Ginsburg-Landau Theory of Solids and Supersolids

Jinwu Ye
Penn State University

Outline of the talk:

1. Introduction: the experiment, broken symmetries…..
2. Ginsburg-Landau theory of a supersolid

3a. SF to NS transition

3b. SF to **SS** transition and global phase diagram

4. Vacancies induced supersolid
5. Excitations in SS
6. Supersolid in other systems
7. Conclusions

Acknowledgement

I thank

P. W. Anderson, T. Clark, M. Cole, B. Halperin, J. K. Jain , T. Leggett and G. D. Mahan especially **Moses Chan** and **Tom Lubensky** for many helpful discussions and critical comments

References:

Jinwu Ye, Phys. Rev. Lett. ***97****,125302, 2006*
Jinwu Ye, cond-mat/0701694
Jinwu Ye, cond-mat/0603269,

1.Introduction

Cosmology
Astrophysics

General Relativity → Gravity

$\sim 10^{19}\, Gev$

String Theory ???

Symmetry and symmetry breaking at different energy (or length) scales

Quantum Mechanics
Quantum Field Theory

Elementary particle physics $\sim 10^{2}\, Gev$

Atomic physics
Many body physics
Condensed matter physics
Statistical mechanics

2d conformal field theory

$\sim mK$

Emergent quantum Phenomena !

Is **supersolid** a new emergent phenomenon ?

P.W. Anderson: More is different !
Emergent Phenomena !

Macroscopic quantum phenomena emerged in ~ 10^{23} interacting atoms, electrons or spins

States of matter break different symmetries at low temperature

- Superconductivity
- Superfluid
- Quantum Hall effects
- Quantum Solids
- **Supersolid ?**
- **……..???**

- High temperature superconductors
- Mott insulators
- Quantum Anti-ferromagnets
- Spin density wave
- Charge density waves
- Valence bond solids
- Spin liquids ?
- …………???

Is the **supersolid** a new state of matter ?

What is a liquid ?

A liquid can flow with some viscosity
It breaks no symmetry, it exists only at high temperatures

At low temperatures, any matter has to have some orders which break some kinds of symmetries.

What is a solid ?

A solid can **not** flow $\vec{G}:$ Reciprocal lattice vectors

Density operator: $n(\vec{x}) = n_0 + \sum_{\vec{G}} n_{\vec{G}} e^{i\vec{G}\bullet\vec{x}}$

Breaking translational symmetry: $\Longrightarrow$ Lattice phonons $\vec{u}$

$H_2O, H_2, \ldots\ldots$

Essentially all substances take solids except $^{4}He,\ ^{3}He$

What is a superfluid ?

A superfluid can flow without viscosity

Complex order parameter: $\psi = |\psi| e^{i\theta}$

Breaking Global U(1) symmetry: $\Rightarrow$ Superfluid phonons θ

What is a **supersolid** ?

A **supersolid** is a new state which has both crystalline order and superfluid order.

Can a supersolid exist in nature, especially in He4 ?

Large quantum fluctuations in He4 make it possible

The **supersolid** state was theoretically speculated in 1970:

Andreev and I. Lifshitz, 1969. Bose-Einstein Condensation (BEC) of vacancies leads to supersolid, classical hydrodynamices of vacancies.

G. V. Chester, 1970, Wavefunction with both BEC and crystalline order, a supersolid cannot exist without vacancies or interstitials

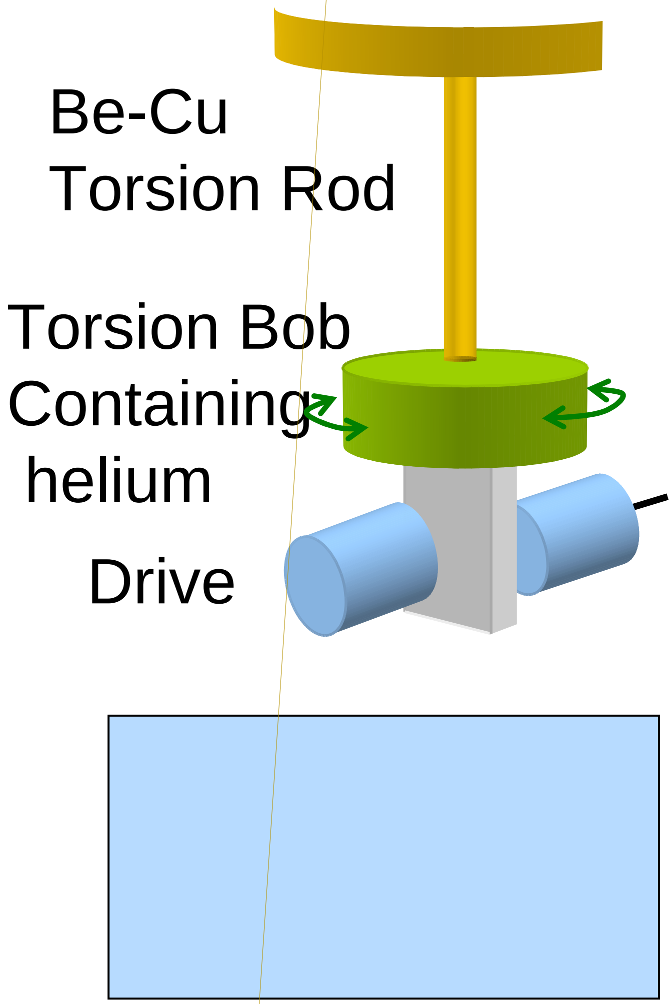
A. J. Leggett, 1970, **Non-Classical Rotational Inertial** (NCRI) of supersolid He4 $I = I_{cl}(1-\rho_s/\rho), \ \rho_s/\rho \leq 10^{-4}$

quantum exchange process of He atoms can also lead to a supersolid even in the absence of vacancies,

