Theory of Phase Transitions in Type II superconductors

李定平

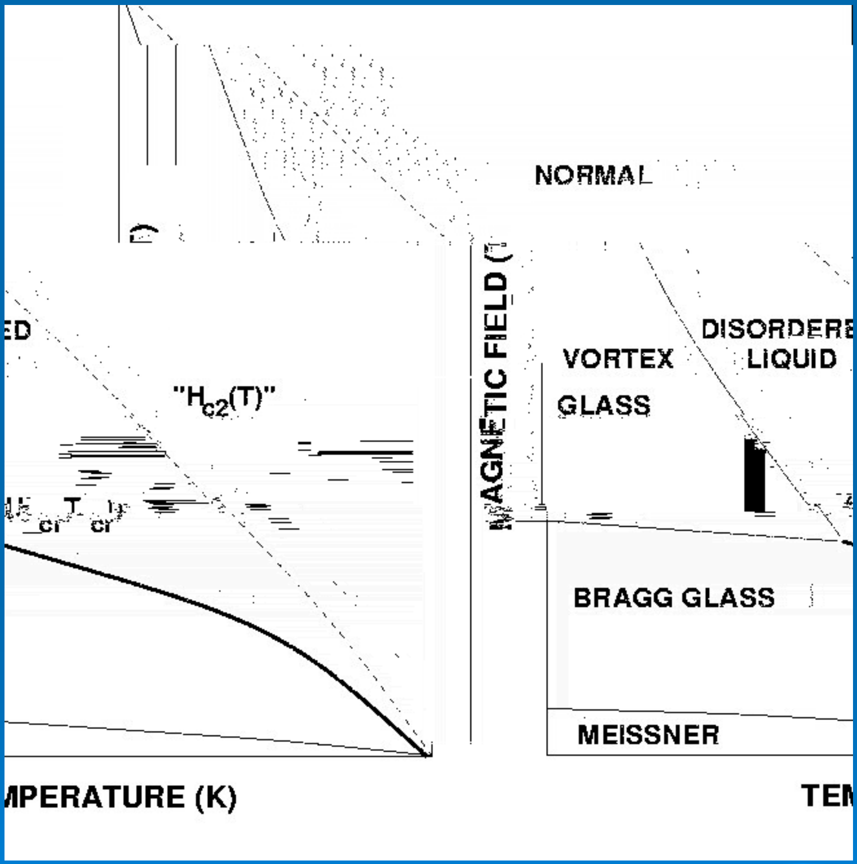
北京大学物理学院理论物理所

Outline

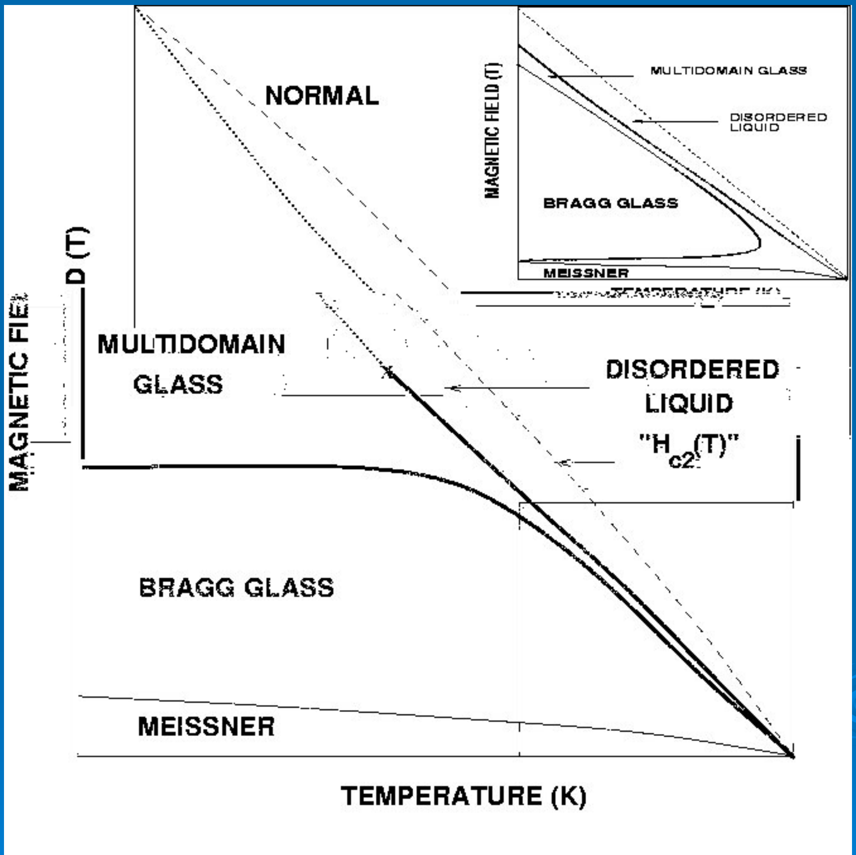
- Introduction a,b,c
- Motivation
- Model
- Theoretical result
- Comparison with experiments
- Conclusion

- Vortex Phase diagram—melting, glass transition, structure phase transition etc, very rich phase diagram
- Traditional theory-using Lindermann criterion to obtain the transition line, not really a theory.
- There are structures in vortices near phase transition.

