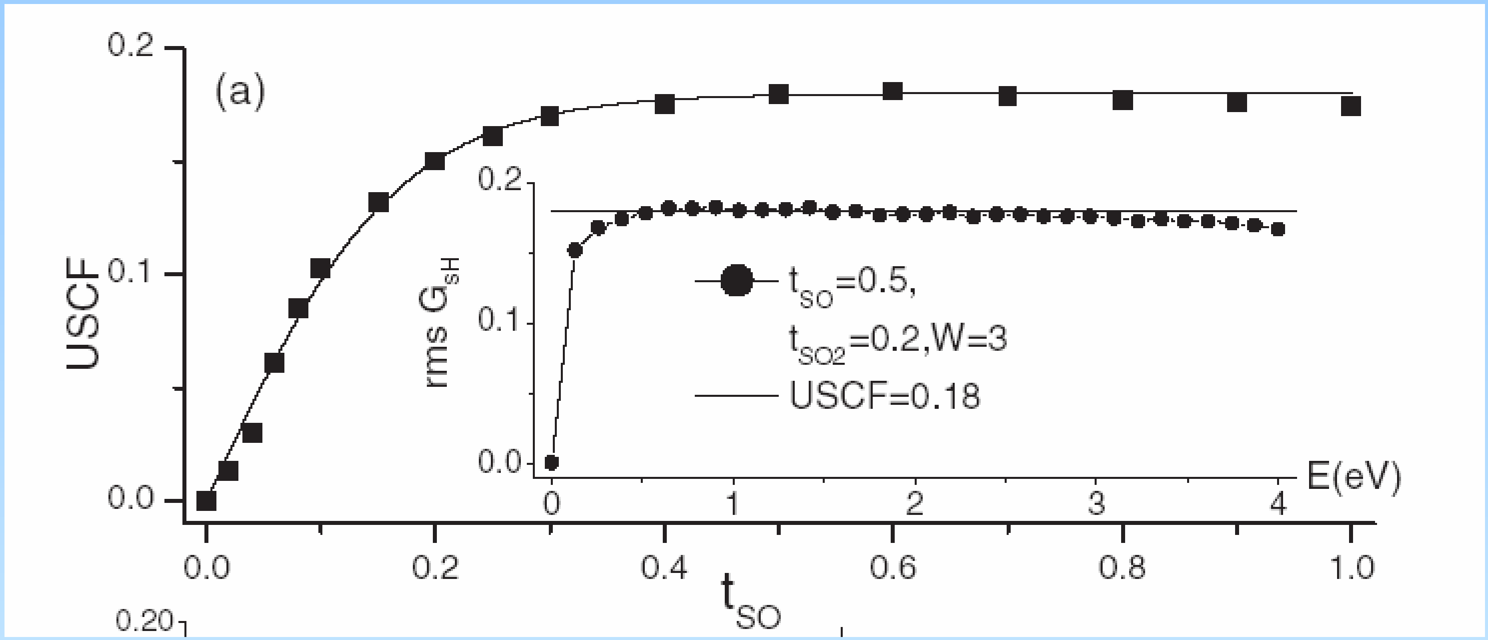


W<2

2<W<4

W>4

tso = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7



(a)
USCF
0.2
0.1
0.0
0.0
0.2
0.4
0.6
0.8
1.0
tSO
rms GsH
0.2
0.1
0.0
0
1
2
3
4
E(eV)
tSO=0.5,
tSO2=0.2,W=3
USCF=0.18
0.20

Hall

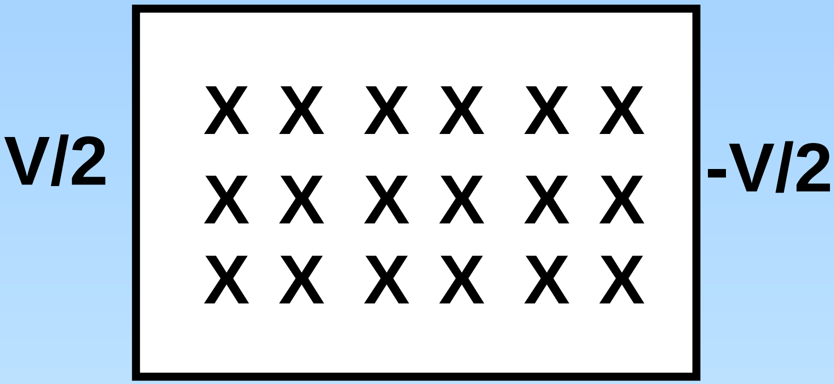
Hall

Hall

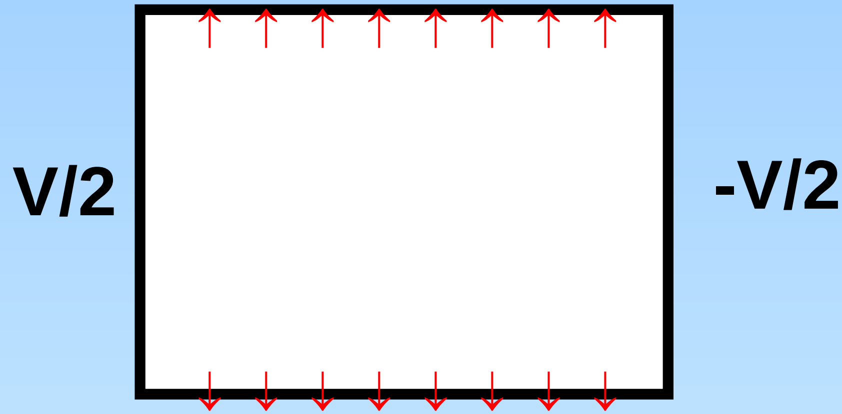
Hall

Hall

Hall

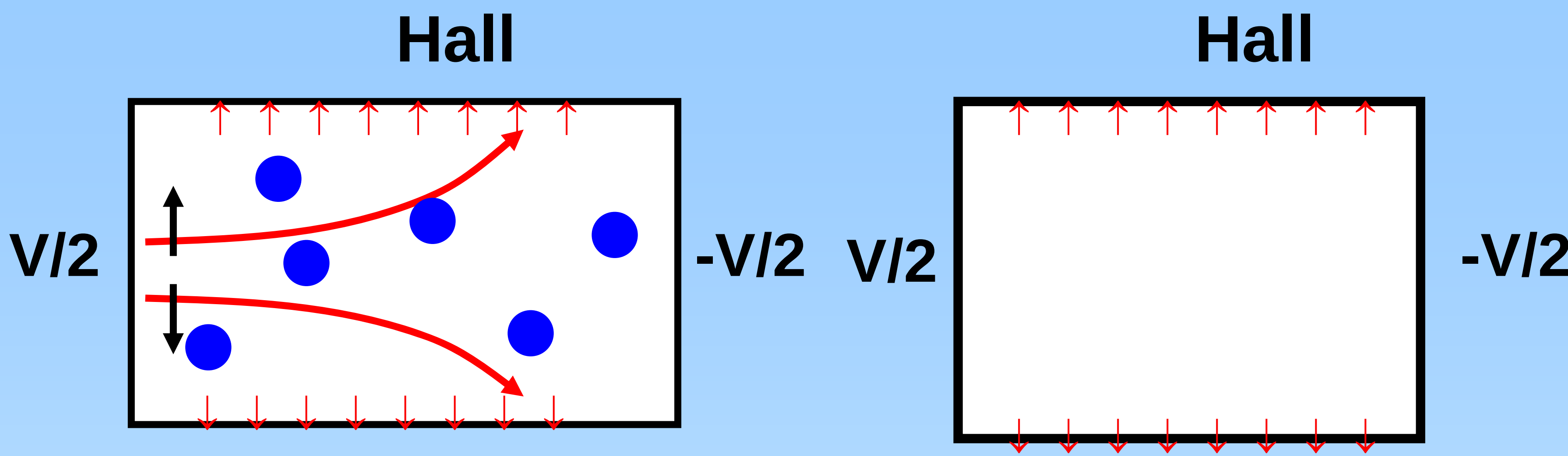


Hall



S. Murakami, et.al. Science 301,1348 (2003);

J. Sinova et.al. Phys.Rev.Lett. 92, 126603 (2004);

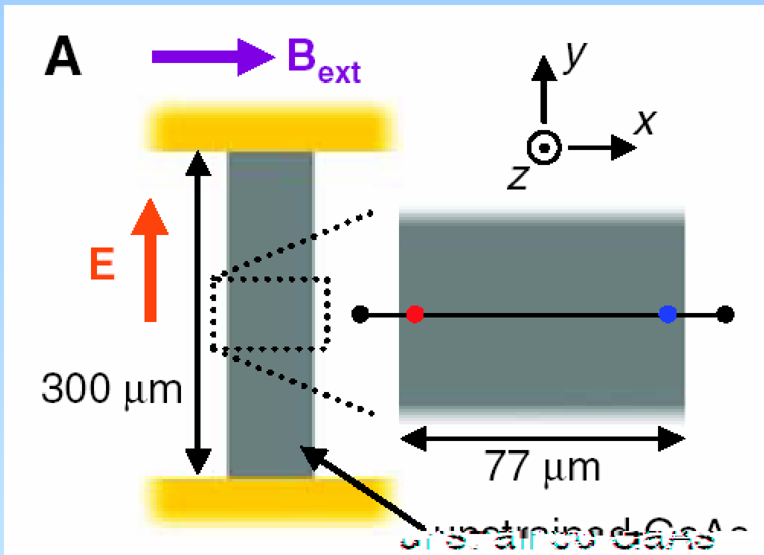


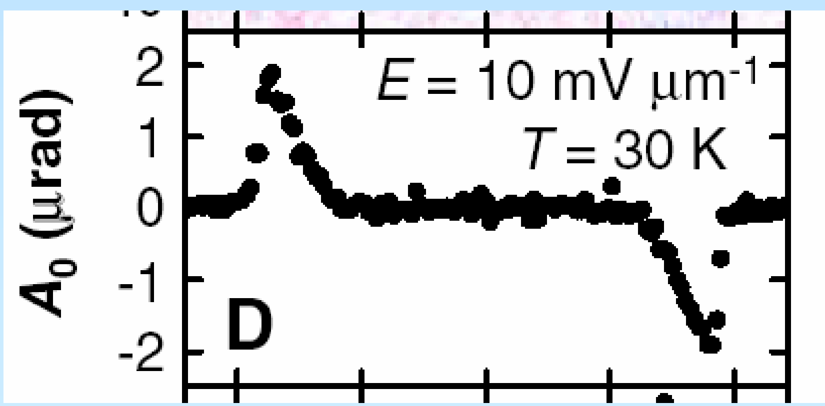
[1] J. E. Hirsch, Phys. Rev. Lett. **83**, 1834 (1999).
[2] M. I. Dyakonov and V. I. Perel, JETP Lett. **13**, 467 (1971); Phys. Lett. **35A**, 459 (1971).