W. M. Saslow , 1976, improve the upper bound $\rho_s / \rho \leq 10^{-2}$

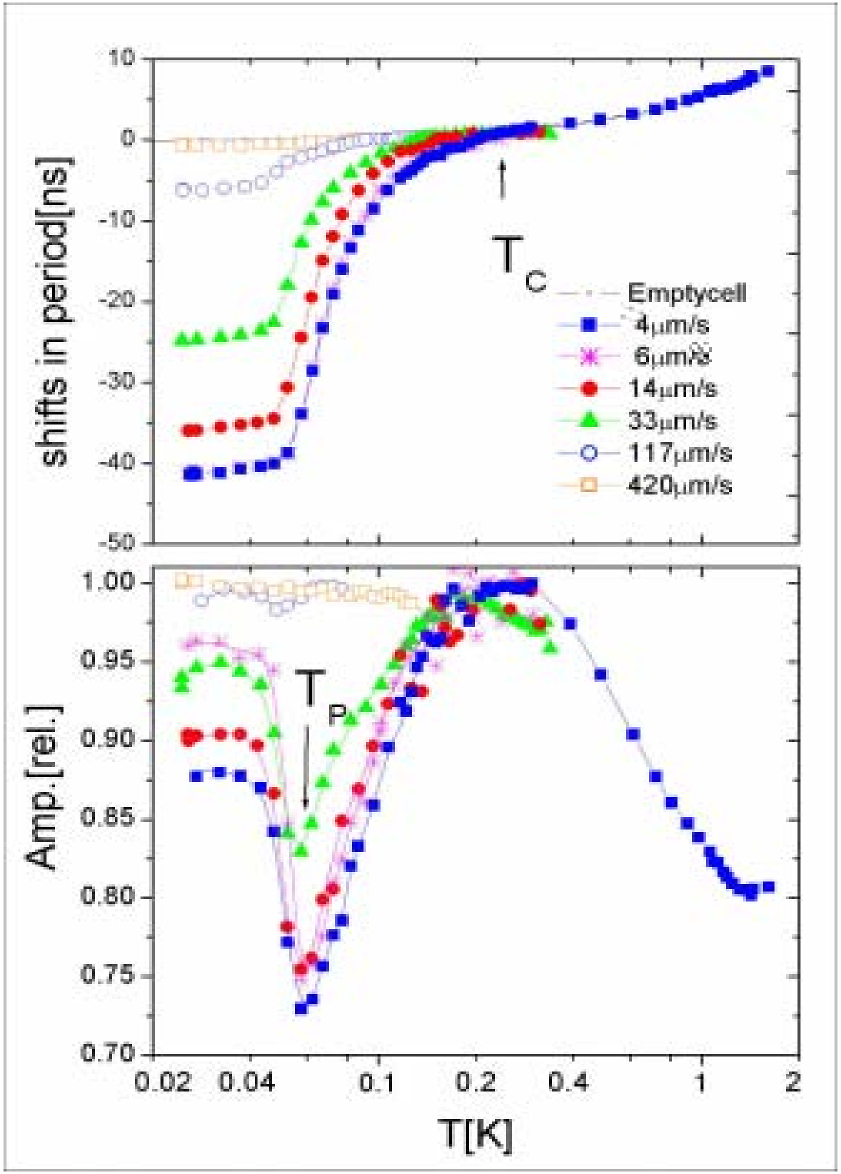
Over the last 35 years, a number of experiments have been designed to search for the supersolid state without success.

Torsional oscillator is ideal for the detection of superfluidity



Kim and Chan

By Torsional Oscillator experiments



Science 305, 1941 (2004)

Soild He4 at 51 bars

1 ~ 2% NCRI appears below 0.25K

Strong $|v|_{max}$ dependence (above 14μm/s)

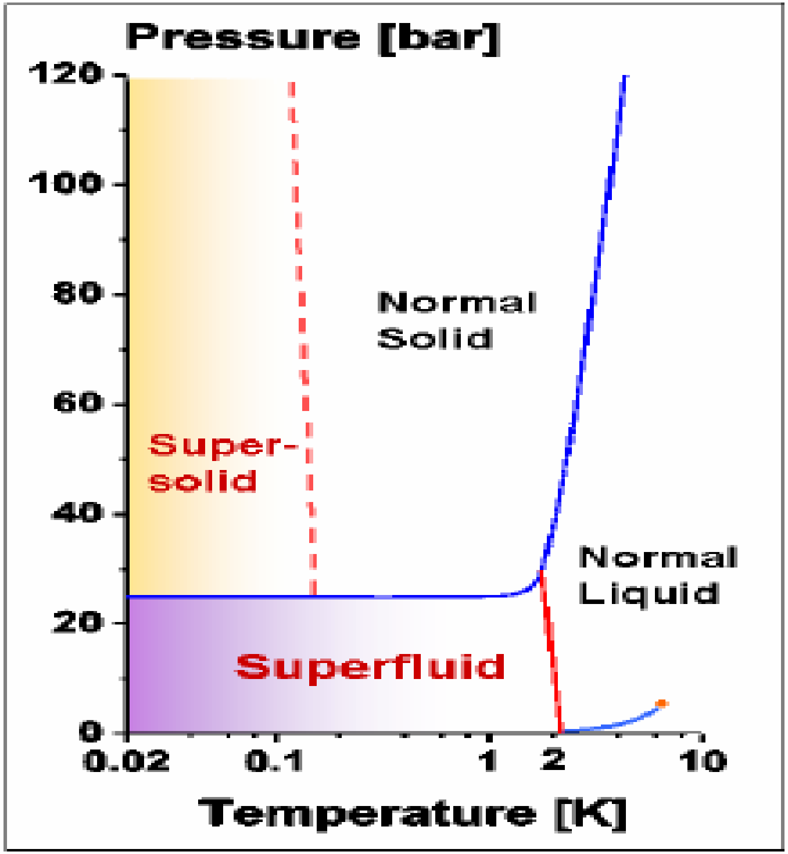
$$\tau = 2\pi\sqrt{\frac{I}{K}}$$

$$I = I_{cl}(1 - \rho_s / \rho)$$

Very recently, there are three experimental groups one in US, two in Japan
Confirmed (?) PSU's experiments.

1. *A.S. Rittner and J. D. Reppy, cond-mat/0604568*
2. *M. Kubota et al*
3. *K. Shirahama et al*

Possible phase diagram of Helium 4



PSU's experiments have rekindled great theoretical interests in the possible supersolid phase of He4

(1) Numerical (Path Integral quantum Monte-Carlo) approach:

- D. M. Ceperley, B. Bernu, Phys. Rev. Lett. 93, 155303 (2004);
- N. Prokof'ev, B. Svistunov, Phys. Rev. Lett. 94, 155302 (2005);
- E. Burovski, E. Kozik, A. Kuklov, N. Prokof'ev, B. Svistunov, Phys. Rev. Lett., 94,165301(2005).
- M. Boninsegni, A. B. Kuklov, L. Pollet, N. V. Prokof'ev, B. V. Svistunov, and M. Troyer,
- Phys. Rev. Lett. {\bf 97}, 080401 (2006).

Superfluid flowing in grain boundary ?

(2) Phenomenological approach:

- Xi Dai, Michael Ma, Fu-Chun Zhang, Phys. Rev. B 72, 132504 (2005)
- P. W. Anderson, W. F. Brinkman, David A. Huse, Phys. Rev. Lett. 96, 055301 (2006)
- A. T. Dorsey, P. M. Goldbart, J. Toner, Science 18 Nov. 2005
-

Supersolid ?

……

Uncertainty Relation: $[x, p] = i, \ \Delta x \bullet \Delta p \geq 1$

Cannot measure position and momentum precisely simultaneously !

$$[N_b, \phi] = i, \quad \Delta N_b \bullet \Delta \phi \geq 1$$

Cannot measure phase and density $n(\vec{x})$ precisely simultaneously !

Superfluid: phase order in ϕ

Solid: density order in $n(\vec{x})$

How to reconcile the two extremes into a supersolid ?