Phase Behaviour in the Mixed Phase

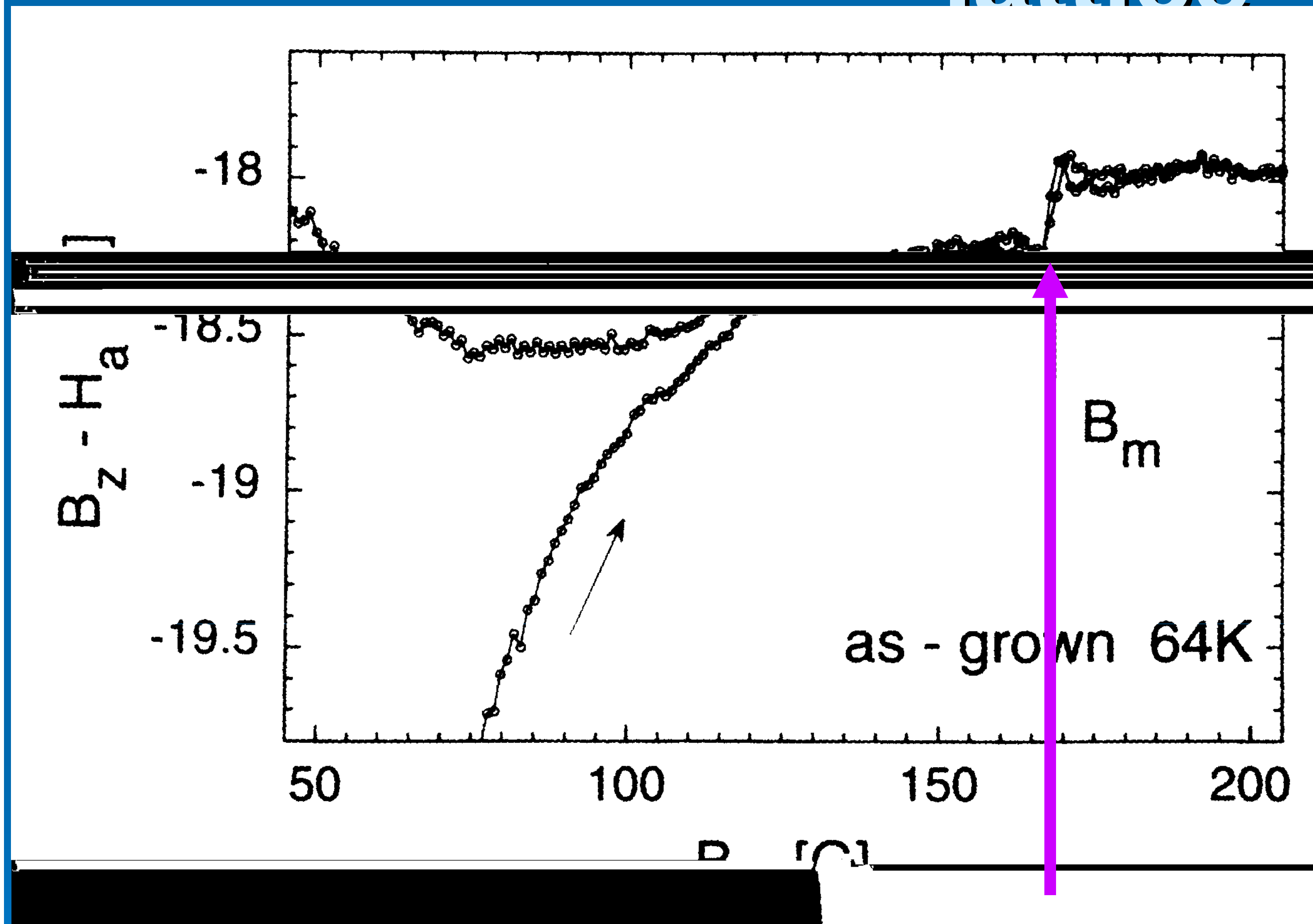


The conventional picture



An alternative view

Melting of the flux line lattice



local magnetisation measurements on BSCCO

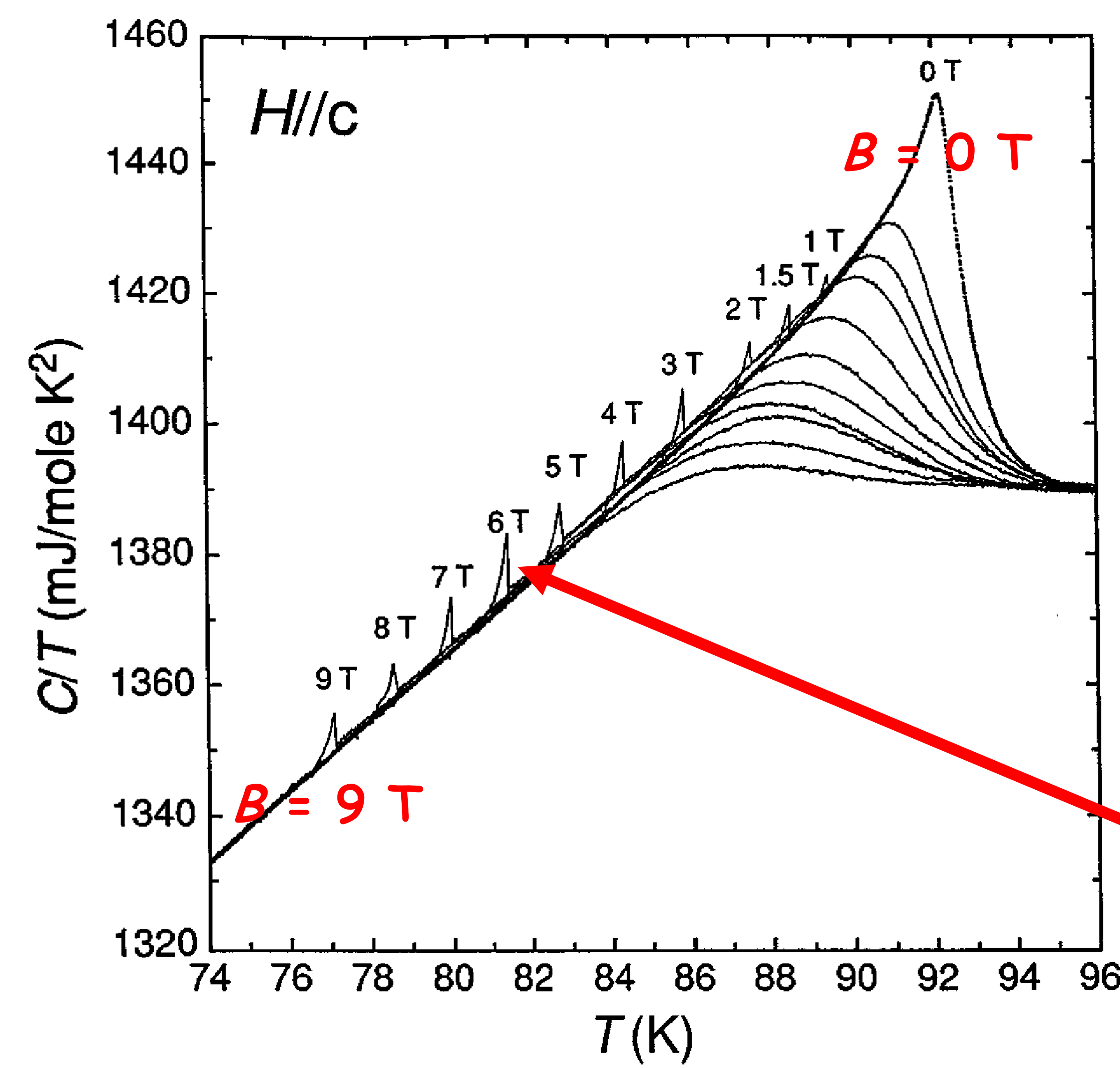
E. Zeldov *et al.*

Nature 1995

1st – order melting transition:-

revealed by jump in magnetisation at B_m

Melting of the flux line lattice



heat capacity measurements on YBCO

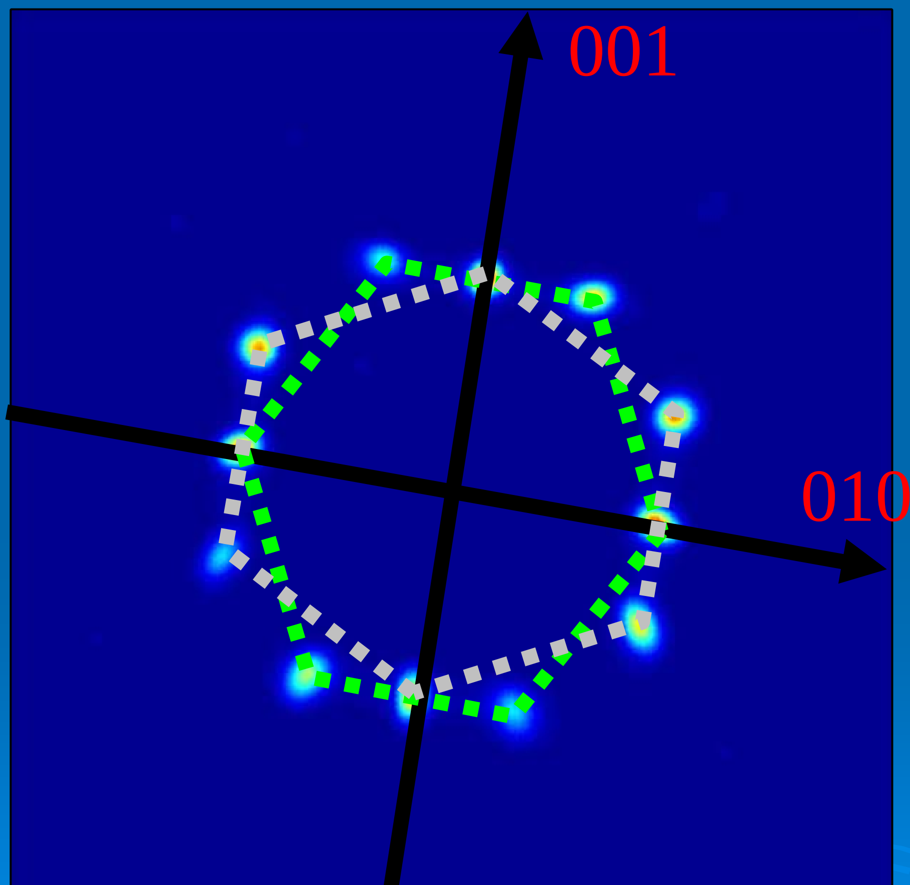
A. Schilling *et al.*

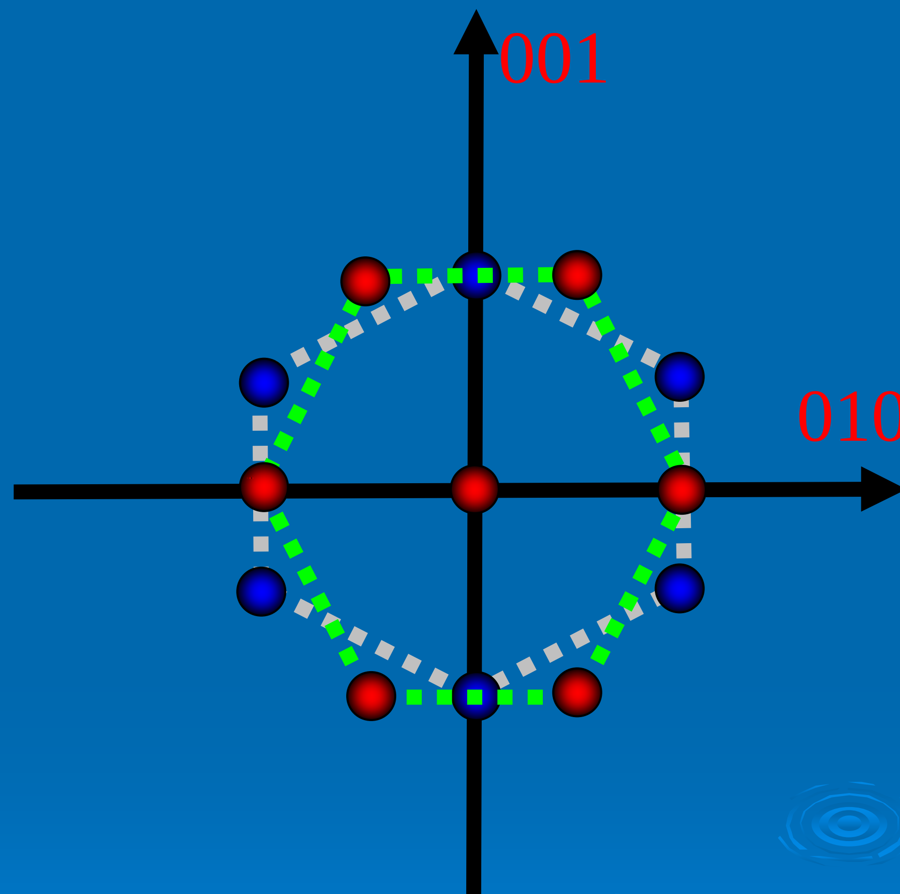
PRL 1997

Entropy jumps at 1st order flux lattice melting

Results in niobium: B // (100)