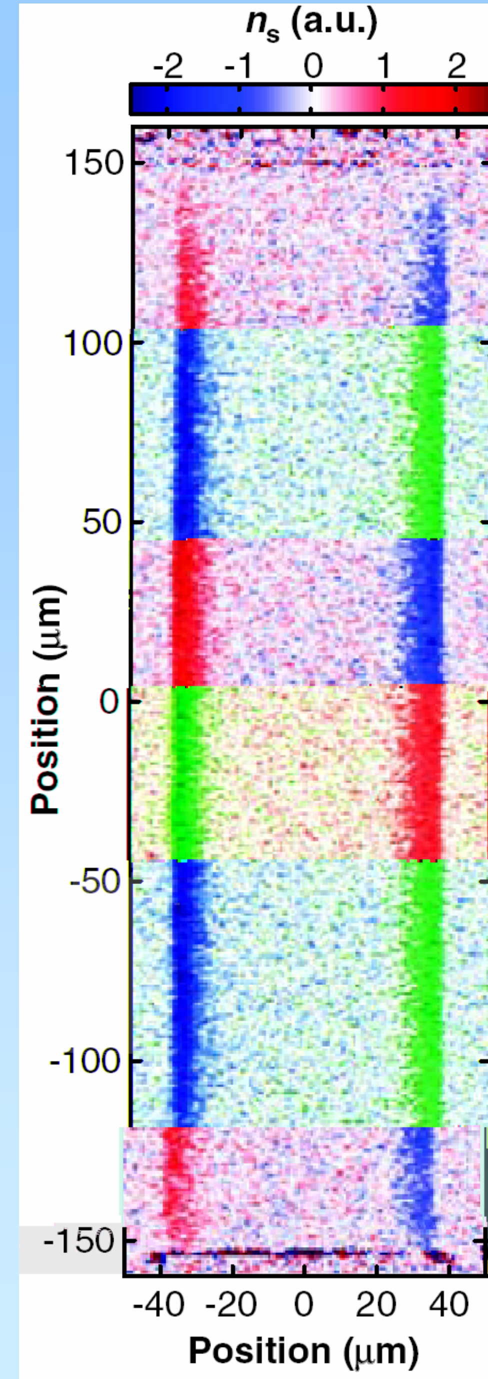
S. Murakami, et.al. Science 301,1348 (2003);

J. Sinova et.al. Phys.Rev.Lett. 92, 126603 (2004);

Y.K. Kato, et.al. Science 306,1910(2004)



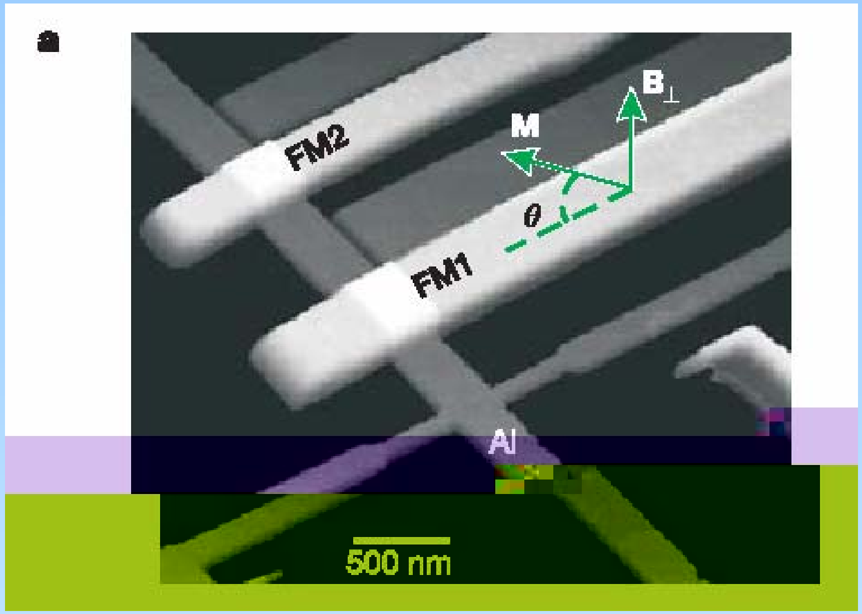


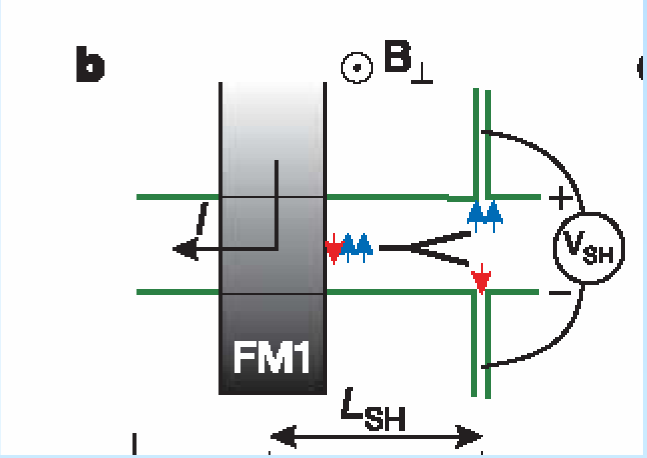


Y. K. Kato, R. C. Myers, A. C. Gossard, and D. D.
Awschalom, Science 306, 1910 (2004); V. Sih, R. C. Myers,
Y. K. Kato, W. H. Lau, A. C. Gossard, and D. D. ers
schalom, Nature Phys. **1**, 31 (2005); V. Sih, W. H. Lau, Aw
C. Myers, V. R. Horowitz, A. C. Gossard, and D. D.
Awschalom, Phys. Rev. Lett. **97**, 096605 (2006).
J. Wunderlich, B. Kaestner, J. Sinova, and T. Jungwirth,
Phys. Rev. Lett. **94**, 047204 (2005).
Phys. Rev. Lett.
a and M. Tinkham, Nature (London) **442**, S. O. Valenzuela
176 (2006).

Hall

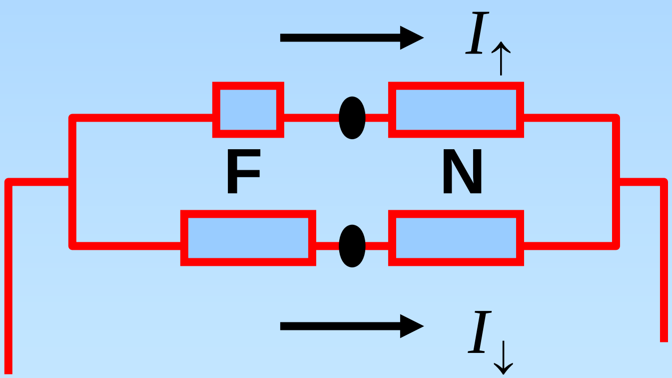
S.O. Valenzuela and M Tinkham, Nature 442, 176(2006)