The solid is **not** perfect: has either vacancies or interstitials whose BEC may lead to a supersolid

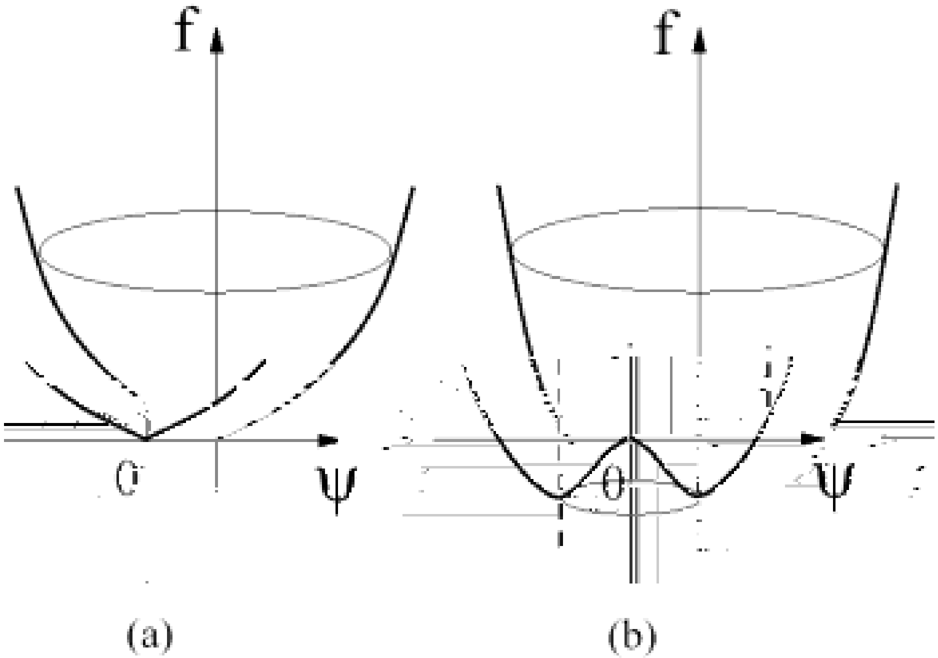
2. Ginsburg-Landau (GL) Theory of a supersolid

(1) GL theory of Liquid to superfluid transition:

Complex order parameter: $\psi = |\psi| e^{i\theta}$

$$f_{L-SF} = K|\nabla\psi|^2 + t|\psi|^2 + u|\psi|^4 + \cdots$$

Invariant under the U(1) symmetry: $\theta \to \theta + \phi$



(a) In liquid:

$$t > 0, < \psi >= 0$$

U(1) symmetry is respected

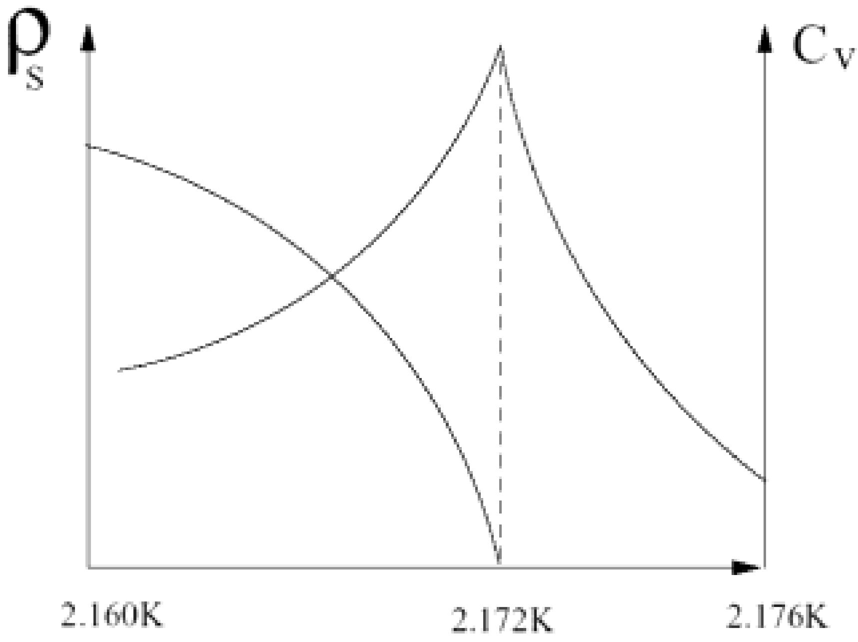
(b) In superfluid:

$$t < 0, < \psi > \neq 0$$

U(1) symmetry is broken

Both sides have the translational symmetry: $\psi(\vec{x}) \rightarrow \psi(\vec{x} + \vec{a})$

3d XY model to describe the λ (cusp) transition



Greywall and Ahlers, 1973
Ahler, 1971

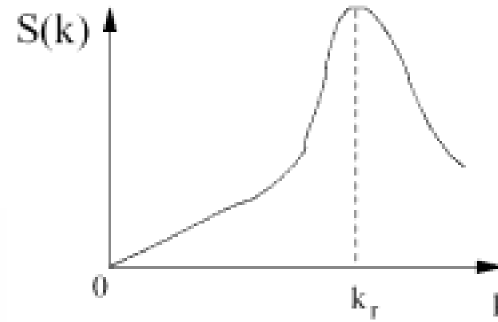
$$C_v \sim |t|^{-\alpha}, \rho_s \sim |t|^{\nu}$$

$$\alpha = -0.013 < 0, \nu = 0.672$$

(2) GL theory of Liquid to solid transition:

Density operator: $$n(\vec{x}) - n_0 = \delta n(\vec{x}) = \sum_{\vec{G}}{}' n_{\vec{G}} e^{i\vec{G}\bullet\vec{x}}$$

Order parameter: $n_{\vec{G}},\ \vec{G} \neq 0$



$$f_{L-NS} = \sum_{\vec{G}} \frac{1}{2} r_{\vec{G}} |n_{\vec{G}}|^2 - w \sum_{\vec{G}_1,\vec{G}_2,\vec{G}_3} n_{\vec{G}_1} n_{\vec{G}_2} n_{\vec{G}_3} \delta_{\vec{G}_1+\vec{G}_2+\vec{G}_3,0} + u \sum_{\vec{G}_1,\vec{G}_2,\vec{G}_3,\vec{G}_4} n_{\vec{G}_1} n_{\vec{G}_2} n_{\vec{G}_3} n_{\vec{G}_4} \delta_{\vec{G}_1+\vec{G}_2+\vec{G}_3+\vec{G}_4,0} + \cdots$$

$r_G = r + c(G^2 - k_r^2)^2,\ S(k) \sim 1/r_G$

$\delta n(\vec{x}) \rightarrow -\delta n(\vec{x})$ (crossed out)

Invariant under translational symmetry:

$n(\vec{x}) \rightarrow n(\vec{x} + \vec{a}),\ n_{\vec{G}} \rightarrow n_{\vec{G}} e^{i\vec{G}\bullet\vec{a}}$ $\vec{a}$ is any vector