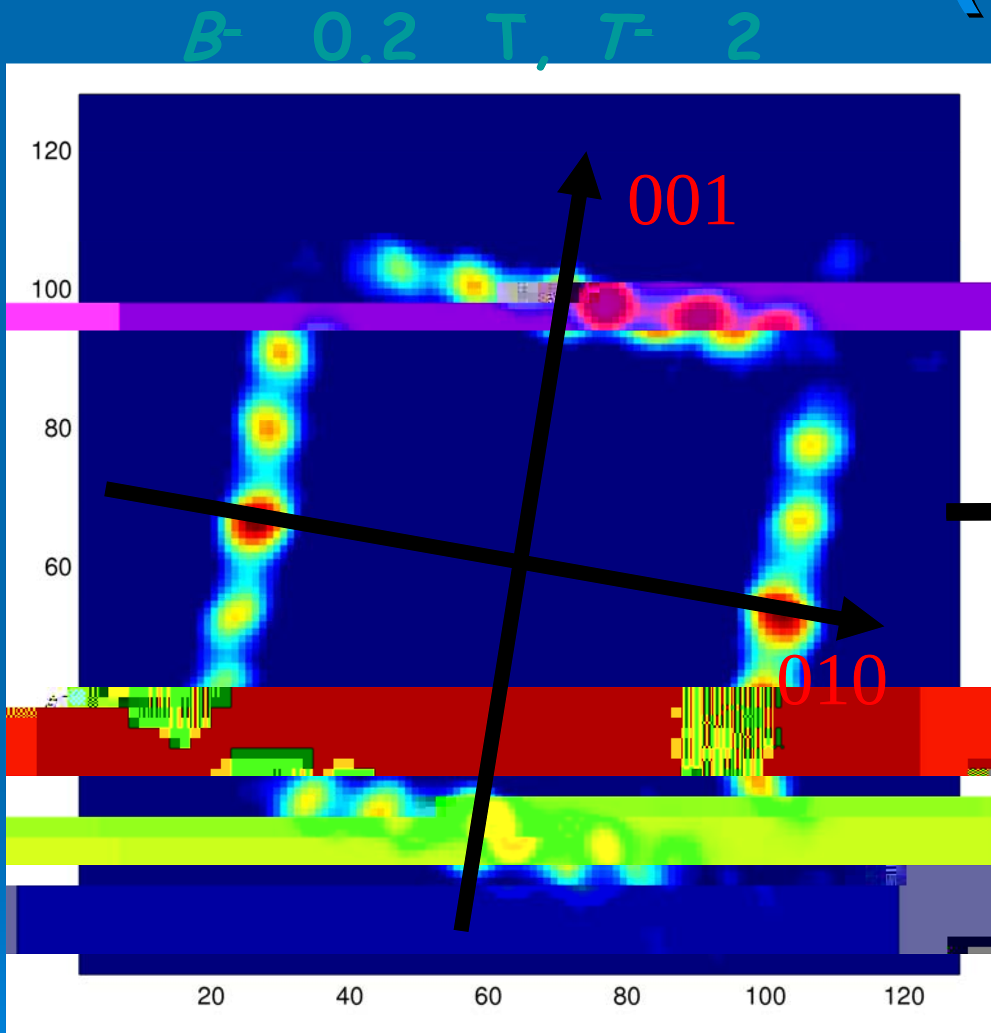
$B = 0.1$ T, $T = 4$

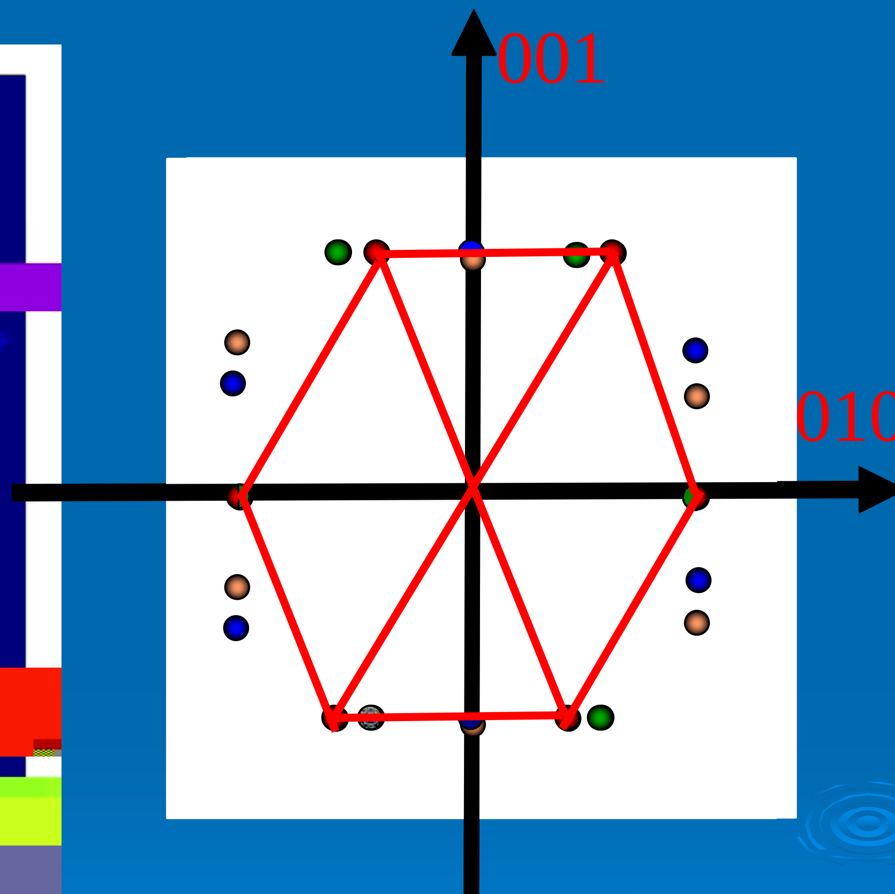




"isosceles phase"

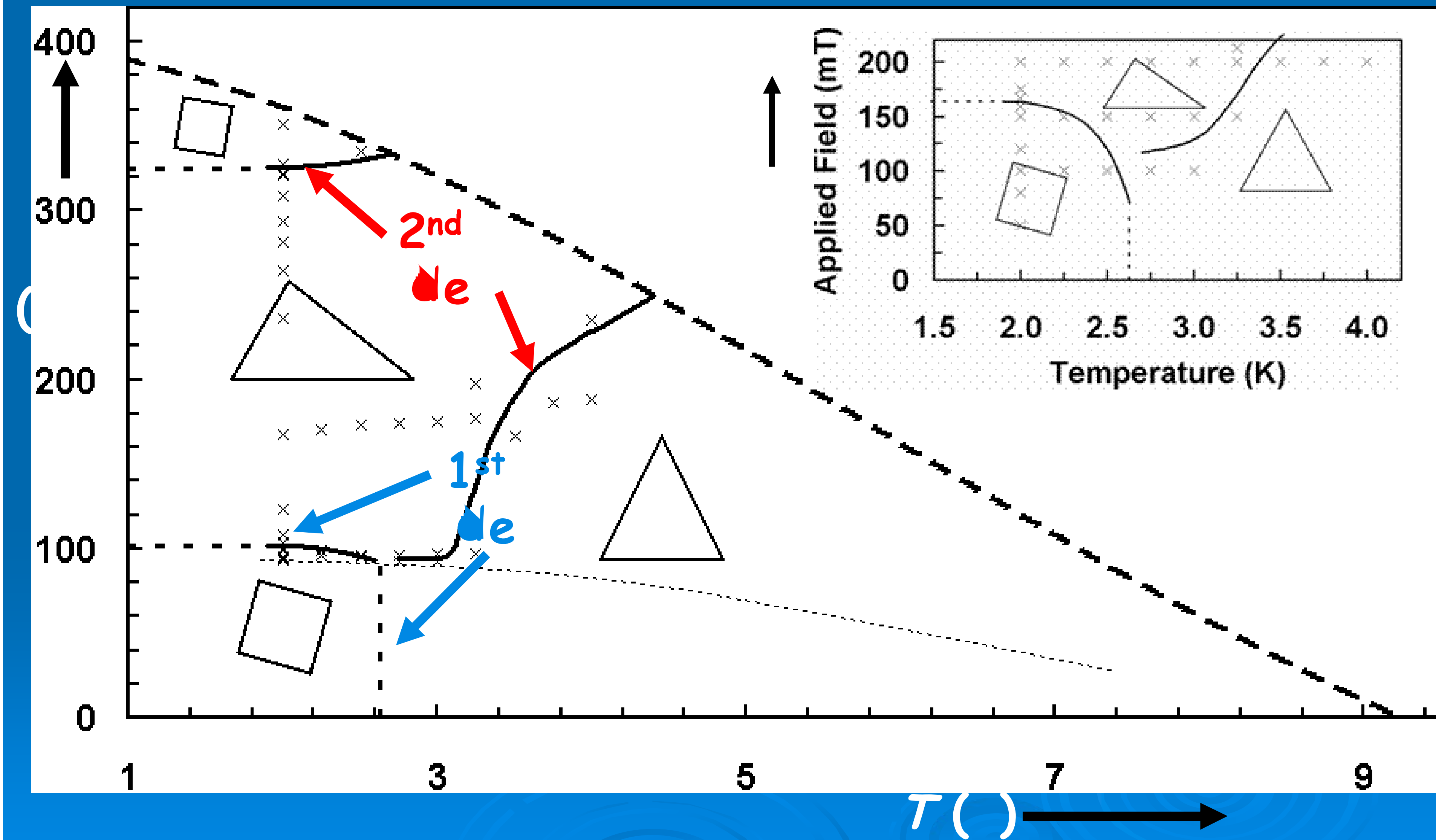
Results in niobium: B // (100)





"scalene phase"

Actual *B* -*T* Phase Diagram

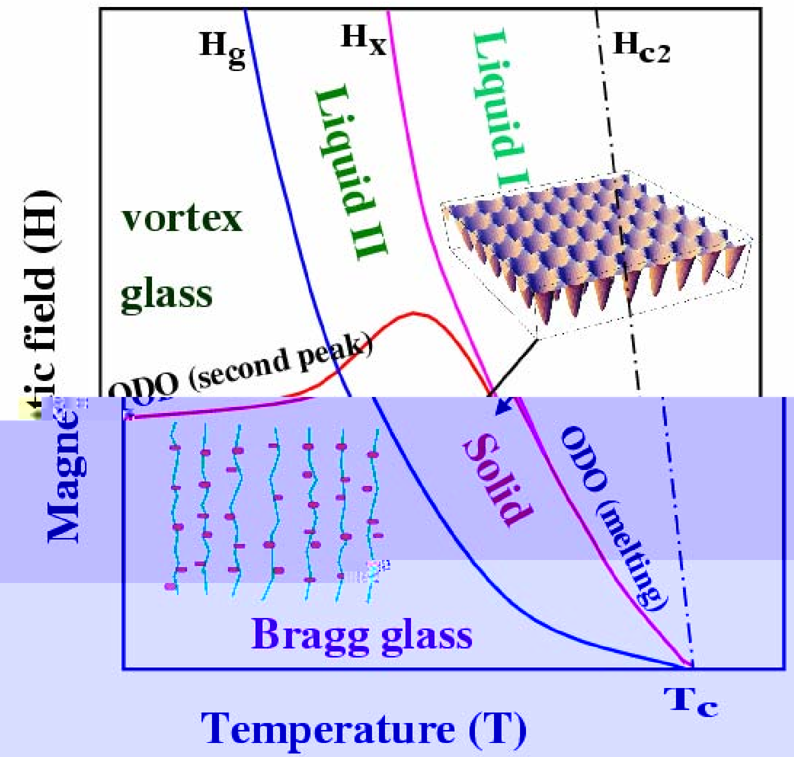


Phase diagram of vortex matter
-Spontaneous symmetry breaking pattern

symmetry	RS (replica symmetric)	RSB (replica symmetry broken)
TS (translation invariant)	liquid	Vortex glass
TSB (translational symmetry broken)	solid	Bragg glass

Different translation symmetry breaking patterns leads square or hexagonal lattice-structure phase transition .