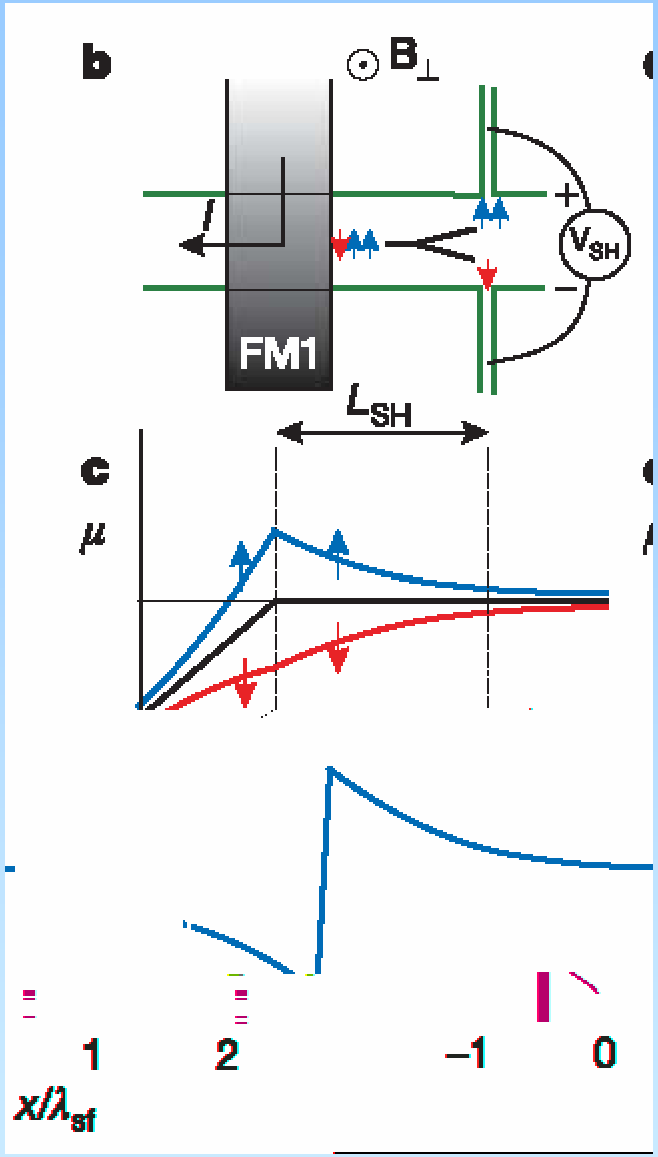


*

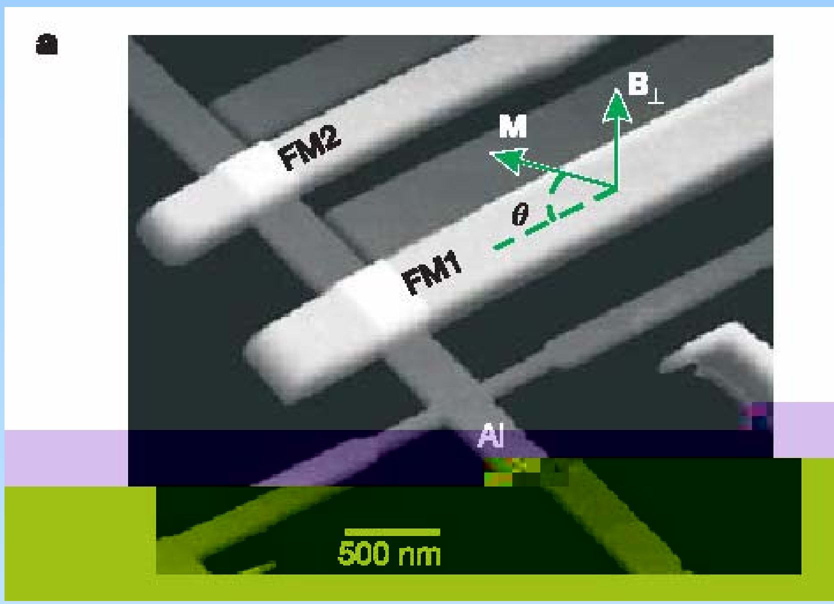
* Hall



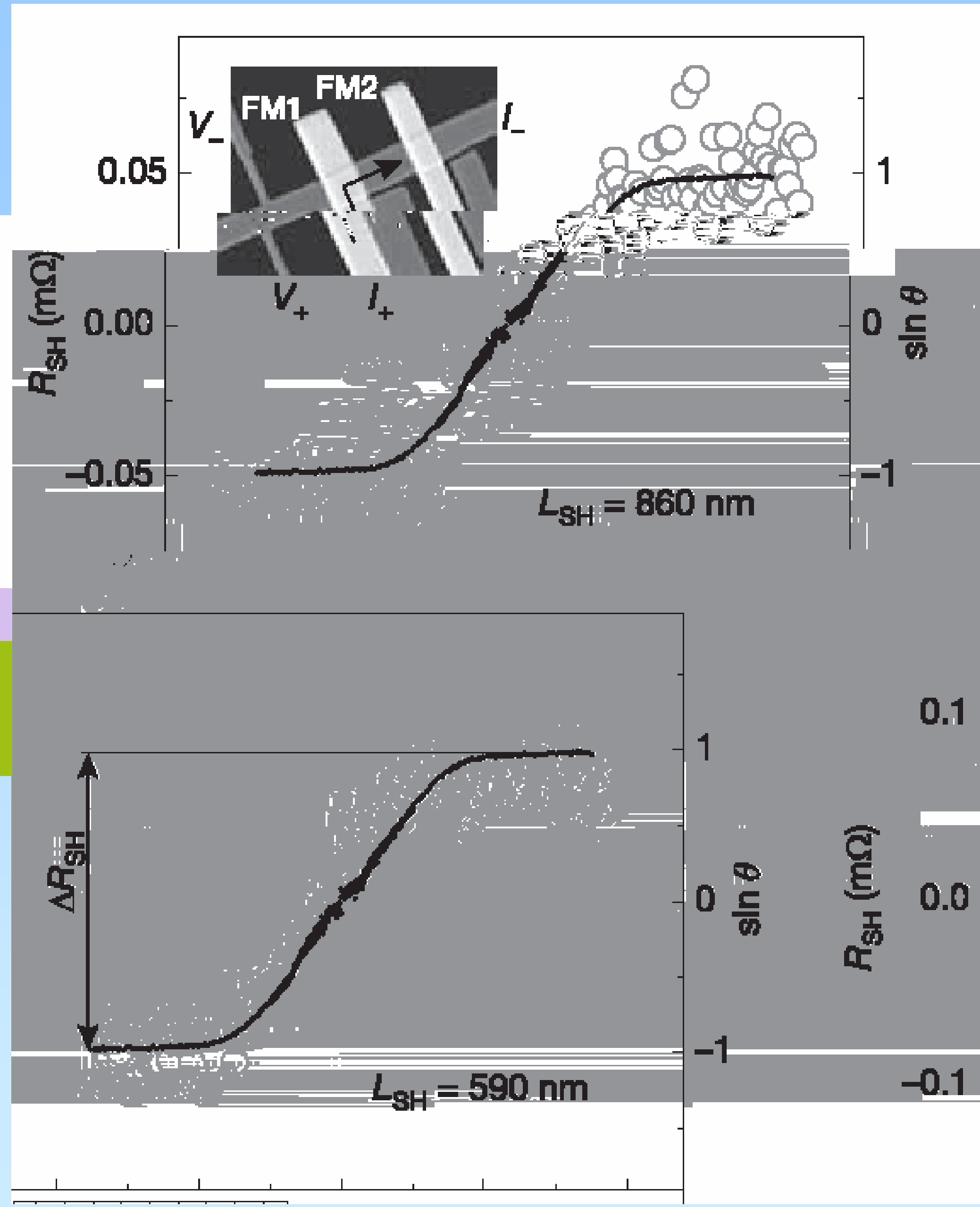
$I_\uparrow$
F
N
$I_\downarrow$



b
$B_\perp$
I
FM1
V_{SH}
+
−
L_{SH}
c
μ
1
2
−1
0
x/λ_{sf}



a
FM2
M
B⊥
θ
FM1
Al
500 nm



V−
FM1
FM2
I−
V+
I+
0.05
0.00
−0.05
1
0
−1
R_{SH} (mΩ)
sin θ
L_{SH} = 860 nm
ΔR_{SH}
1
0
−1
sin θ
R_{SH} (mΩ)
0.1
0.0
−0.1
L_{SH} = 590 nm

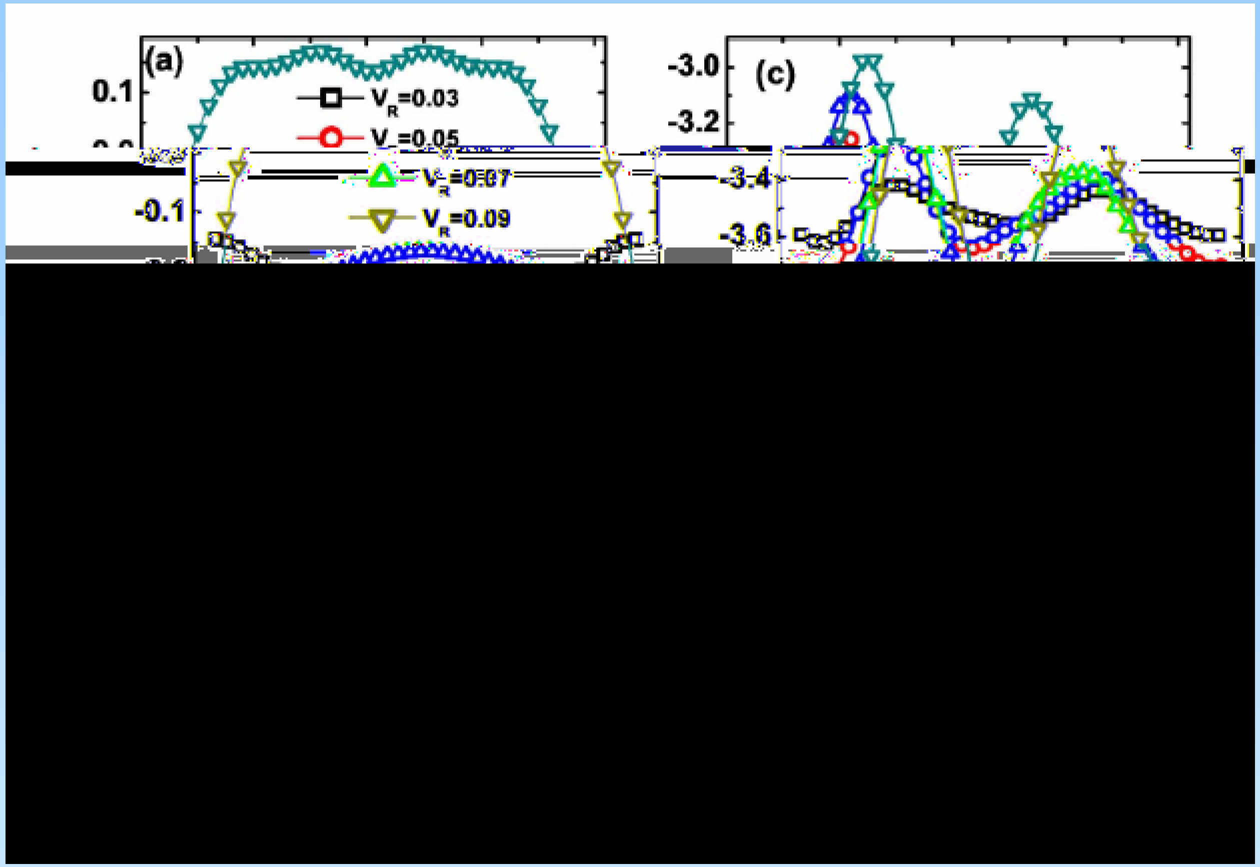
Hall **Hall**

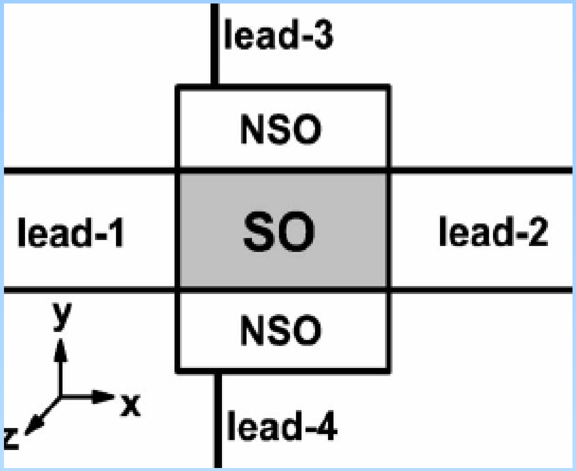
Hall

Hall

Hall **Hall**

Hall

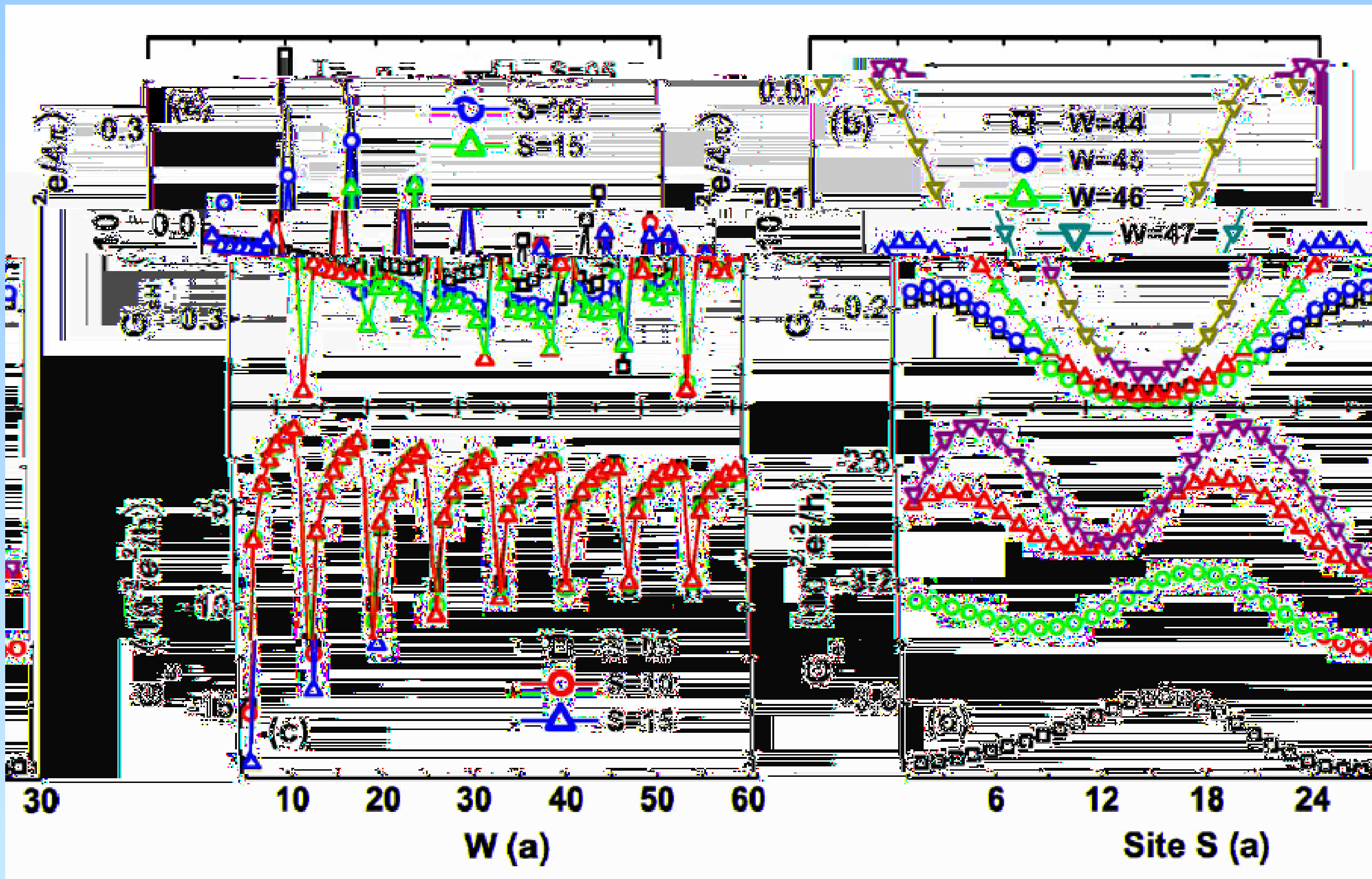


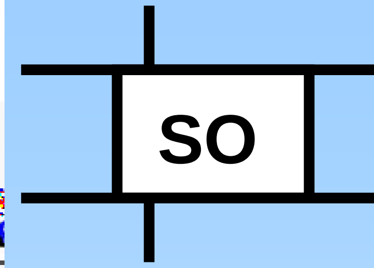


Hall

Hall **SO**

Y.X. Xing, Q.f. Sun, and J. Wang, PRB 73, 205339(2006)