In liquid: $r > 0, \langle n_{\vec{G}} \rangle = 0$

In solid: $r < 0, \langle n_{\vec{G}} \rangle \neq 0$

Translational symmetry is broken down to the lattice symmetry:

$\vec{a} = \vec{R},\ \vec{G} \bullet \vec{R} = 2\pi$ $\vec{R}$ is any lattice vector

$$n(\vec{x}) \rightarrow n(\vec{x} + \vec{R}),\ n_{\vec{G}} \rightarrow n_{\vec{G}} e^{i\vec{G}\bullet\vec{R}} = n_{\vec{G}}$$

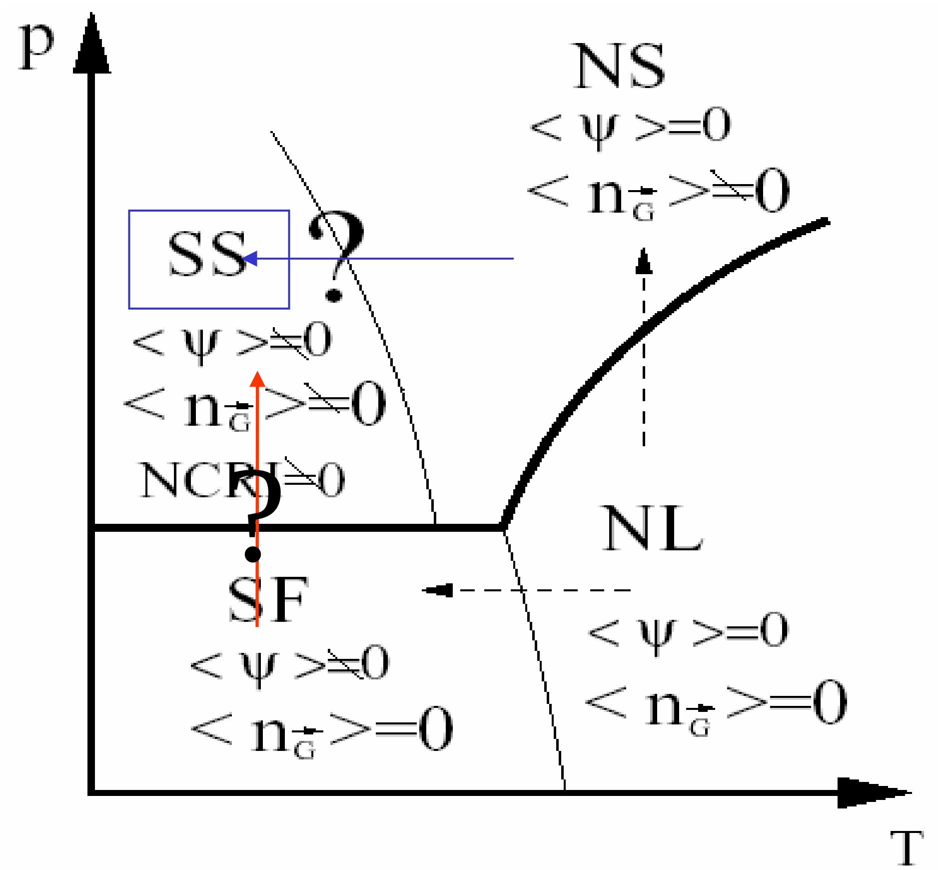
NL to SF transition
C22 0]rex

NL to NS transition
Density OP
at reciprocal lattice vector $\vec{G}$
cubic term,
first order

In supersolid: $< n_{\vec{G}} > \neq 0, < \psi > \neq 0$

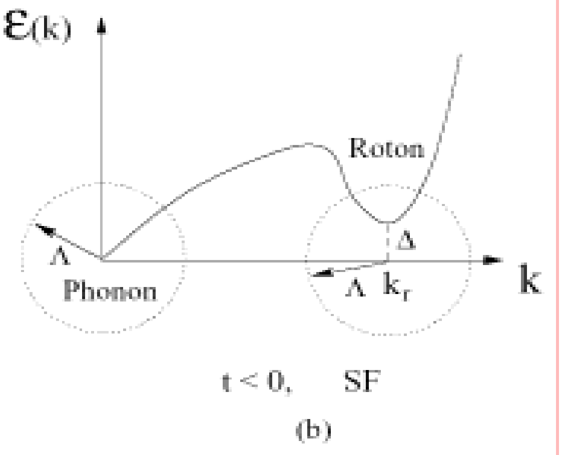
In the NL, $t > 0$, ψ has a gap, can be integrated out.

In the NL, $< n(x) > = n_0, \delta n(x)$ has a gap, can be integrated out



3. The SF to NS or SS transition

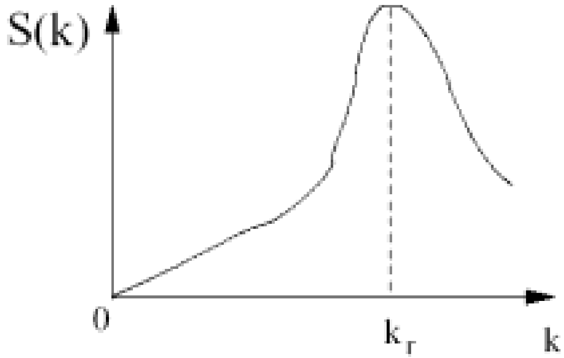
In the SF state, $t < 0, \Delta_r > 0$



BEC: $<\psi_1>= a \neq 0$ breaks U(1) symmetry

$$k_r \sim \frac{2\pi}{a} \sim \frac{2\pi}{3.17\,\mathring{A}}$$

As $P \uparrow$, the roton minimum gets deeper and deeper, $\Delta_r \downarrow$ as detected in neutron scattering experiments



Feymann relation: $$\omega(q) \sim \frac{q^2}{2mS_n(q)}$$

The first maximum peak in $S_n(q) \Rightarrow$ the roton minimum in $\omega(q)$

Two possibilities:

(1) $< \psi >= 0$

Commensurate Solid, SF to NS transition No SS

(2) $< \psi > \neq 0$

In-Commensurate Solid with either vacancies or interstitials

There is a SS ! SF to SS transition

Which scenario will happen *depends on* the sign and strength of the coupling g , will be analyzed further in a few minutes

Let’s focus on **case (1) first:**

I will explicitly construct a Quantum Ginzburg-Landau (QGL) action to describe **SF to NS transition**

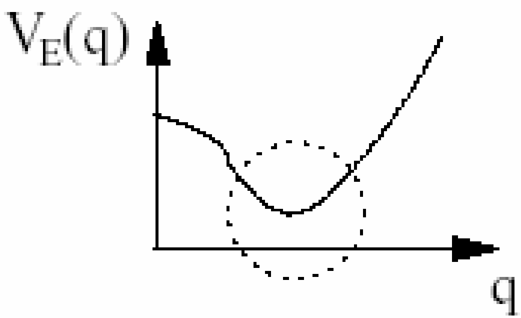
3a. The SF to NS transition

Inside the SF: $$\mathcal{L}[\delta n, \theta] = i\delta n \partial_\tau \theta + \frac{1}{2}\rho_s (\nabla\theta)^2 + \frac{1}{2}\delta n V_n(\vec{q})\delta n$$