Generic Phase Transition



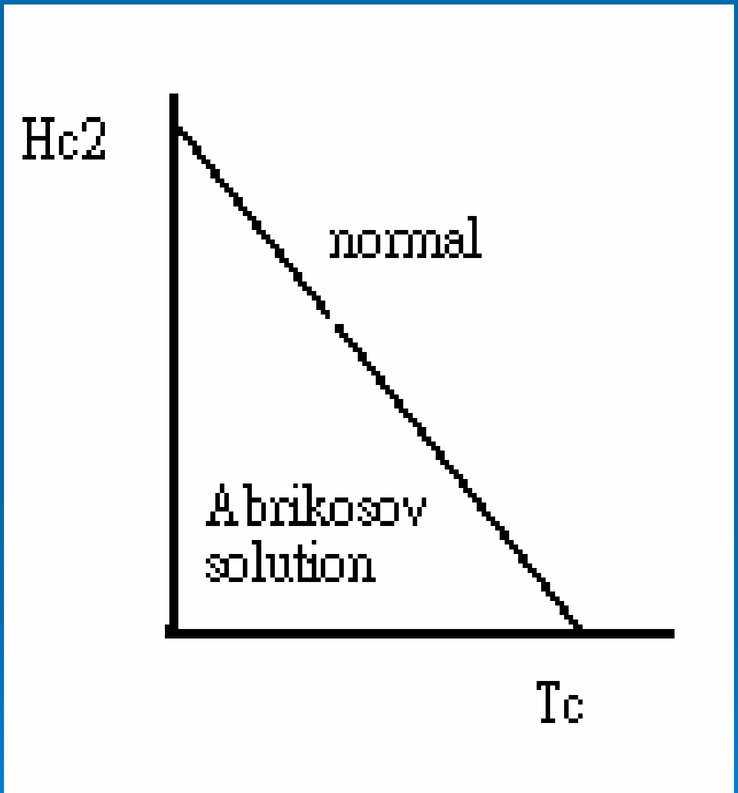
Ginzburg-Landau theory for superconductiv

$$F = \int d^3x \frac{\hbar^2}{2m_{ab}} \left| (\nabla - \frac{2ie}{\hbar c}\mathbf{A})\psi \right|^2 + \frac{\hbar^2}{2m_c} \left| \partial_z \psi \right|^2 + \alpha(T - T_c) \left| \psi \right|^2 + \frac{b'}{2} \left| \psi \right|^4 + \frac{B^2}{8\pi}$$

In equilibrium under fixed external magnetic field, the relevant thermo-dynamical quantity is the Gibbs free energy:

$$G = F - \int dx^3 \frac{1}{4\pi} \vec{B} \cdot \vec{H}$$

Mean field phase diagram



Near Hc2, it is convenient to do follow scaling: length in unit of ξ , magnetic field in unit of Hc2. in Unit of

$$— \quad — \quad ^{3} \quad —\Big| \quad \Big|^{2} \quad —\Big| \quad \Big|^{2} \quad ——\Big| \quad \Big|^{2} \quad —\Big| \quad \Big|^{4} \quad \frac{2 \qquad 2}{\qquad}$$

$$\frac{}{23}$$

With following parameters:

$$\begin{cases} \omega \equiv \sqrt{2Gi}\,\pi^2 t; \\ Gi \equiv \dfrac{1}{2}\left(\dfrac{32\pi e^2 \lambda^2 T_c \gamma}{c^2 h^2 \xi}\right)^2 \propto \dfrac{\kappa^4 T_c^2 \gamma^2}{H_{c2}(0)}. \end{cases}$$

Gi is the so called Ginzburg parameters which characterize the thermal fluctuation strength.

ω **is a better number which can be used to judge the fluctuation effect near Hc2(T) line. If this number is large enough, the fluctuation effect could be observed.**

$$\omega > 0.08 \rightarrow Gi = 3 \cdot 10^{-5}$$

Vortex melting (magnetization jumps) is observable.

Near Hc2(T), magnetic field is nearly constant and nonfluctuating when κ is large

$$b-h\propto\frac{1}{\kappa^2}\rightarrow\frac{\kappa^2(b-h)}{4}\propto\frac{1}{\kappa^2}$$

Near Hc2(T), the lowest Landau level (LLL) approximation also can be used(Abrikosov,57).

We will study the phase diagram of the superconductors with not too big thermal fluctuation (where the LLL approximation can be justified).

Near Hc2, we can use the LLL approximation

$$\int \frac{1}{2}|D\psi|^2 \approx \int \frac{b}{2}|\psi|^2 \rightarrow$$

$$g = \frac{1}{\omega}\int d^3x \left[\frac{1}{2}|D\psi|^2 + \frac{1}{2}|\partial_z\psi|^2 - \frac{1-t}{2}|\psi|^2 + \frac{1}{2}|\psi|^4 + \frac{\kappa^2(b-h)^2}{4} \right]$$

$$\approx \frac{1}{\omega}\int d^3x \left[\frac{1}{2}|\partial_z\psi|^2 - \frac{1-t-b}{2}|\psi|^2 + \frac{1}{2}|\psi|^4 + \frac{\kappa^2(b-h)^2}{4} \right]$$

LLL model and LLL scaling

In the LLL limit, the model simplifies after further rescaling and large kappa approximation:

$$g = \frac{F}{T} = \frac{1}{4\pi\sqrt{2}} \int d^3x \left[\frac{1}{2} \left| \partial_z \psi \right|^2 + a_T \left| \psi \right|^2 + \frac{1}{2} \left| \psi \right|^4 \right]$$

With the only parameter being LLL scaled temperature:

$$a_T \equiv -\left(\frac{\pi t b \sqrt{Gi}}{4} \right)^{-2/3} \frac{1-t-b}{2}, \quad b = H / H_{c2}, \quad t = T / T_c$$

Vortex Melting Theory

1 Requires accurate vortex solid free energy. There is Infrared problem due to soft phonon modes (goldstone modes).

2 Requires liquid free energy. Usually Gaussian or Hatree-Fock is not enough for the precision of the free energy. Nonperturbative method is needed in calculating the liquid free energy

Traditional melting theory using Lindermann criterion based on elasticity theory is purely phenomenogical.

Theory of Vortex glass

Replica symmetry breaking will be used to determine the transition as Mezard and Parisi did for other models using variational method.

Thermal fluctuations

Thermal fluctuations are taken into account

via statistical sum

$$Z = \int D\psi^*(x) D\psi(x) e^{-\frac{G[\psi]}{T}}$$

Gaussian Variational Calculation for Vortex lattice

Gaussian variational method,
2D as an example:

$$\psi(x,y) = v\varphi(x,y) + \frac{1}{\sqrt{2}\left(2\pi\right)} \int_{k\in B.Z,k_z} \mathrm{e}^{-\frac{\theta_k}{2}} \varphi_k(x,y)(O_k + iA_k)$$

$$g = K + V;$$

$$K = \frac{1}{\omega}\left[O_k G_{OO}^{-1}(k) O_{-k} + A_k G_{AA}^{-1}(k) A_{-k} + A_k G_{AO}^{-1}(k) O_{-k} + O_k G_{OA}^{-1}(k) A_{-k}\right]$$

One found after analyzing gap equations that:

$$G^{-1} = \begin{pmatrix} E(k) + \Delta|\gamma_k| & 0 \\ 0 & E(k) - \Delta|\gamma_k| \end{pmatrix}$$

Finally the equations:

( )

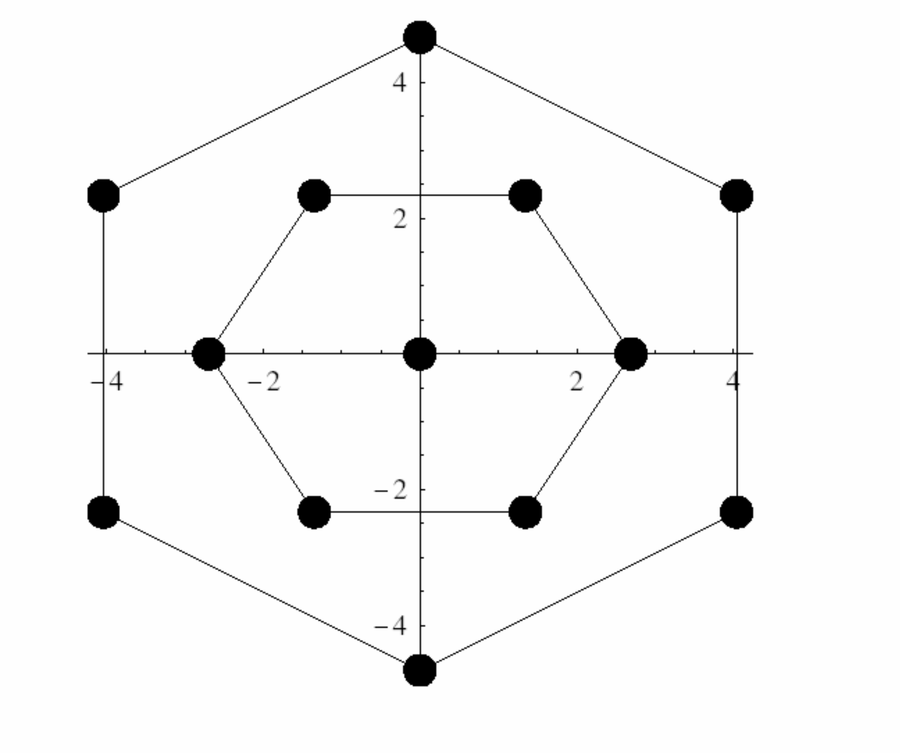
( ) | | ( )