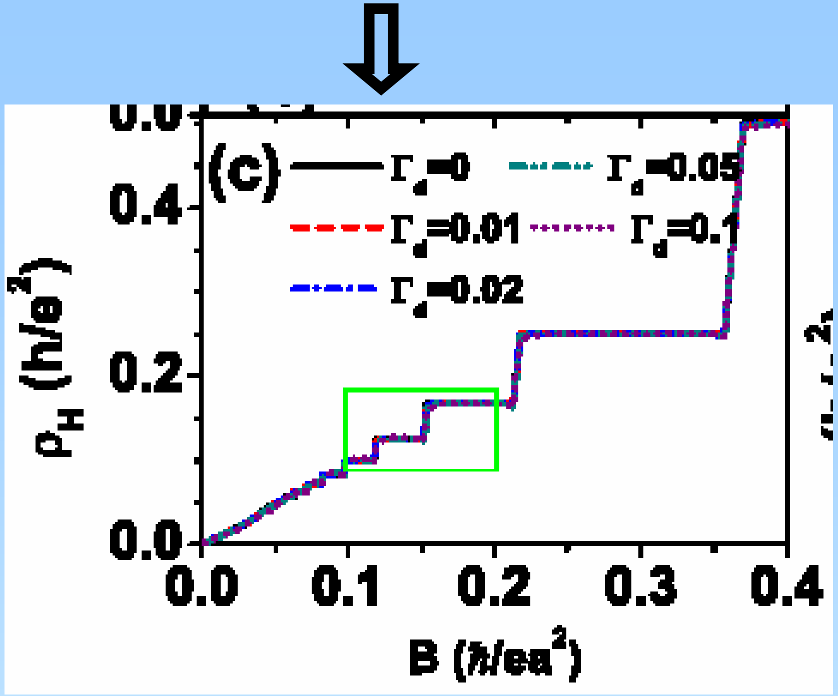


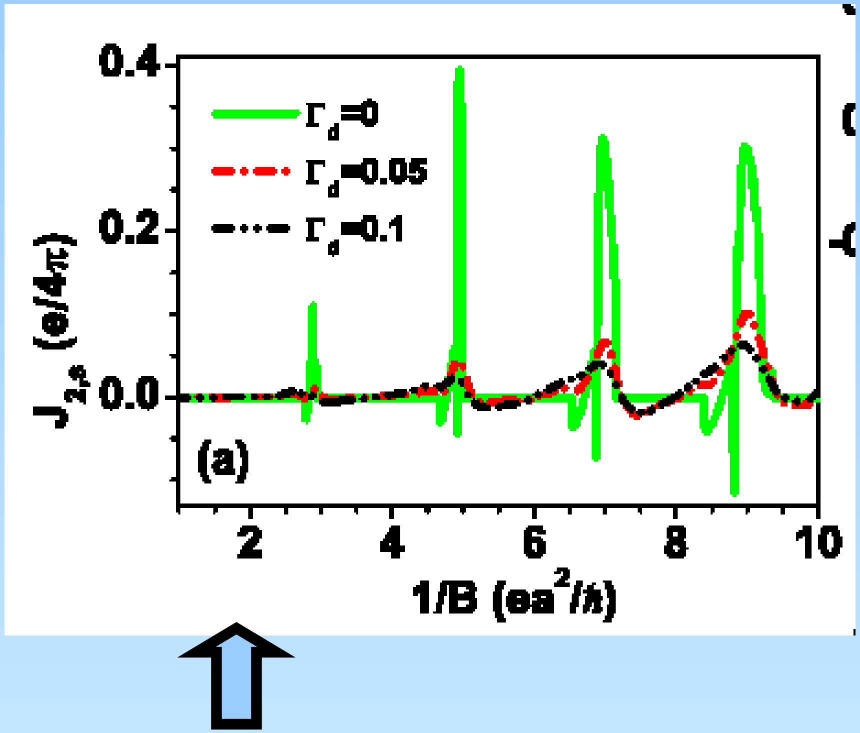


Hall

Y.X. Xing, Q.f. Sun, and J. Wang, PRB 73, 205339(2006)

Hall





Hall

Y.X. Xing, Q.f. Sun, and J. Wang, PRB 77, 115346(2008)

Hall

Hall

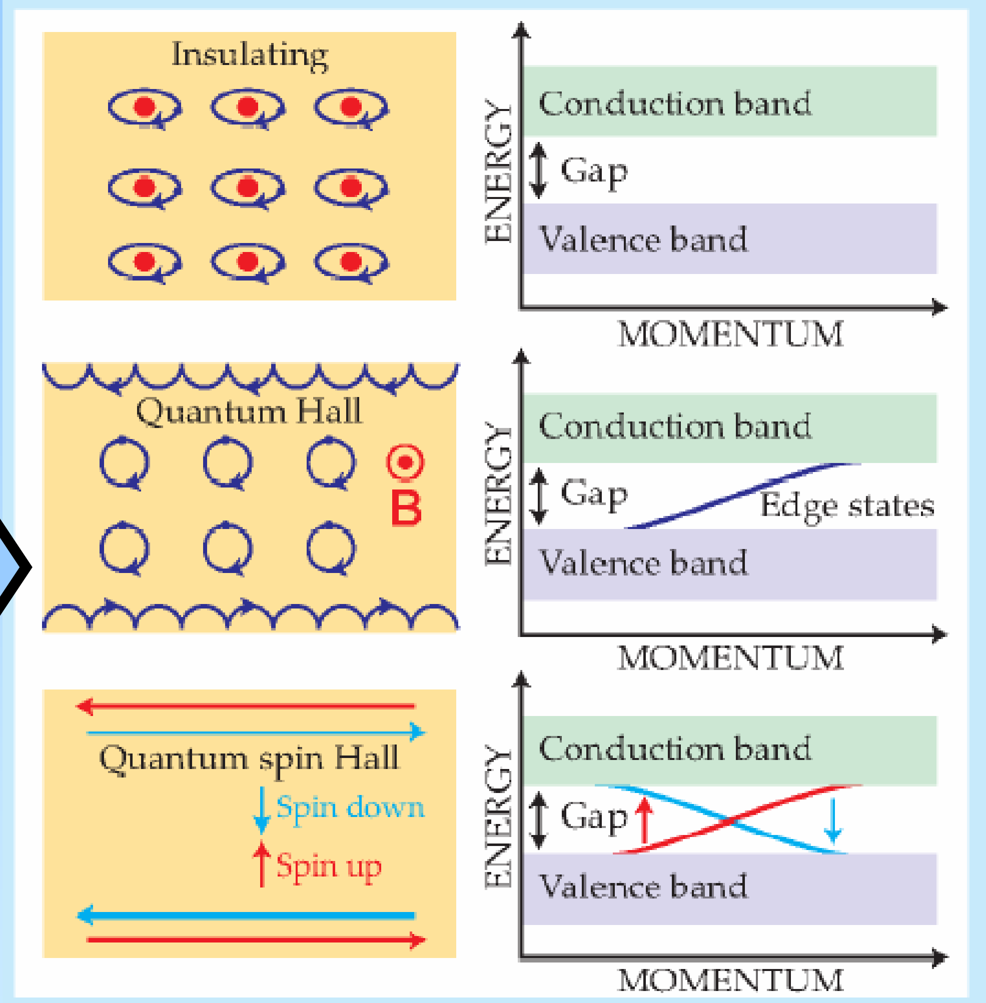
Hall

Hall

Hall

Hall

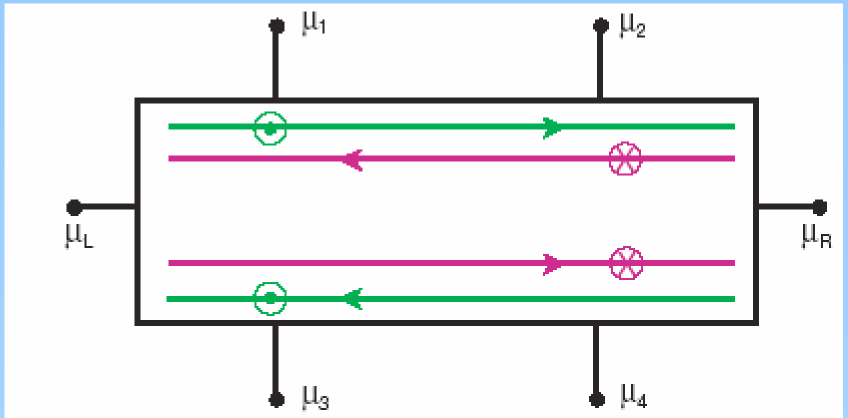
Hall



Hall

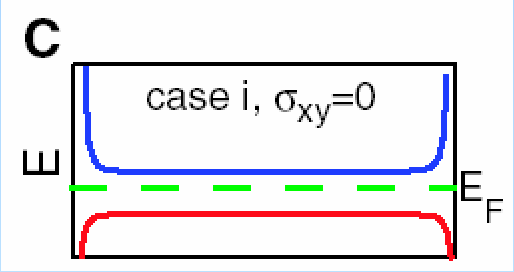
Hall Kane Mele

C.L. Kane and E.J. Mele, PRL 95,226801(2005); PRL 95,146802 (2005).