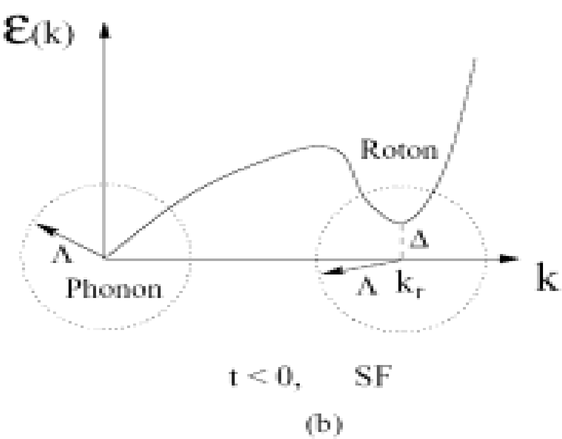
Where: $$V_n(q) = a - bq^2 + cq^4$$

Phase representation: $$\mathcal{L}[\theta] = \frac{1}{2V_n(\vec{q})}(\partial_\tau\theta)^2 + \frac{1}{2}\rho_s(\nabla\theta)^2$$

Dispersion Relation: $$\omega^2 = [\rho_s V_n(\vec{q})]q^2 = \rho_s q^2 (a - bq^2 + cq^4)$$



$\Rightarrow$

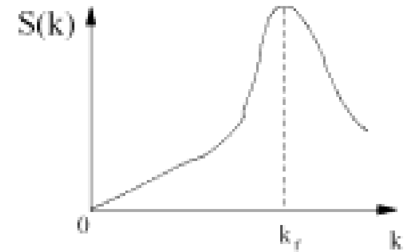


Neglecting vortex excitations:

Density representation: $$\mathcal{L}[\delta n] = \frac{1}{2}\delta n(-\vec{q}, -\omega_n)[\frac{\omega_n^2}{\rho_s q^2} + V_n(\vec{q})]\delta n(\vec{q}, \omega_n)$$

Feymann Relation: $$\omega(q) = \frac{\rho_s \pi q^2}{2S_n(q)}$$

Structure function: $$S_n(q) = \rho_s q\pi / 2v(q)$$



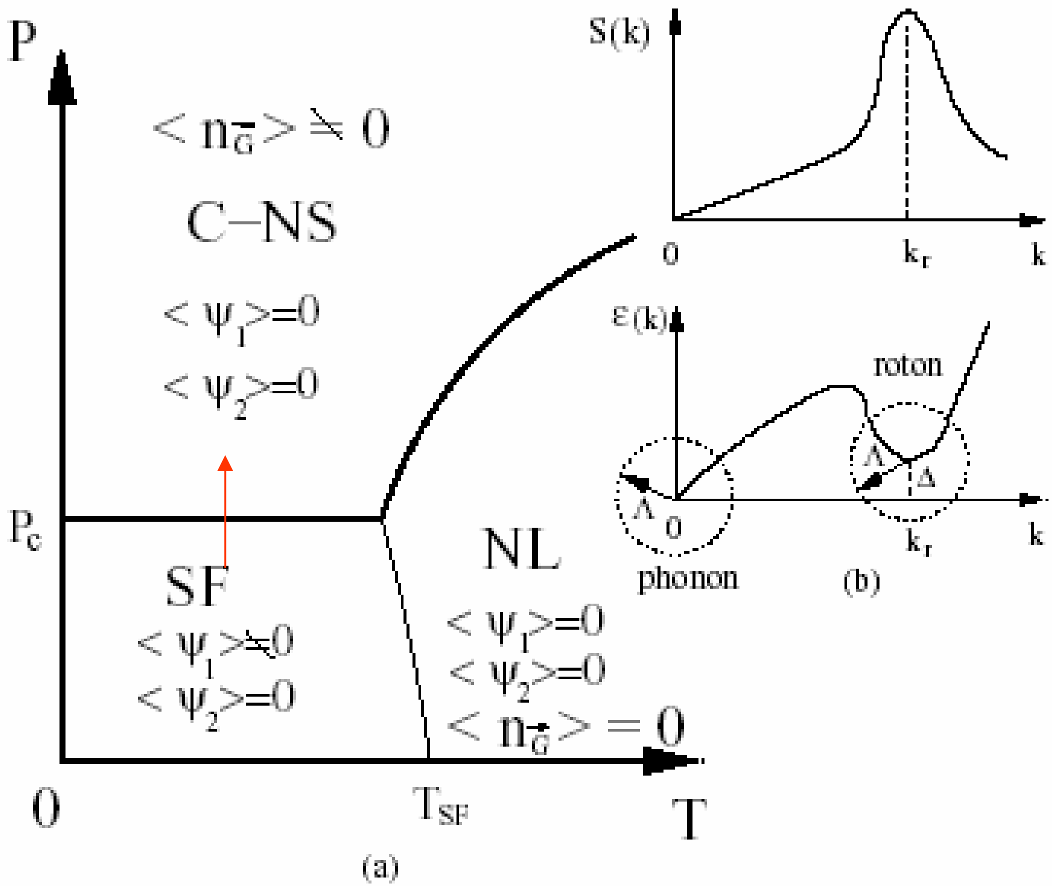
Including vortex excitations: QGL to describe SF to NS transition:

$$\mathcal{L}[\delta n] = \frac{1}{2}\delta n[A_n\omega_n^2 + r + c(q^2 - q_0^2)^2]\delta n - w(\delta n)^3 + u(\delta n)^4 + \cdots$$

$r \sim p_{c1} - p$ $\qquad$ $A_\rho \sim \frac{1}{\rho_s q_0^2}$

SF: $r > 0, < \psi > \neq 0, < \delta n >= 0$

NS: $r < 0, < \psi >= 0, < \delta n >= \sum'_{\vec{G}} n_{\vec{G}} e^{i\vec{G}\cdot\vec{x}}$