$$\beta_k = \sum_{n=0}^{\infty} \chi^n \beta_n(k)$$

$$\beta_n(k) = \sum_{|X|^2 = n a_\Delta^2} \exp(ik \bullet X)$$

$$E(k) = \sum_{n=0}^{\infty} E_n \beta_n(k)$$

with



$$\chi = \exp\left[-\frac{a_\Delta^2}{2}\right]$$

$$= \exp\left[-\frac{2\pi}{\sqrt{3}}\right]$$

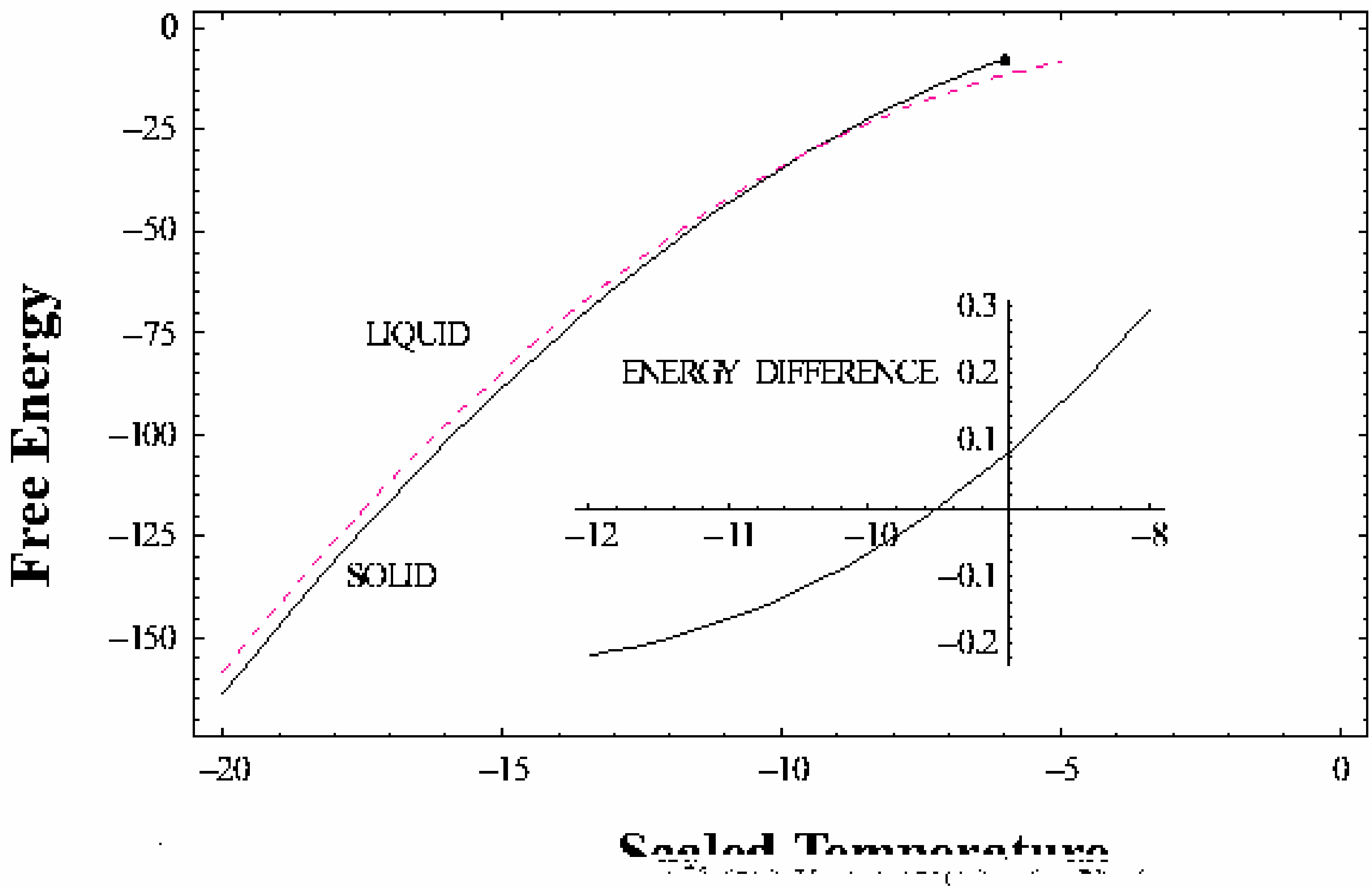
So we have an algebraic equation for $\Delta, \; E_n$!!!

becomes smaller with bigger n. We can solve the equation very easily by iterations along with shift equation

Mode expansion, 3D.

1 mode	2 modes	3 modes
−446023. 8395	−431171. 9948	−431171. 9757
−40131. 29217	−38796. 0277	−38796. 02297
−4450. 41636	−4303. 28685	−4303. 28593
−1106. 51575	−1070. 63806	−1070. 63791
−171. 678045	−166. 690727	−166. 690827
−39. 292885	−38. 433571	−38. 433645
−7. 3153440	−7. 2237197	−7. 2237422

a_T
−100
−300
−100
−50
−20
−10
−5



3D calculations

Dingping Li,B.Rosenstein

PRL90,167004 (2003).

Spinodal Point

Our guassian calculation showed that there is a spin nodal point (or line) for vortex solid. There exists the solid above melting temperature down to $a_T \approx -5.5$

Dingping Li,B.Rosenstein PRB65,220504 (2002).

The experiments done by Z.L. Xiao, O. Dogru, E.Y. Andrei, P. Shuk, M. Greenblatt also confirmed that there is superheated solid to $a_T \approx -5.5$ **, and then stop at this line.**

Z.L. Xiao et.al,PRL(2004)

VOLUME 92, NUMBER 22 PHYSICAL REVIEW LETTERS week ending 4 JUNE 2004

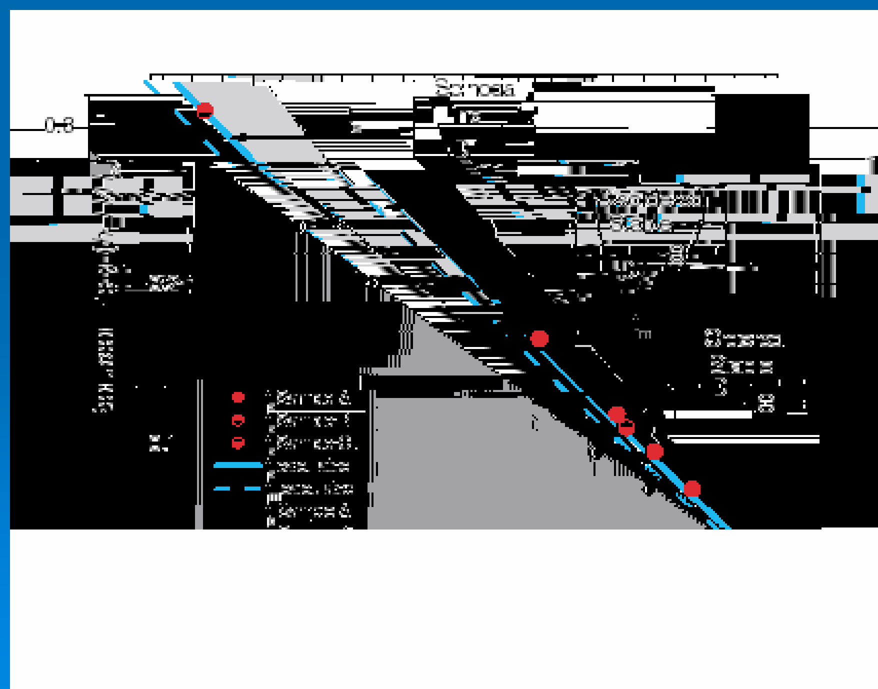
Observation of the Vortex Lattice Spinodal in $NbSe_2$

Z. L. Xiao,[1,3] O. Dogru,[1] E. Y. Andrei,[1] P. Shuk,[2,4] and M. Greenblatt[2]

[1]*Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08855, USA*
[2]*Department of Chemistry, Rutgers University, Piscataway, New Jersey 08855, USA*
[3]*Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[4]*Emerson Process Management, Rosemount Analytical Inc., Orrville, Ohio 44667, USA*

(Received 28 September 2003; published 4 June 2004)

Metastable superheated and supercooled vortex states in $NbSe_2$ crystals were probed with fast transport measurements over a wide range of field and temperature. The limit of metastability of the superheated vortex lattice defines a line in the phase diagram that lies below the superconducting transition and is clearly separated from it. This line is identified as the vortex lattice spinodal, and is in good agreement with recent theoretical predictions by Li and Rosenstein [Phys. Rev. B 65, 220504 (2002); cond-mat/0305258]. By contrast, no limit of metastability is observed for the supercooled disordered state.

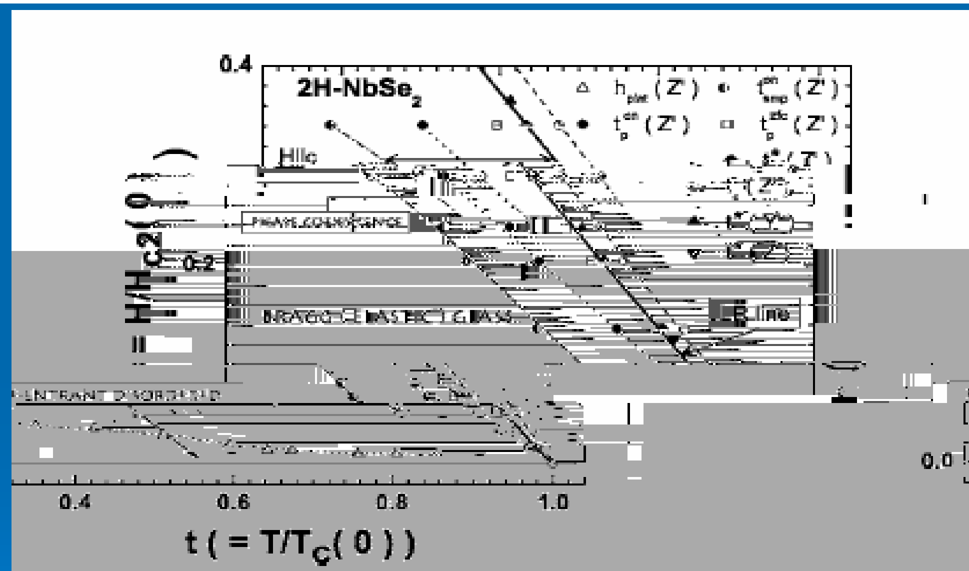


PHYSICAL REVIEW B **72**, 134524 (2005)

Exploring metastability via third harmonic measurements in single crystals of 2*H*-$NbSe_2$ showing an anomalous peak effect

A. D. Thakur,[1,*] S. S. Banerjee,[2] M. J. Higgins,[3] S. Ramakrishnan,[1] and A. K. Grover[1,†]
[1]*Department of Condensed Matter Physics and Materials Science, Tata Institute of Fundamental Research, Mumbai 400005, India*
[2]*Department of Physics, India Institute of Technology-Kanpur, Kanpur 208076, India*
[3]*NEC Research Institute, Princeton, New Jersey 08540, USA*
(Received 6 May 2005; revised manuscript received 5 July 2005; published 27 October 2005)

We explore the metastability effects across the order-disorder transition pertaining to the peak effect phenomenonon in critical current density (J_c) via the first and the third harmonic ac susceptibility measurements in the weakly pinned single crystals of 2*H*-$NbSe_2$. An analysis of our data suggests that an imprint of the limiting (spinodal) temperature above which J_c is path independent can be conveniently located in the third harmonic data ($\chi'_{3\omega}$).



...gnetic phase diagram in a typically weakly pinned ... of 2*H*-$NbSe_2$ drawn in terms of reduced field (*h*) ... (*t*). The theoretical spinodal line based on the work ...stein (Ref. 31) has also been drawn as a solid line. ...nes passing through various data sets are to guide ...ng Ref. 11, we have also included the data related ...spinodal) temperature in the other 2*H*-$NbSe_2$ crys-

FIG. 5. A ma... single crystal *Z*′ ... and temperature ... of Li and Rosens... The rest of the li... the eye. Followi... to the limiting (s... tals (*Y*′ and *Z*).