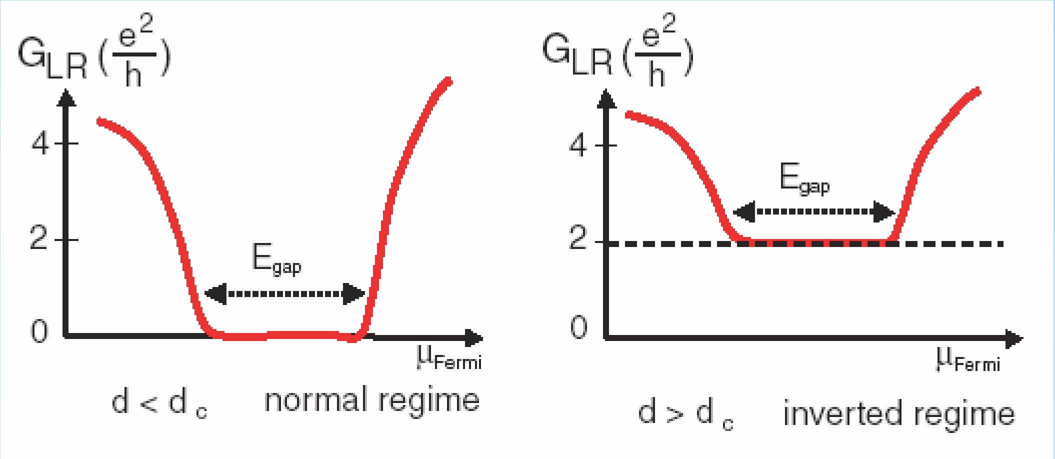


μ1
μ2
μL
μR
μ3
μ4

QSHE

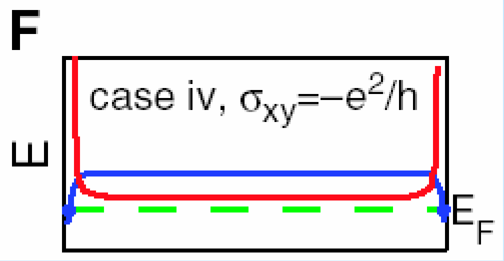


C
case i, σxy=0
E
EF

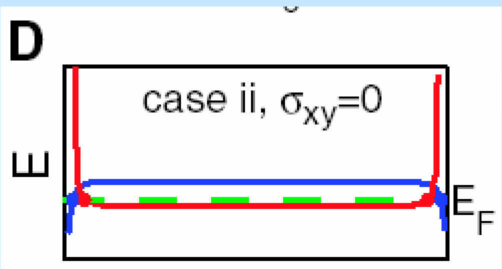


GLR (e2/h)
4
2
0
Egap
μFermi
d < d c
normal regime
GLR (e2/h)
4
2
0
Egap
μFermi
d > d c
inverted regime

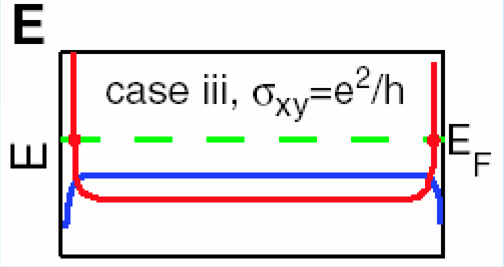
QSHE



F
case iv, σxy=−e2/h
E
EF

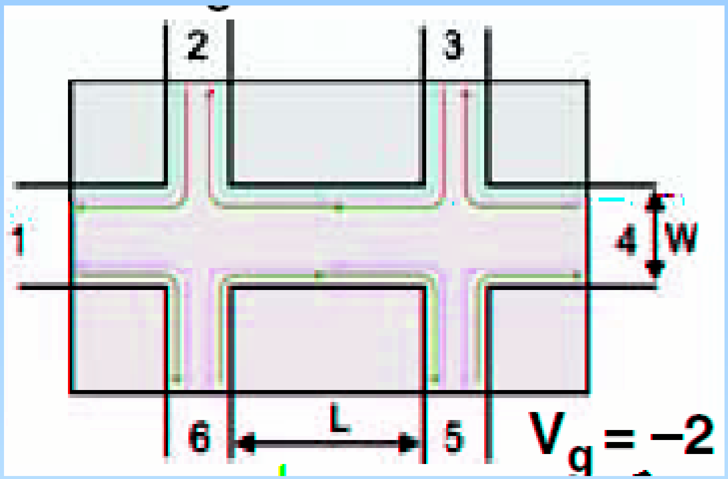


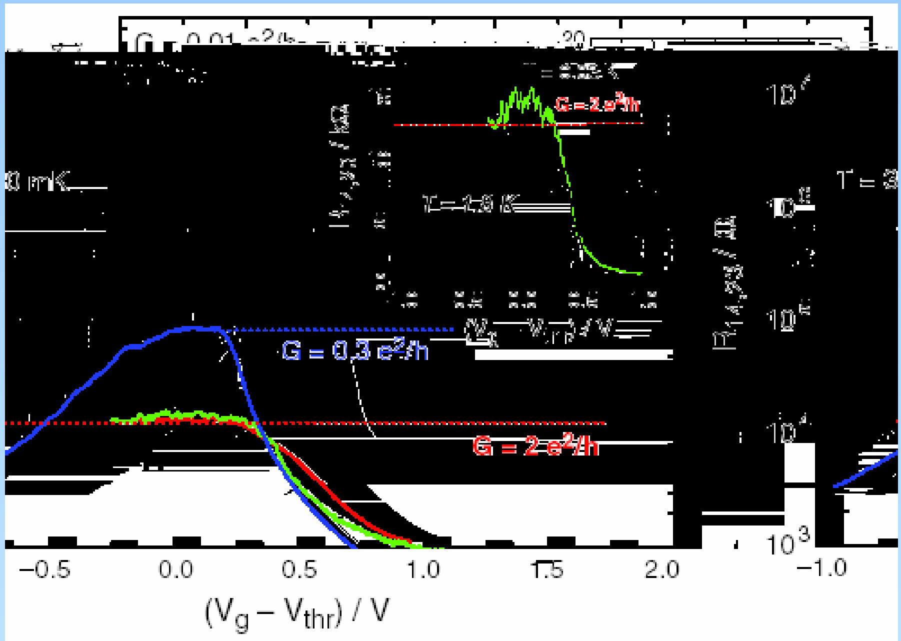
D
case ii, σxy=0
E
EF



E
case iii, σxy=e2/h
E
EF

Konig … Science 318, 766(2007).



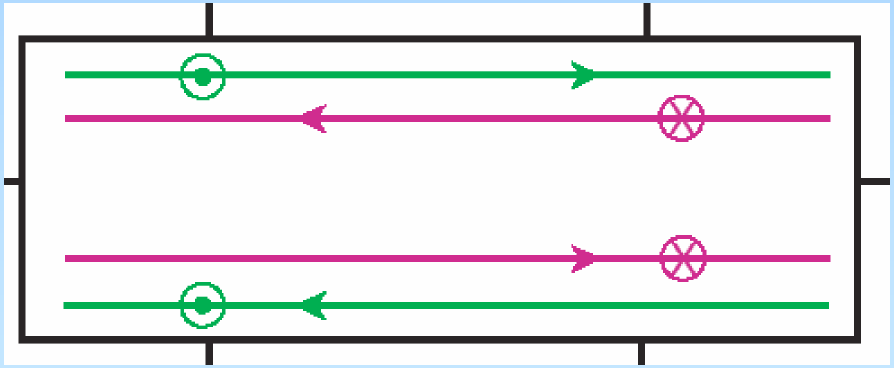


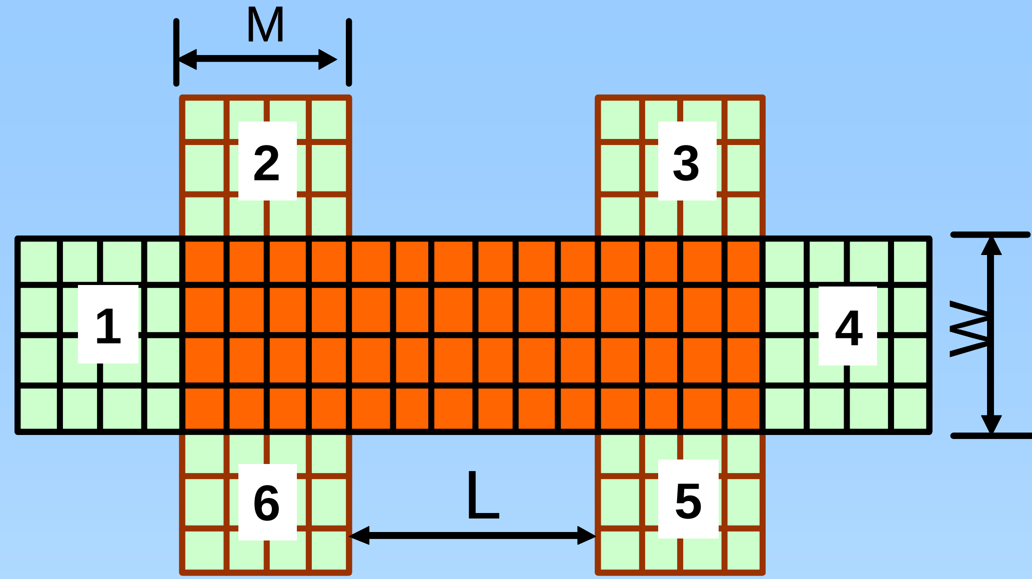
* 1X1 μm^2 1X0.5 μm^2

20X13.3 μm^2

* Hall Hall

Hall

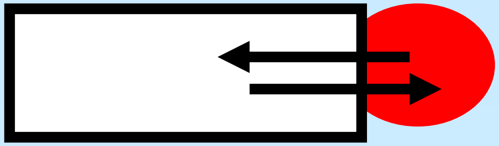




$$H = -[\sum_{<\mathbf{ij}>\sigma} t e^{i\eta(\sigma)\phi_{\mathbf{ij}}} c^{\dagger}_{\mathbf{i}\sigma} c_{\mathbf{j}\sigma} + H.c.] + [\sum_{\mathbf{i}k\sigma} \epsilon_{k\sigma} a^{\dagger}_{\mathbf{i}k\sigma} a_{\mathbf{i}k\sigma} + (t_{k\sigma} a^{\dagger}_{\mathbf{i}k\sigma} c_{\mathbf{i}\sigma} + H.c.)]$$

QF Sun, et al PRB71,165310(2005)

Buttiker



$$T_{pq} = Tr[\Gamma_{\mathbf{p}\sigma} G^{r} \Gamma_{\mathbf{q}\sigma} G^{a}]$$

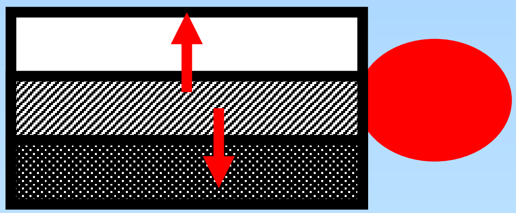
$$G^{r} = [G^{a}]^{\dagger} = [E\mathbf{I} - \mathbf{H}_{cen} - \sum_{p\sigma} \Sigma^{r}_{p\sigma}]^{-1}$$

$$\Gamma_{p\sigma} = i[\mathbf{\Sigma^{r}_{p\sigma}} - \mathbf{\Sigma^{r+}_{p\sigma}}]$$

$$J_{p\sigma} = \frac{e}{\hbar} \sum_{q \neq p} T^{\sigma}_{pq} (V_{p\sigma} - V_{q\sigma}),$$