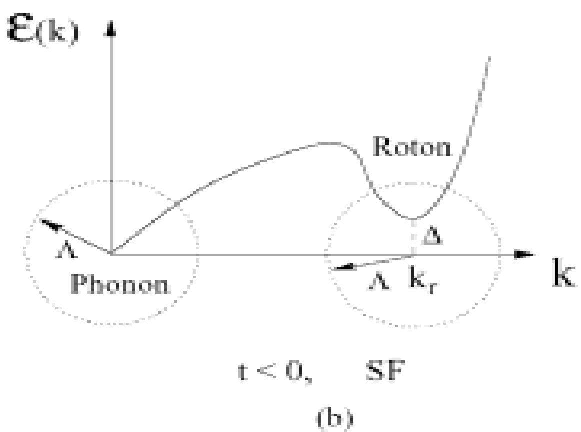


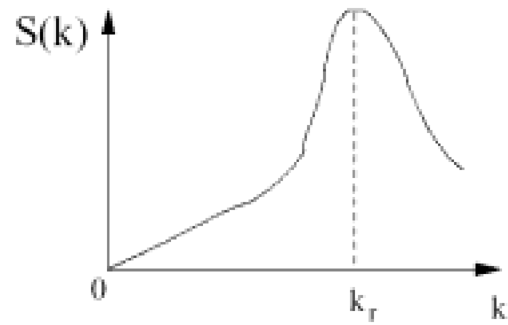
P
$< n_{\vec{G}} > \neq 0$
C–NS
$< \psi_1 > = 0$
$< \psi_2 > = 0$
P_c
SF
$< \psi_1 > \neq 0$
$< \psi_2 > = 0$
NL
$< \psi_1 > = 0$
$< \psi_2 > = 0$
$< n_{\vec{G}} > = 0$
0
T_{SF}
T
(a)
S(k)
0
k_r
k
ε(k)
roton
Λ
Δ
Λ
0
k_r
k
phonon
(b)

3b. **The SF to SS transition**

(2) $< \psi > \neq 0$ There is a SS ! SF to SS transition

ψ : Vacancies or Interstitials !





Feymann relation: $$\omega(q) \sim \frac{q^2}{2mS_n(q)}$$

The first maximum peak in $S_n(q) \Rightarrow$ the roton minimum in $\omega(q)$

The n lattice and ψ Superfluid density wave (SDW) formations happen simultaneously

$$\Delta_r > 0, \ <\psi_1> \neq 0, \ <\psi_2> = 0, <n_G> = 0$$

SF phase

$$\Delta_r < 0, <\psi_1> \neq 0, <\psi_2> \neq 0, <n_{\vec{G}}> \neq 0$$

SS phase

$$n(\vec{x}) = n_0 + \sum_{\vec{G}} n_{\vec{G}} e^{i\vec{G}\bullet\vec{x}}$$

$$G = k_n = k_r$$

$$k_r \sim \frac{2\pi}{a} \sim \frac{2\pi}{3.17 \overset{\circ}{A}}$$

$$\psi(\vec{x}) = \psi_1 + \sum_{\vec{G}} \psi_{\vec{G}} e^{i\vec{G}\bullet\vec{x}}, \quad \psi_2 = \sum_{\vec{G}} \psi_{\vec{G}} e^{i\vec{G}\bullet\vec{x}}$$

SS is still invariant under translations by lattice vectors: $\vec{R}$

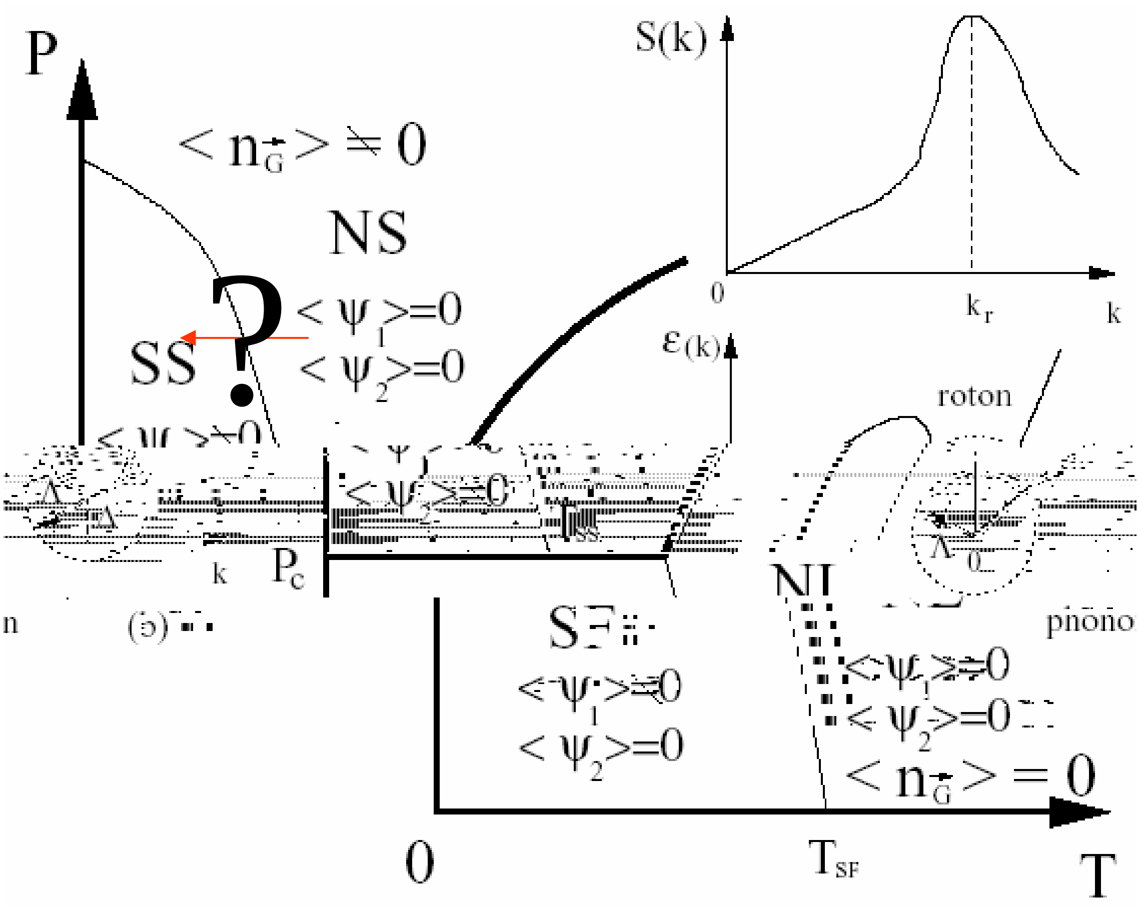
$$n(\vec{x}) \to n(\vec{x} + \vec{R}), \ n_{\vec{G}} \to n_{\vec{G}} e^{i\vec{G}\bullet\vec{R}} = n_{\vec{G}}$$

$$\psi(\vec{x}) \to \psi(\vec{x} + \vec{R}), \ \psi_{\vec{G}} \to \psi_{\vec{G}} e^{i\vec{G}\bullet\vec{R}} = \psi_{\vec{G}}$$

Superfluid Density Wave : $\rho = |\psi|^2 = |\psi_1 + \psi_2|^2$

Vacancy SDW or Interstitial SDW

Global phase diagram of He4 in case (2)



(a)

$$\omega_q = \frac{q^2}{2mS_q}$$

The phase diagram will be confirmed from the NS side

4. Vancancy induced supersolid and NS to SS transition

Looking the effects of g and v

$g > 0,$ solid is the vacancy type, NS-v, $\varepsilon_v < \varepsilon_i$

$g < 0,$ solid is the interstitial type, NS-i, $\varepsilon_v > \varepsilon_i$

$g \to 0,$ solid has the P-H symmetry, NS-PH

$$v > 0 \Rightarrow \boxed{t \to t + v(p)\sum_{\vec{G}} |n(\vec{G})|^2}$$

$$\boxed{t \sim T - T_{XY}}$$

$$\boxed{t = T + \Delta(p), \Delta(p) = v(p)\sum_{\vec{G}} |n(\vec{G})|^2 - T_{XY} > 0}$$