Perturbation Theory of LLL GL Model Gaussian Expansion for liquid

Ruggeri and Thouless developed high temperature perturbation theory around

Borel-Pade Summation

Borel-Pade summation of $g(x)=\sum c_n x^n,$

$$g_k(x) = \int_0^\infty dt\, g_k'(xt)\exp(-t),$$

$$g_k'(x) = Pade_{(k,k-1)}\left[\sum_{n=1}^{2k-1} \frac{c_n}{n!} x^n\right]$$

$g_k(x)$ **converge** $k=4,5.$ **so**

$f_{liq}=4\varepsilon^{1/2}[1+g(x)]$ **converges. The result is consistent with *MC* and OPT.**

Overcooled liquid state

BP calculation shows that there is a meta stable liquid state down to very lower temperature

We speculate that there is a meta stable liquid state down to very lower temperature for any repulsive interaction system.

Recent experiments done by Z.L. Xiao et.al. (PRL2004) confirmed that there is a meta stable liquid state down to very lower temperature ($NbSe_2$).

Application of the metastable liquid and solid theory-Melting of Vortex Lattice

Plotting f_{sol}, f_{liq} **,one found that for 3D** $a_T^m = -9.5$

Similar calculation in 2D, one found that $a_T^m = -13.2$ **.Magnetization curves etc. are consistent with *MC* (Kato, Nagaosa, Phy. Rev. B31,7336 (1992))and OPT in 2D too.**

One can use the above theory to calculate various quantities

- Magnetization Jump
- Entropy Jump
- Parameter Fitting
- LLL Scaling Function

Disorder effects in type-II superconductors

$$G = L_z \int dx^2 \left[\frac{\hbar^2}{2m^*} \left| \vec{D}\psi \right|^2 + a'\psi^*\psi + \frac{b'}{2}\left(\psi^*\psi\right)^2 \right]$$

Disorders are introduced via:

$$\left(m^{*}\right)^{-1} \rightarrow \left(m^{*}\right)^{-1}\left(1+U\left(x\right)\right),$$

$$\overline{G} = -T\,\overline{\log\left\{G\left[\psi, U, W, V\right]/T\right\}}$$

Disorder average can be done using replica trick:

$$\overline{Z^n} = \int_{\psi_a} \exp\left[-\sum_a g(\psi_a) + \frac{1}{2(4\pi)^2}\sum_{a,b}\left[r'|\psi_a|^2|\psi_b|^2 + \frac{q'}{4}(\psi_a^*\psi_a)^2(\psi_b^*\psi_b)^2\right]\right]$$

There is a replica symmetry breaking solution in part of phase diagram!! It is a glass phase region!

Equilibrium First-Order Melting and Second-Order Glass Transitions of the Vortex Matter in $Bi_2Sr_2CaCu_2O_8$

H. Beidenkopf,[1,*] N. Avraham,[1] Y. Myasoedov,[1] H. Shtrikman,[1] E. Zeldov,[1] B. Rosenstein,[1,2] E. H. Brandt,[3] and T. Tamegai[4]

[1]*Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel*
[2]*National Center for Theoretical Sciences and Electrophysics Department, National Chiao Tung University, Hsinchu 30050, Taiwan, Republic of China*
[3]*Max-Planck-Institut für Metallforschung, Heisenbergstrasse 3, D-70506 Stuttgart, Germany*
[4]*Department of Applied Physics, The University of Tokyo, Hongo, Bunkyo-ku, Tokyo 113-8656, Japan*

(Received 16 July 2005; published 16 December 2005)

The thermodynamic H-T phase diagram of $Bi_2Sr_2CaCu_2O_8$ was mapped by measuring local *equilibrium* magnetization $M(H, T)$ in the presence of vortex shaking. Two equally sharp first-order magnetization steps are revealed in a single temperature sweep, manifesting a liquid-solid-liquid sequence. In addition, a second-order glass transition line is revealed by a sharp break in the equilibrium $M(T)$ slope. The first- and second-order lines intersect at intermediate temperatures, suggesting the existence of four phases: Bragg glass and vortex crystal at low fields, glass and liquid at higher fields.

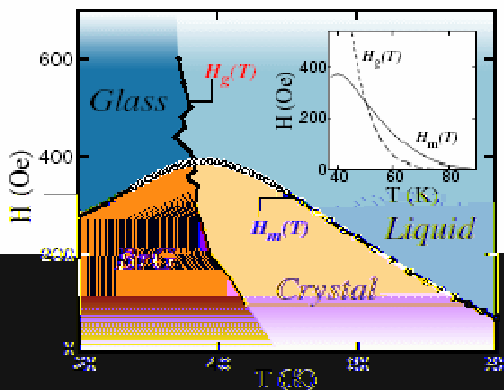
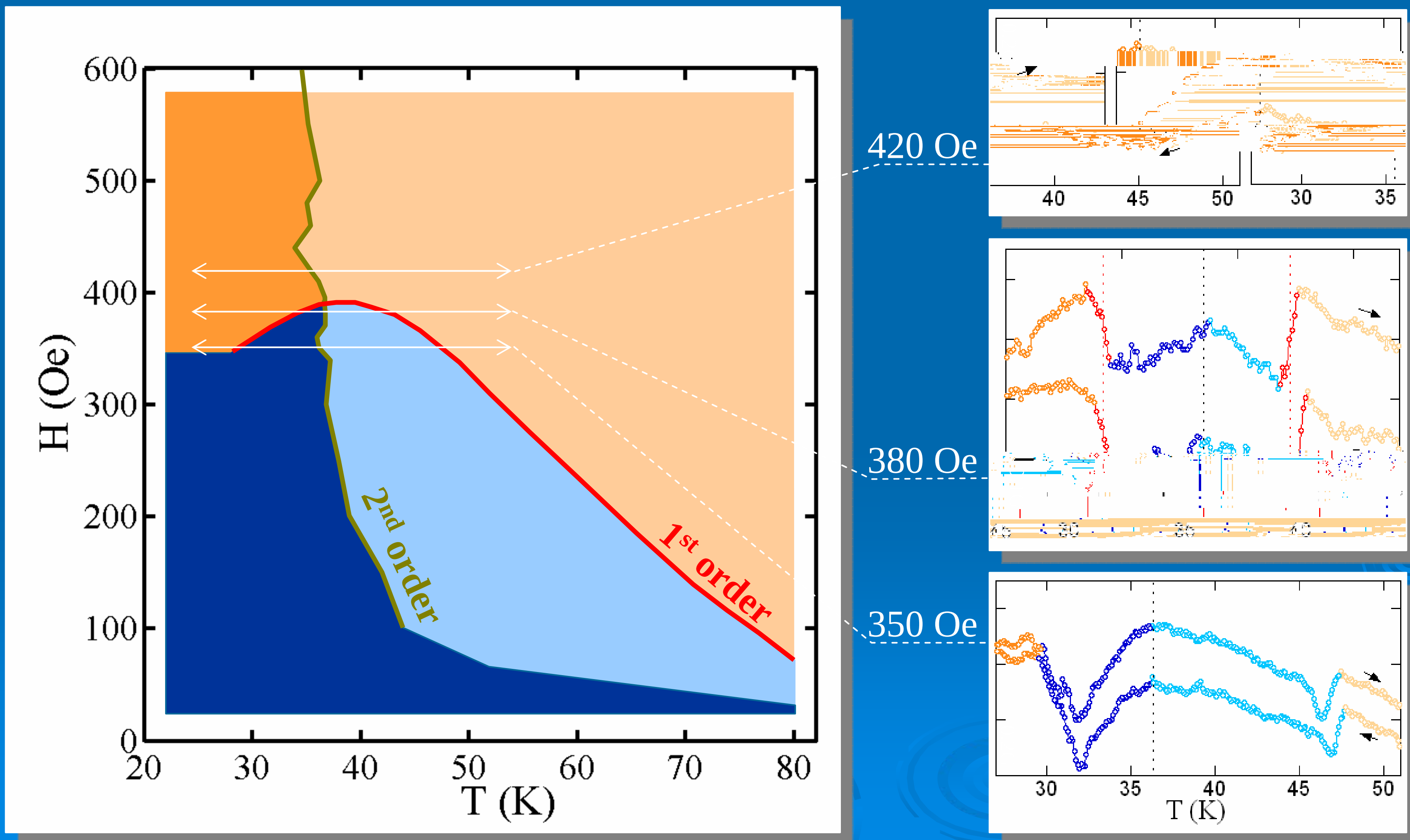


FIG. 4 (color online). The thermodynamic phase diagram of BSCCO accommodates *four* distinct phases, separated by a first-order melting line $H_m(T)$ (open circles), which is intersected by the second-order glass line $H_g(T)$ (solid dots). The inset: the ... equivalent phase diagram, calculated based on Ref. [31], consisting of a second-order replica symmetry breaking line $H_g(T)$ both above (dotted line) and below (dashed line) the first-order transition $H_m(T)$ (solid line).

[21] T. Shibauchi *et al.*, Phys. Rev. Lett. **83**, 1010 (1999).
[22] R. Sugano *et al.*, Physica (Amsterdam) **357C**, 428 (2001).
[23] F. Bouquet *et al.*, Nature (London) **411**, 448 (2001).
[24] H. Safar *et al.*, Phys. Rev. Lett. **70**, 3800 (1993).
[25] W. K. Kwok *et al.*, Phys. Rev. Lett. **84**, 3706 (2000).
[26] K. Shibata *et al.*, Phys. Rev. B **66**, 214518 (2002).
[27] B. J. Taylor *et al.*, Phys. Rev. B **68**, 054523 (2003).
[28] T. Giamarchi and P. Le Doussal, Phys. Rev. B **52**, 1242 (1995).
[29] T. Nattermann, Phys. Rev. Lett. **64**, 2454 (1990).
[30] B. Horovitz and T. R. Goldin, Phys. Rev. Lett. **80**, 1734 (1998); B. Horovitz, Phys. Rev. B **72**, 024519 (2005).
[31] D. Li and B. Rosenstein, Phys. Rev. Lett. **90**, 167004 (2003); cond-mat/0411096.
[32] B. Rosenstein and D. Li (unpublished).