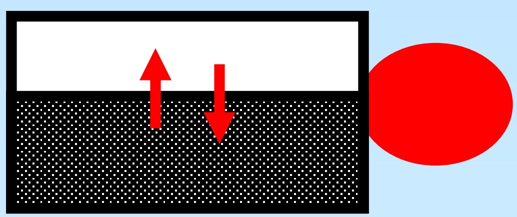
1 $J_{p\sigma}$ **0**

$V_{p\sigma}$



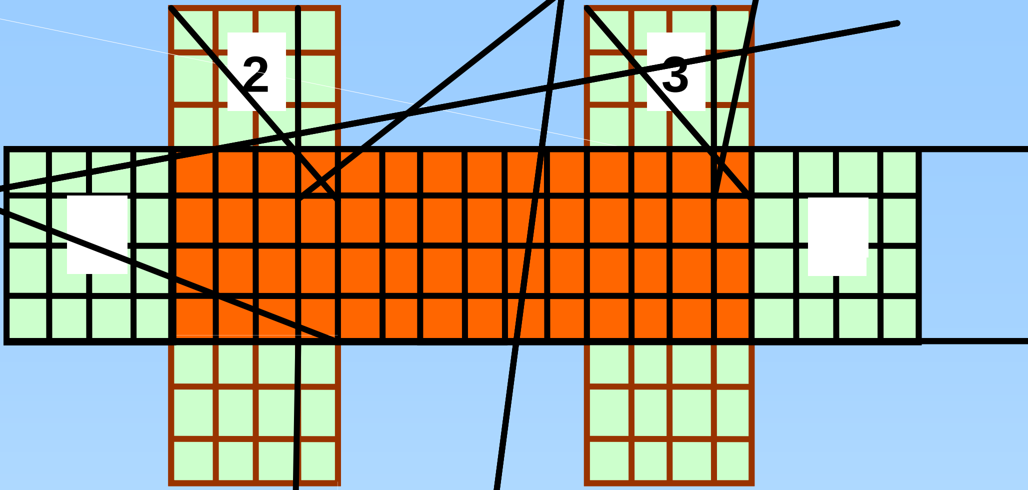
2 **0**

$V_{p\sigma}$

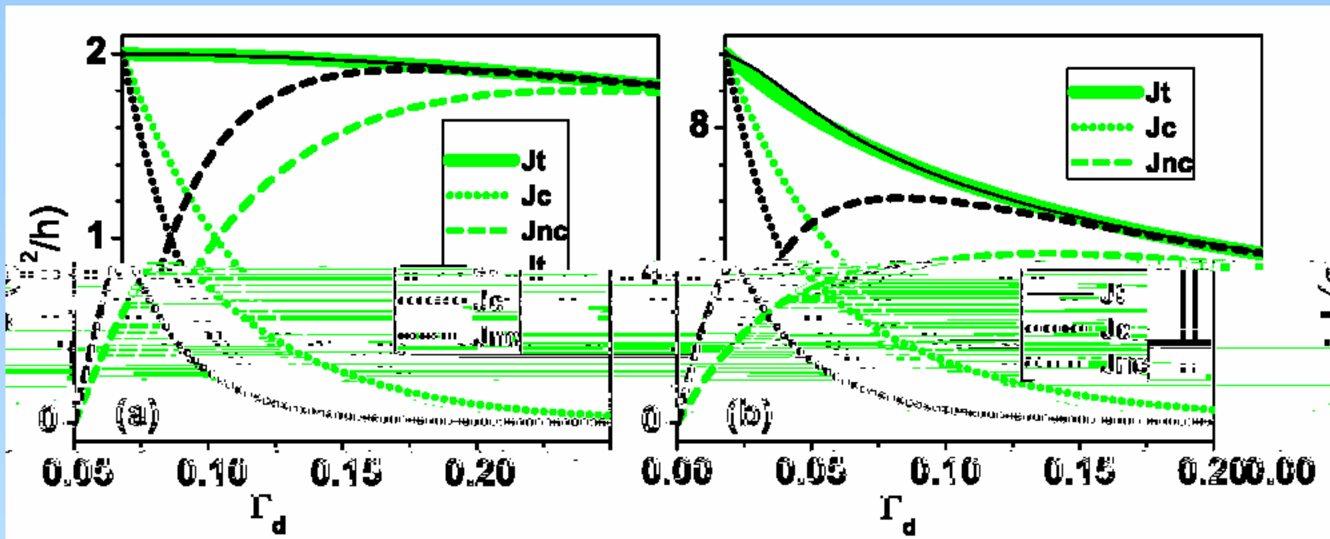


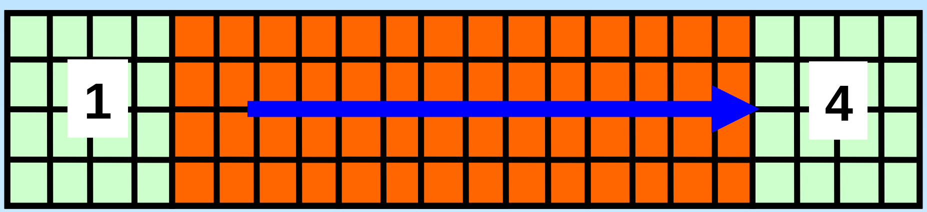
$V_{p\sigma}$ $J_{p\sigma}$

2,3,5,6

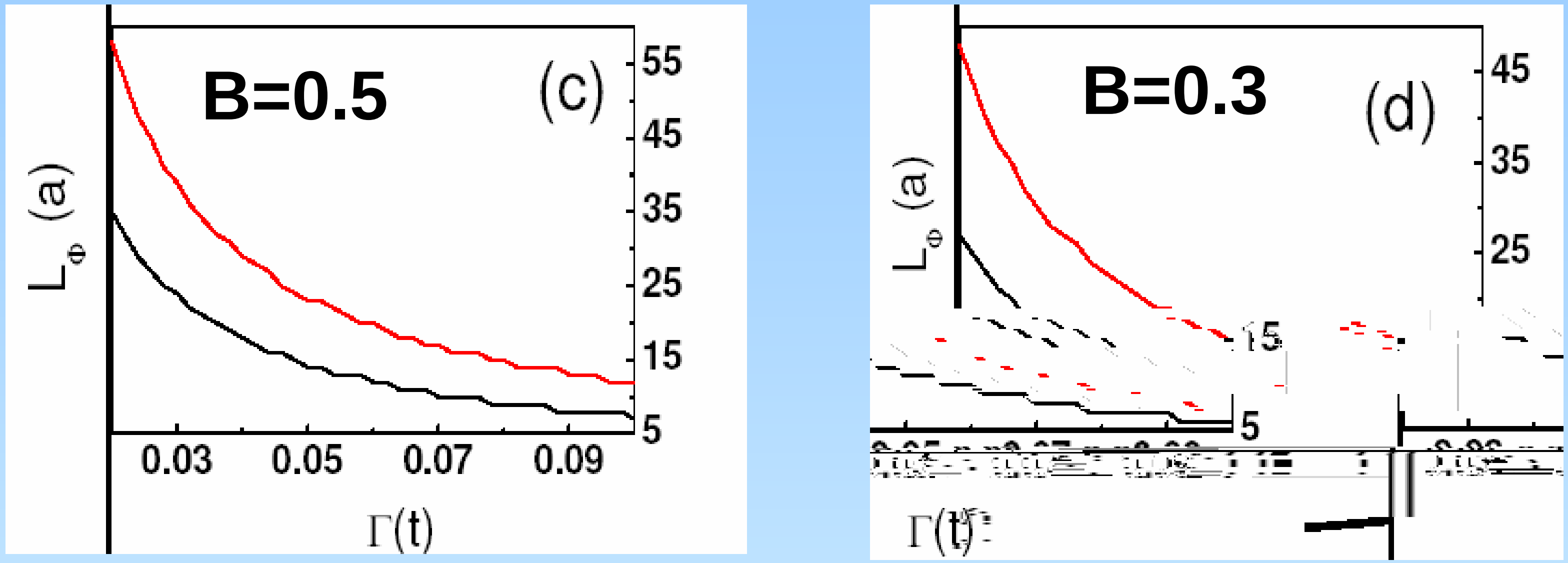


Γ

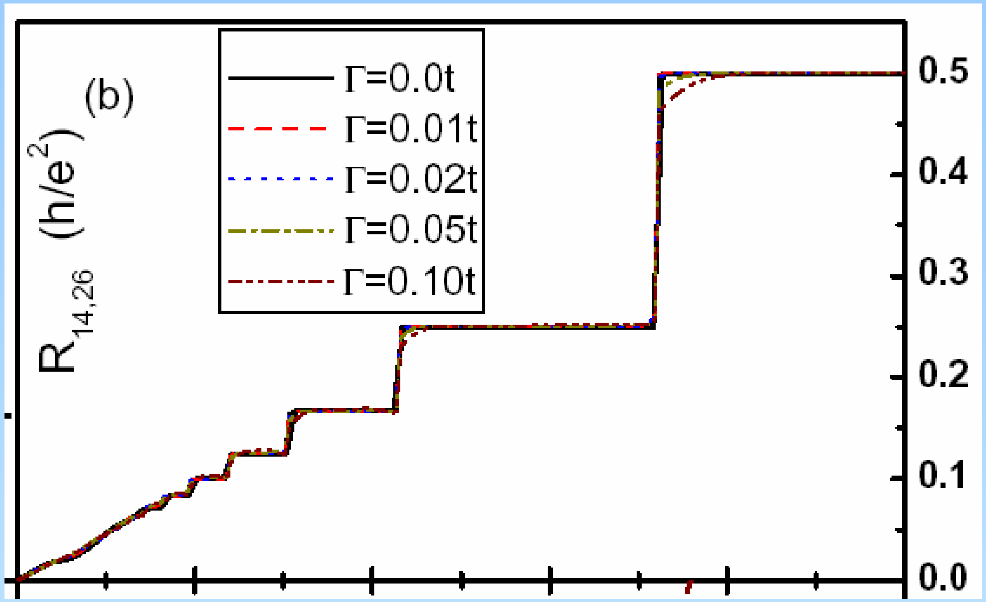




Γ

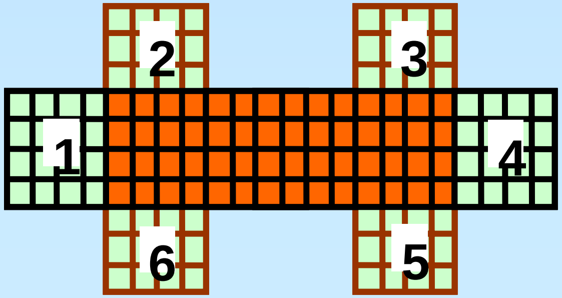

B=0.5
(c)
L_Φ (a)
55
45
35
25
15
5
0.03
0.05
0.07
0.09
Γ(t)
B=0.3
(d)
L_Φ (a)
45
35
25
15
5
Γ(t)