We assume the gap $\Sigma(h)$ is the increasing function of

For vacancies, $g_v < 0$

$$\epsilon_v(0) = t - g_v^2 - |g_v|^3 - |g_v|^4 + \cdots = t - f_v(g_v)$$

$f_v(g_v) > f_v(0) = 0$ is an increasing function of g_v

For interstitials, $g_i > 0$

$$\epsilon_i(0) = t - g_i^2 + g_i^3 - g_i^4 + \cdots = t - f_i(g_i)$$

It is hard to judge the behavior of $f_i(g_i)$ except $f_i(0) = 0$

Where:

$$t = T + \Delta(p), \Delta(p) = v(p) \sum_{\vec{G}} |n(\vec{G})|^2 - T_{XY} > 0$$

Vacancies induced supersolids:

$$t_{\psi_v} = T + \Delta(p) - f_v(g_v) = T - T_{SS-v}$$

$$T_{SS-v} = f_v(g_v) - \Delta(p)$$

Defining a critical value g_{vc} : $f_v(g_{vc}) = \Delta(p_{c1})$

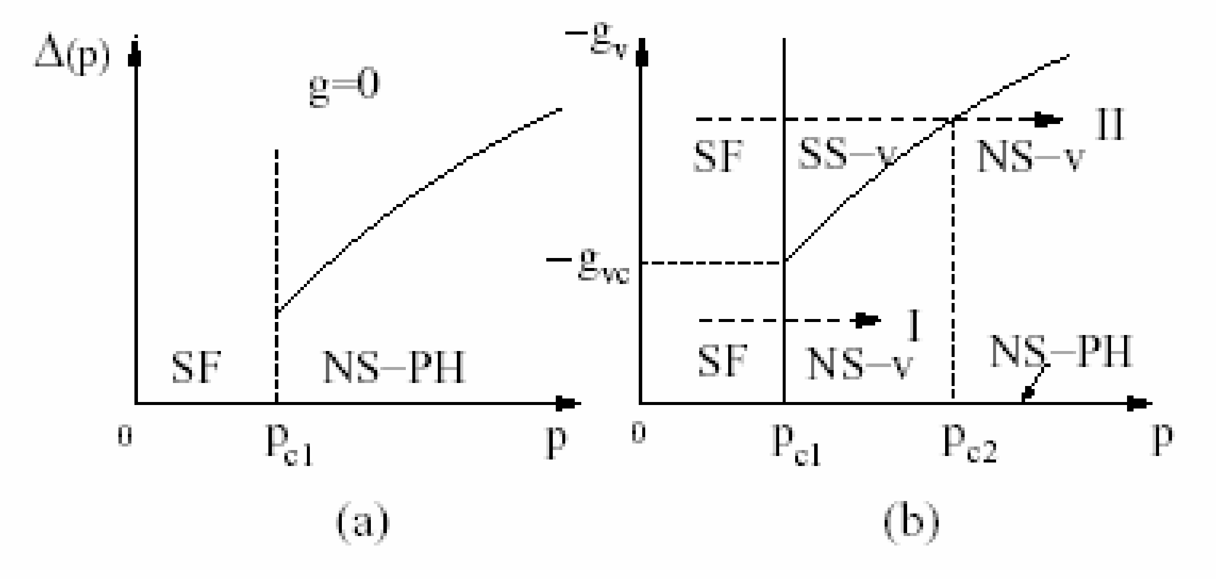
(a) $|g_v| < |g_{vc}|, \; T_{SS-v} < 0, \; <\psi_v> = 0, \; NS - v$

(b) $|g_v| > |g_{vc}|, \; T_{SS-v}(p_{c1}) = f_v(g_v) - \Delta(p_{c1}) > 0$

$p \uparrow$ $\quad T_{SS-v}(p_{c2}) = f_v(g_v) - \Delta(p_{c2}) = 0$

$$T_{SS-v}(p) = f_v(g_v) - \Delta(p) = \Delta(p_{c2}) - \Delta(p)$$

is an effective measure of the gap $\Delta(p)$



(a) (b)

Setting $<\psi_1>= a\,e^{i\theta_1},\ <\psi_2>= e^{i\theta_2}\sum_{m=1}^{P}\psi_m e^{i\vec{Q}_m\bullet\vec{x}}$, $Q_m \sim G$ into the GL,

We study the effects of n lattice on $\psi = \psi_1 + \psi_2$

$$f_{\text{int}} = g\delta n(x)\,|\,\psi(x)\,|^2 + \ldots\ldots$$

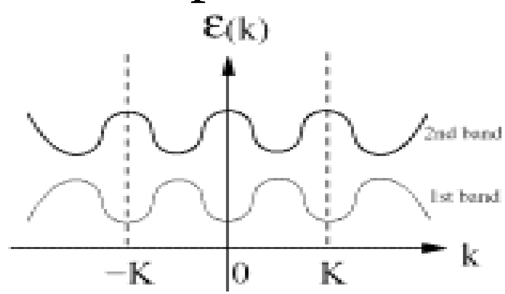
The lowest energy ground state must satisfy:

(1) ψ has to be real, $\vec{Q}_m$ has to be paired as anti-nodal points

(2) $\vec{Q}_m, m = 1, \ldots, P$ are the P shortest reciprocal lattice vectors

Bloch theorem:

$$\varepsilon(\vec{K} = 0) = \varepsilon(\vec{K} = \vec{Q}_m)$$



ψ_1 and ψ_2 have to condense at the same time

(3) Point group symmetry: $\Delta_m = \Delta = real$

(4) The attractive $g_v < 0$ favors $\psi(x = \vec{R}/2) \sim 0$

$$\Rightarrow \quad \psi_{ss-v} = \psi_0 \left(1 + \frac{2}{P} \sum_{m=1}^{P/2} \cos \vec{Q}_m \cdot \vec{x}\right) \qquad \psi_0 = ae^{i\theta}$$

Similarly, Interstitials induced supersolid SS-i:

$$\psi_{ss-i} = \psi_0(1 - \frac{2}{P}\sum_{m=1}^{P/2} \cos \vec{Q}_m \cdot \vec{x})$$

For both SS-v (+) or SS-i (-):