Glass transition

measured by Beidenkopf, Zeldov, etc. PRL,2005



Interplay of Anisotropy and Disorder in the Doping-Dependent Melting and Glass Transitions of Vortices in $Bi_2Sr_2CaCu_2O_{8+\delta}$

H. Beidenkopf,[1,*] Y. Myasoedov,[1] H. Shtrikman,[1] E. Zeldov,[1] B. Rosenstein,[2] D. Li,[3] and T. Tamegai[4]

[1] [illegible]

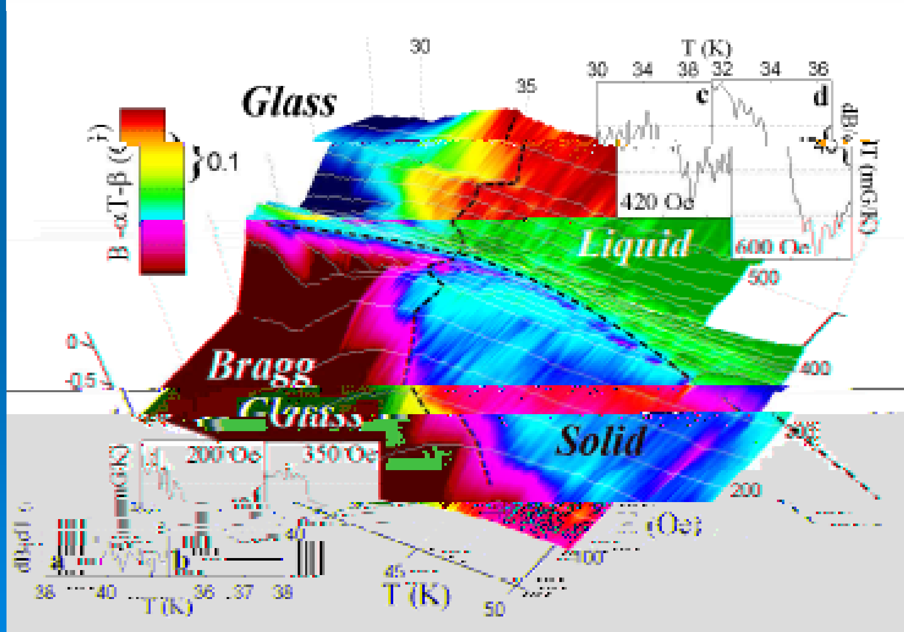
[2] *Electrophysics Department, National Chiao Tung University, Hsinchu 30050, Taiwan, Republic of China*

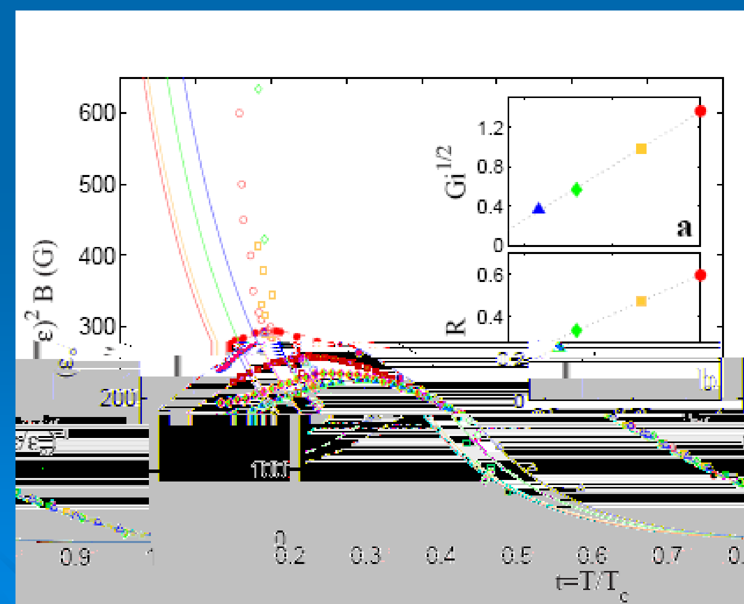
[3] *Department of Physics, Peking University, Beijing 100871, China*

[4] *Department of Applied Physics, The University of Tokyo, Hongo, Bunkyo-ku, Tokyo 113-8656, Japan*

(Dated: September 7, 2006)

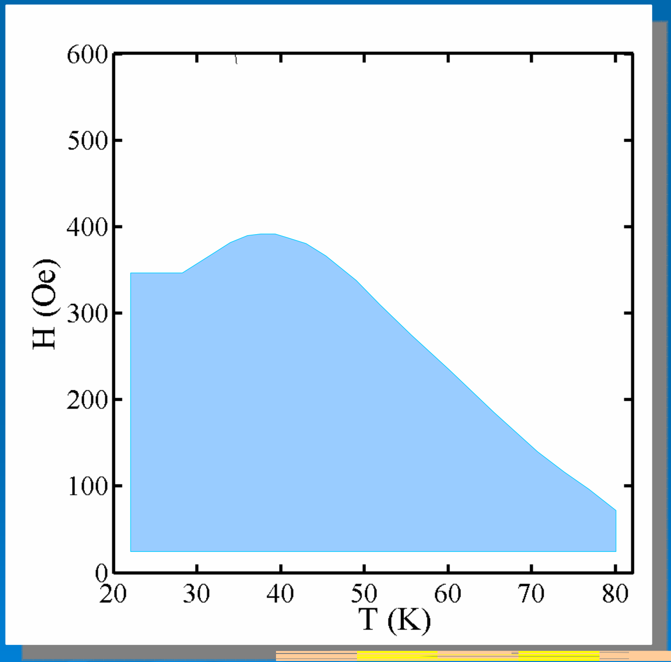
We study the oxygen doping dependence of the equilibrium first-order melting and second-order glass transitions of vortices in $Bi_2Sr_2CaCu_2O_{8+\delta}$. Doping affects both anisotropy and disorder. Anisotropy scaling is shown to collapse the melting lines only where thermal fluctuations are dominant. Yet, in the region where disorder breaks that scaling, the glass lines are still collapsed. A quantitative fit to melting and replica symmetry breaking lines of a 2D Ginzburg-Landau model further reveals that disorder amplitude weakens with doping, but to a lesser degree than thermal fluctuations, enhancing the relative role of disorder.





[32] D. Li and B. Rosenstein, to be published.

[33] D. Li and B. Rosenstein, Phys. Rev. Lett. **90**, 167004 (2003).

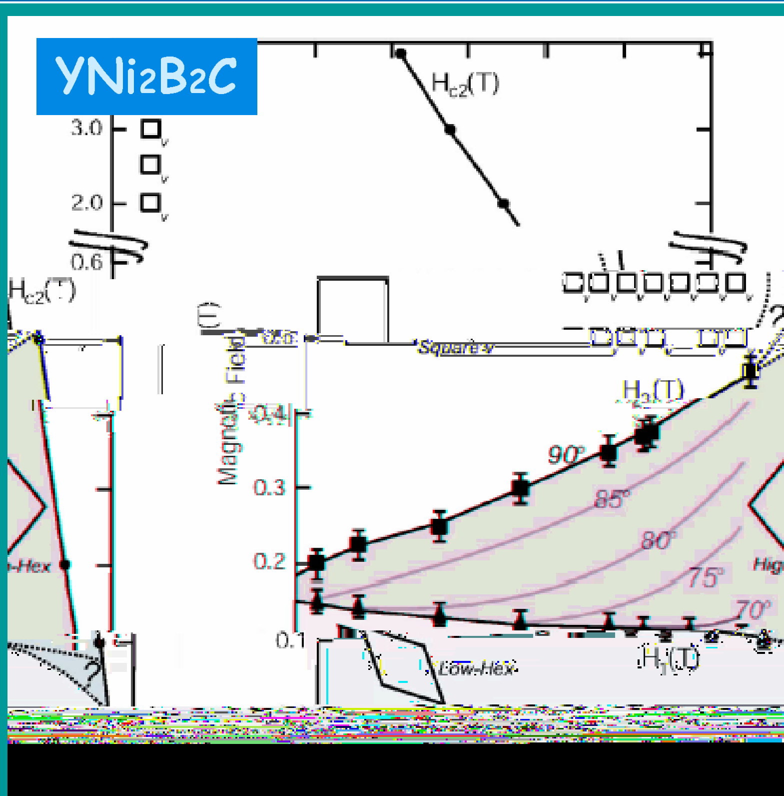

600
500
400
300
200
100
0
H (Oe)
20
30
40
50
60
70
80
T (K)

Disorder effects to the melting line

A universal melting line, smoothly interpolating from low temperature where disorder is important, to high temperature where disorder is negligible.

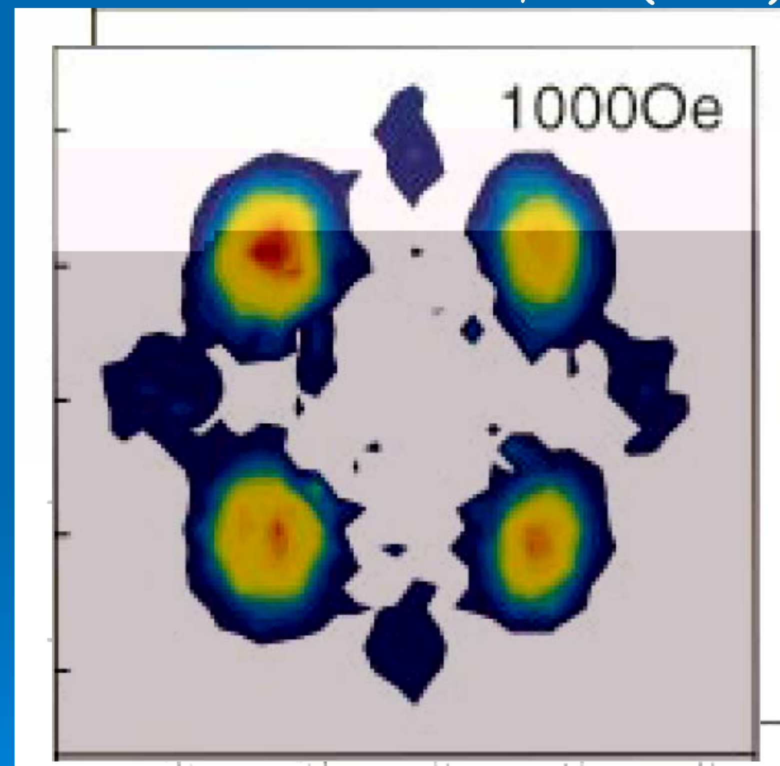
Observation of a Field-Driven Structural Phase Transition in the FLL

$ErNi_2B_2C$: triangular to **square!**



Dewhurst et al
PRB **72**, 014542 (2005)

Eskildsen et al. PRL 78,1968(1997)



Small-angle neutron scattering

Mechanism

Symmetry and reorientation of FLL relative to the crystallographic axes is determined by:

- **Anisotropy of gap parameter** (Obst 1971)
- **Anisotropy of Fermi surface** (Ullmaier 1973)

It can be analyzed *phenomenologically*

$$F_{Aniso}[\psi]=\eta\left[\left|\left(D_x^2-D_y^2\right)\psi\right|^2-\left|\left(D_xD_y+D_yD_x\right)\psi\right|^2\right]$$

- Affleck et al. PRB **55**, R704 (1997)

  Mixture of *s-wave and d*-wave coupling

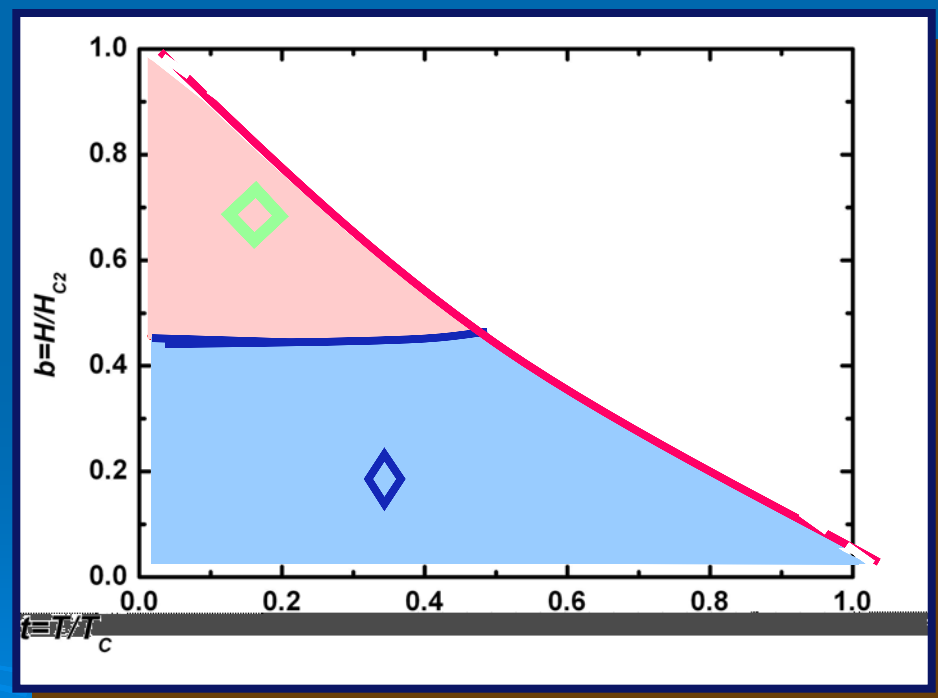
- Rosenstein et al. PRL **80**, 145 (1998)

  Anisotropic contribution to GL free energy

Thermal fluctuation

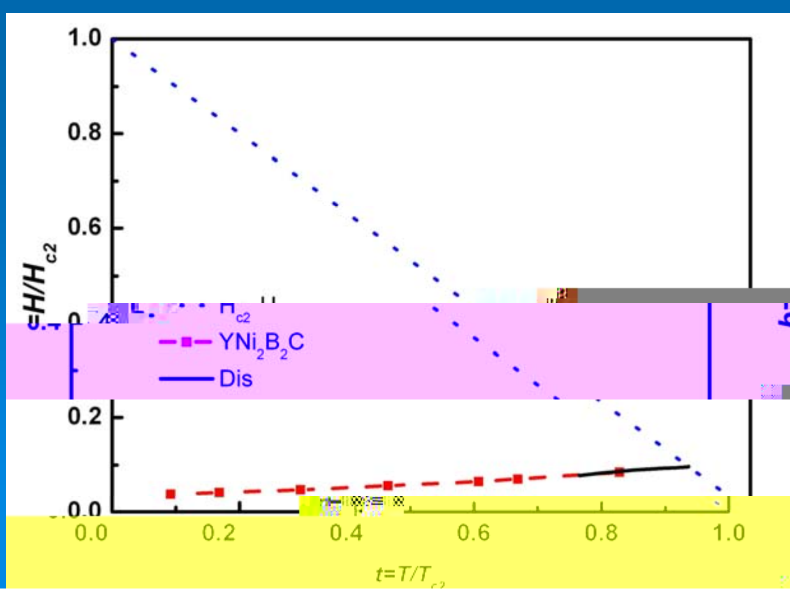
- **Without thermal fluctuation, the SPT line is parallel to the x axis.**
- **It is a 2nd order phase transition.**

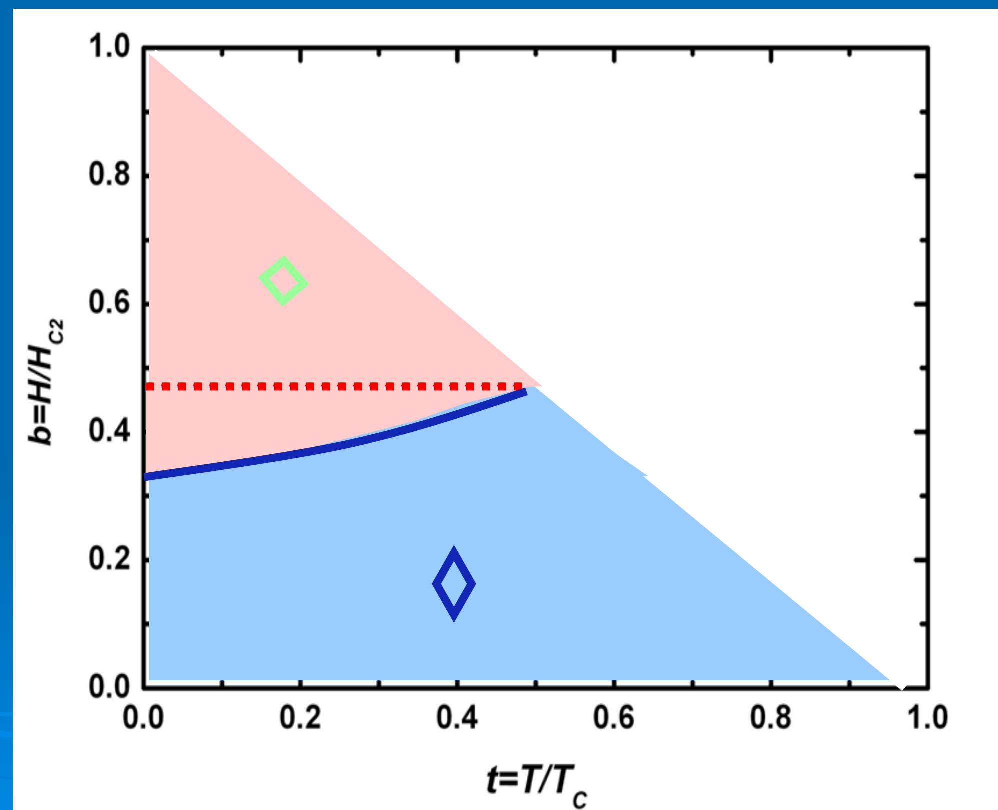
- With weak thermal fluctuation, perturbation method has ***negative*** slope.
- Near melting line, the thermal fluctuation became stronger, Gaussian method tells that the slope became ***positive.***
- However, the thermal fluctuation is not important for LTSC.



Quenched Disorder effect-result

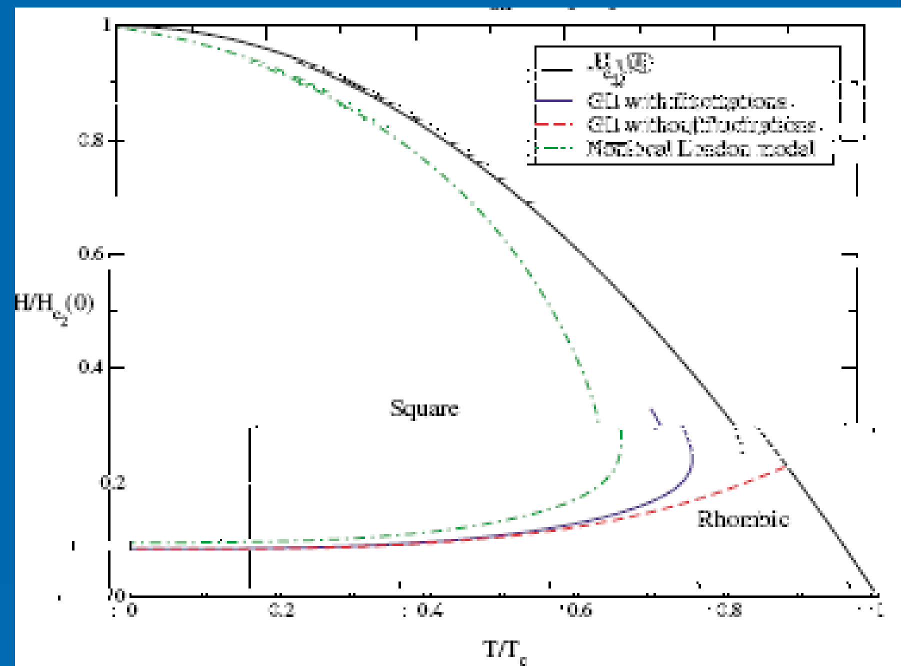
- Slope of SPT is always positive
- Disorder strongly influences the system at ***low temperature***
- ***The SPT is 1st order phase transition.***





Comments on other theories

Those theories can only obtain experimental phase diagrams by using unjustified assumption of some parameters dependence on temperature



Dorsey et al,PRL91,097002(2003)

Conclusion

The LLL GL model is solved and used to explain the experimental results.

Our theory is the quantitative theory. The theory can explain the experimental results well.

- 李　平，　Rosenstein，Phys.Rev.Lett . 90,167004 (2003).
- 李　平，　Rosenstein，　Phys.　Rev.　B 65，220504 (2002)，Rapid Communication
- 李　平，Rosenstein，Phy. Rev.　B70，144521 （2004）.
- 李　平，　Rosenstein，Vinokur，**Journal of Superconductivity: Incorporating Novel Magnetism, vol 19, 369 (2006);** review article　in honor of the festival of 90th birthday of　Nobel Laureate Professor V. Ginzburg.

- 李定平, P. Lin, B. Rosenstein, B. Ya. Shapiro, I. Shapiro **Phys. Rev. B 74, 174518 (2006) .**

- H. Beibdenkopf, Myasoedov, Shtrikman, Zeldov, Rosenstein, 李定平, Tamegai, to be published in PRL**07**