Hall



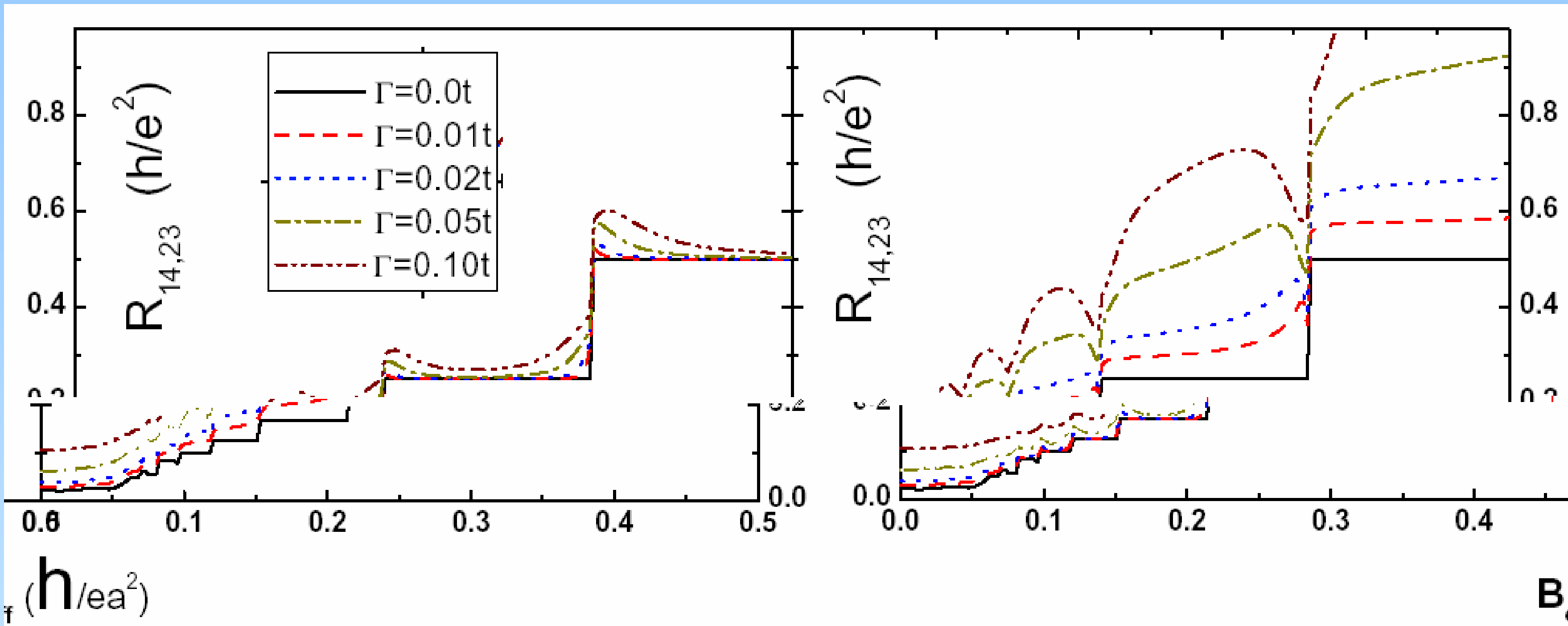
(b)
$R_{14,26}$ (h/e^2)
Γ=0.0t
Γ=0.01t
Γ=0.02t
Γ=0.05t
Γ=0.10t
0.5
0.4
0.3
0.2
0.1
0.0

Hall



2
3
1
4
6
5

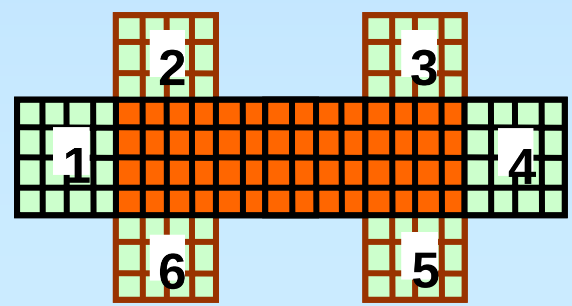
Hall



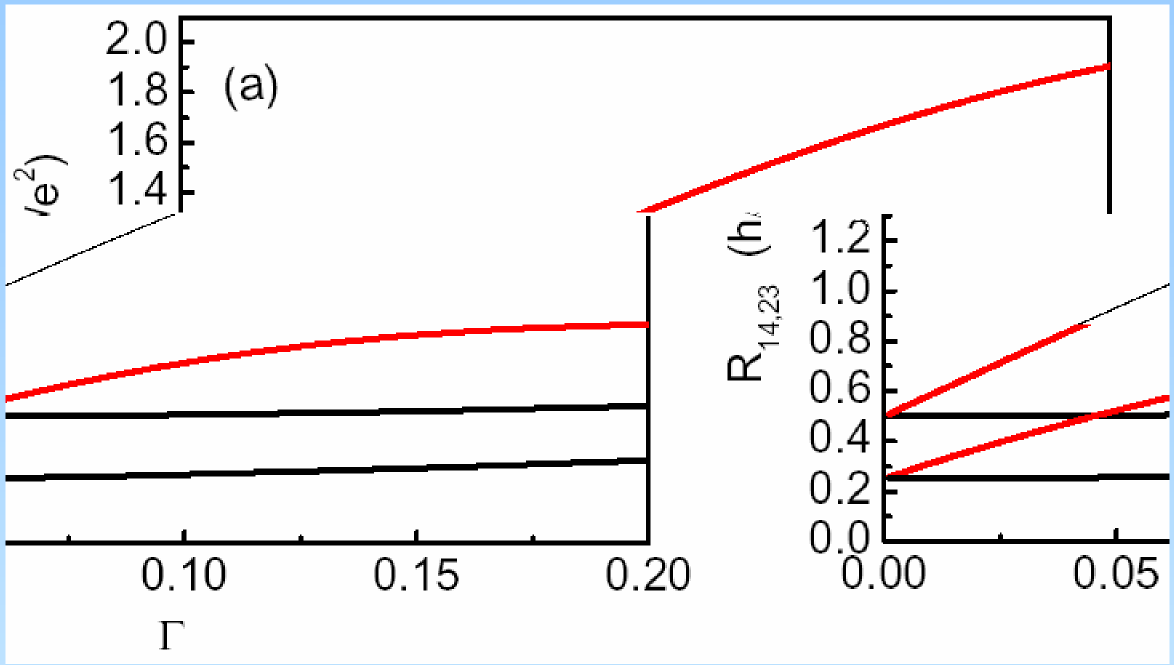
Hall

Hall

Hall

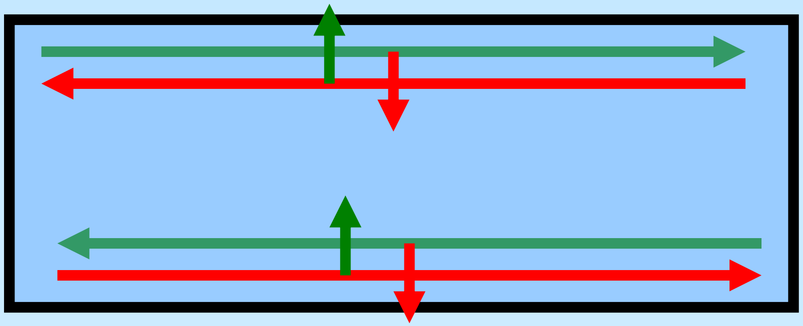


Hall



Hall

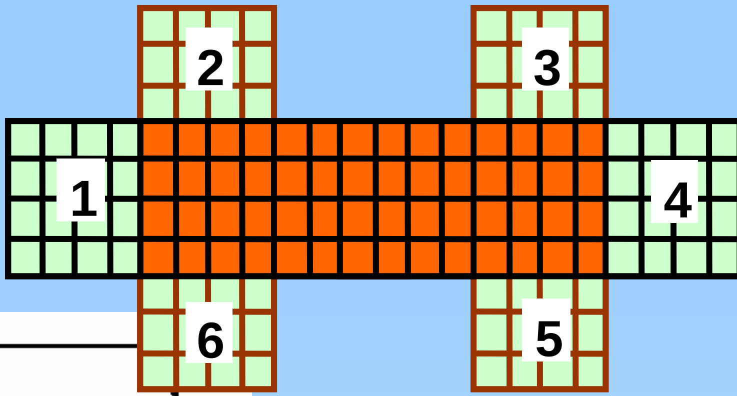
Hall

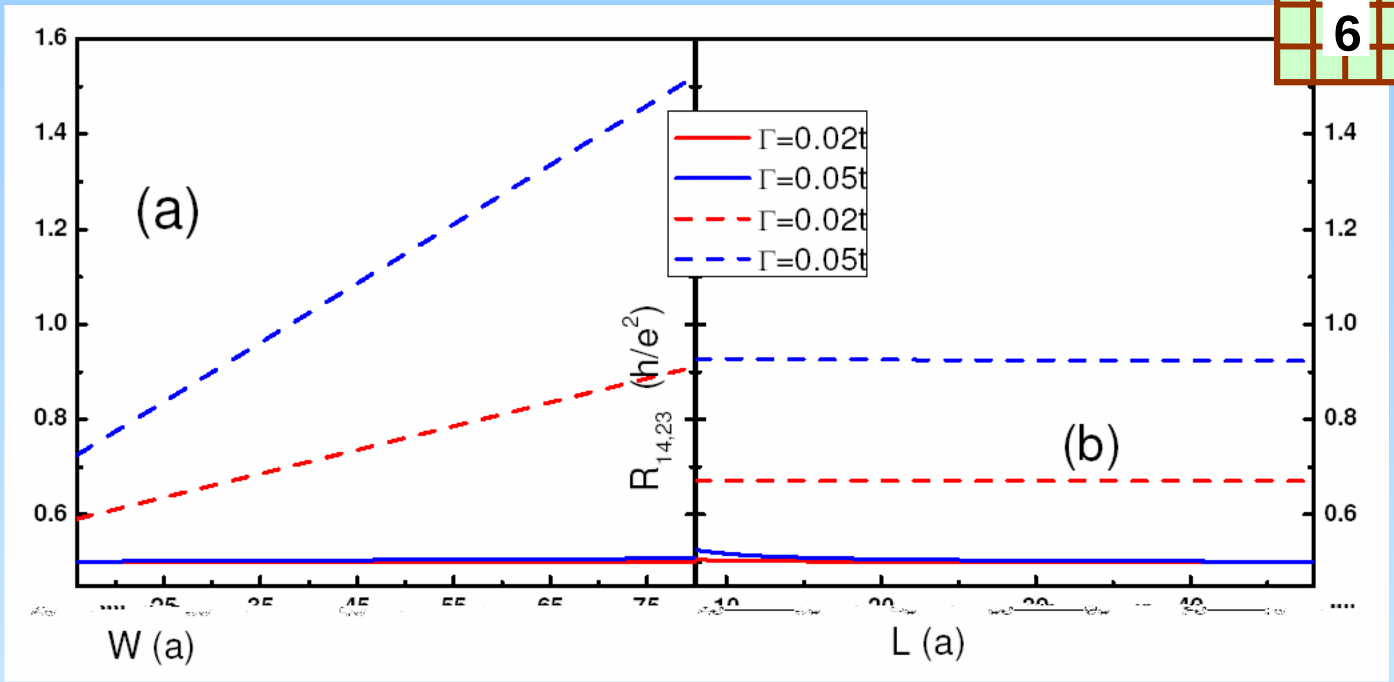


*****

***** **QSHE** **QHE**

Hall





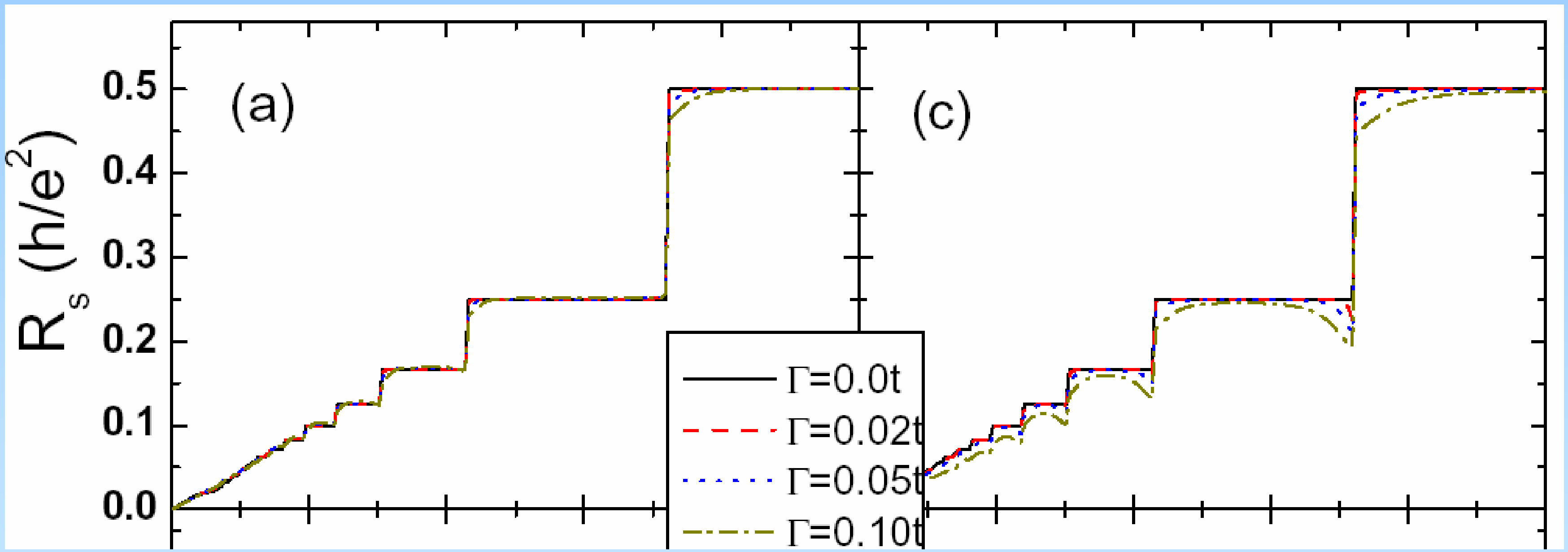
Hall

Hall

QSH

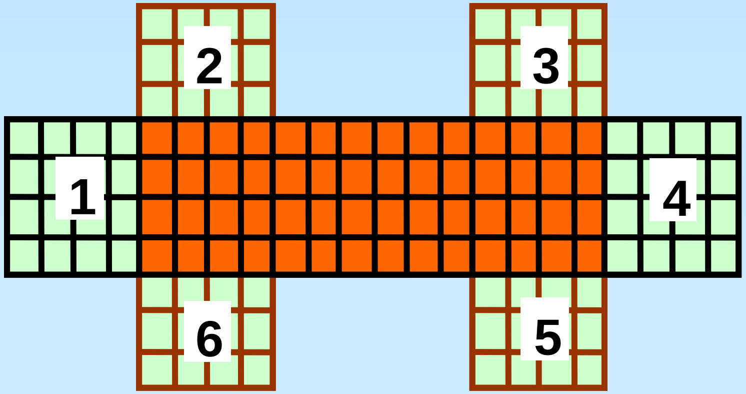
Hall

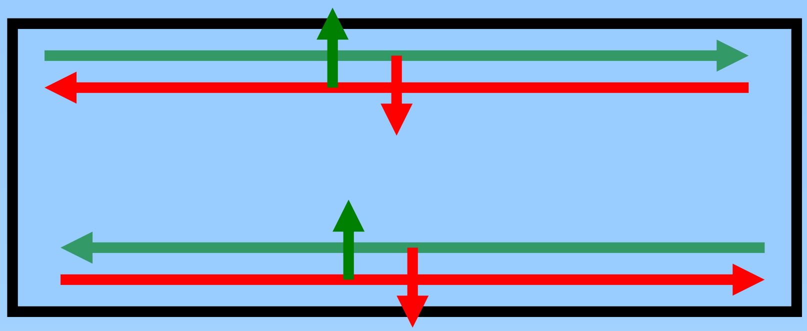
Hall

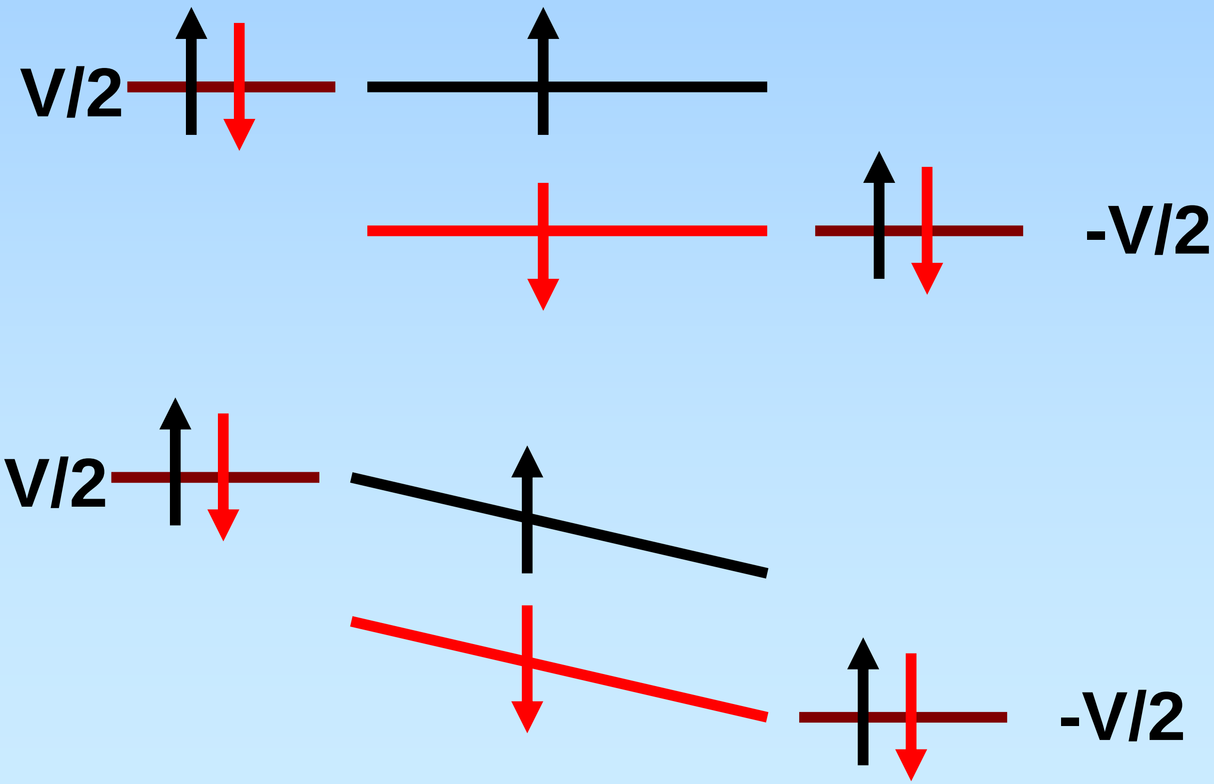


$$R_s = (V_{2\uparrow} - V_{2\downarrow})/I_{14}$$

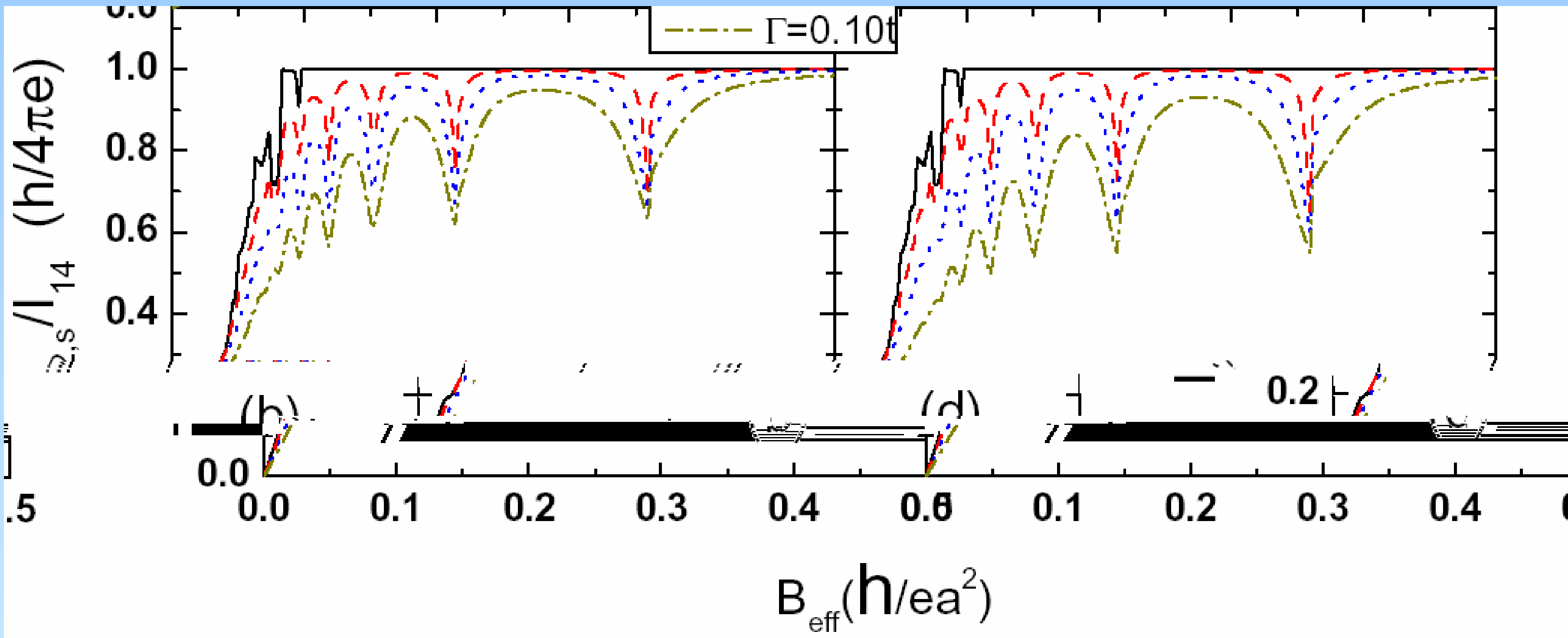
vs



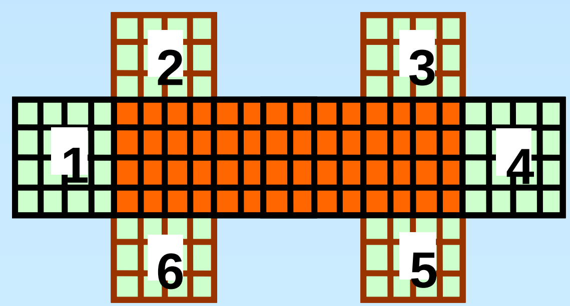




$$R_s = (V_{2\uparrow} - V_{2\downarrow})/I_{14}$$



Γ=0.10t
$I_{e,s}/I_{14}$ (h/4πe)
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.1
0.2
0.3
0.4
(b)
(d)
B_{eff}(h/ea^2)



1
2
3
4
5
6

Hall Hall

( )

:

Oklahoma State Univ.

McGill