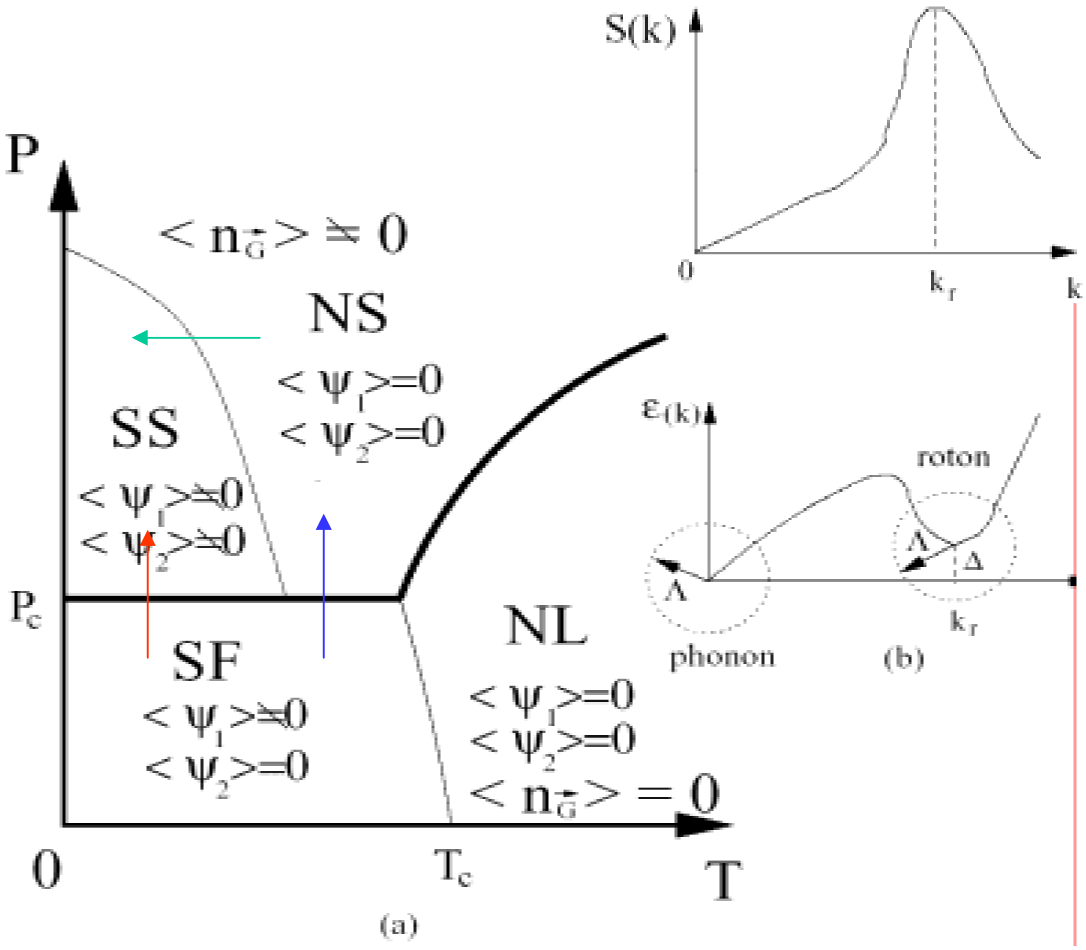
$$\psi_{ss} = \psi_0(1 \pm \frac{2}{P}\sum_{m=1}^{P/2} \cos \vec{Q}_m \cdot \vec{x}), \qquad \psi_0 = |\psi_0|e^{i\theta}$$

At mean field theory level:

The X-ray scattering from SS-v is the same as the NS-v, but will be modified by Debye-Waller factor

The X-ray scattering from SS-i has the even-odd modulation

Global phase diagram of He4 in case (2)



$$\omega_q = \frac{q^2}{2mS_q}$$

5. Excitations in a Supersolid

(1) Superfluid phonons in the ψ sector: θ

Topological vortex excitations in θ

(2) Due to the couplings, the lattice phonons $\vec{u}$ in ψ sector are locked together with the n lattice phonons.

Only one kind of transnational symmetry breaking leads to one kind of lattice phonons $\vec{u}$.

A low energy effective action is *under* construction:

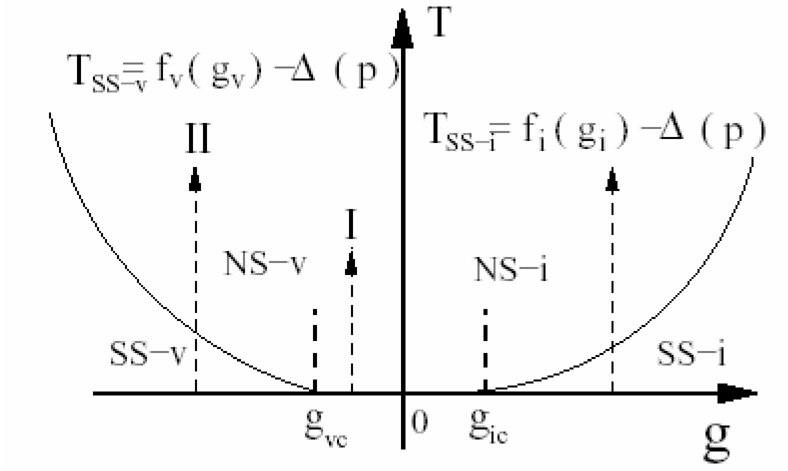
$$L_{eff}(\theta,\vec{u}) = L_1(\theta) + L_2(\vec{u}) + L_{\text{int}}(\theta,\vec{u})$$

If ignoring the coupling term $L_{\text{int}}(\theta,\vec{u})$

NS to SS transition is in 3D XY universality class:

$$f_{\psi_0} = K_{NS}|\nabla\psi_0|^2 + t_{NS}|\psi_0|^2 + u_{NS}|\psi_0|^4 + \cdots$$

$$t_{NS} = T - T_{SS}$$



Very recent very refined Specific heat experiment at PSU detected:

Excess specific heat exhibits a peak $\sim 100\ mK$

Ongoing Experiments at PSU:

High precision X-ray scattering at [illegible]

6. Supersolids in other systems

Supersolid on Lattices can be realized on optical lattices !

Jinwu, Ye, cond-mat/0503113, bipartite lattices;
cond-mat/0612009, frustrated lattices

Cooper pair supersolid
in high temperature SC ?

The physics of lattice SS is *different* than that of He4 SS

Possible excitonic supersolid in electronic systems ?

Electron + hole $\Longrightarrow$ Exciton is a boson

There is also a roton minimum in electron-hole bilayer system

Excitonic supersolid in EHBS ?

There is a magneto-roton in Bilayer quantum Hall systems,
for very interesting physics similar to He4 in BLQH, see:

Jinwu, Ye and Longhua Jiang, cond-mat/0606639 *Jinwu, Ye, PRL, 2006*

7. Conclusions

1. A **simple** GL theory to study all the 4 phases from a unified picture
2. If a SS exists depends on the sign and strength of the coupling g
3. Construct explicitly the QGL of SF to NS transition
4. SF to SS is a **simultaneous** formation of SDW and normal lattice
5. Vacancy induced supersolid is certainly possible
6. NS to SS is a 3d XY with much narrower critical regime than NL to SF
7. Excitations in SS, phonon spectra, Debye-waller factors…..
8. X-ray scattering patterns from SS-v and SS-i

No matter if He4 has the SS phase, the SS has deep and wide scientific interests. It could be realized in other systems: