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Benford定理

报告人：Lijing Shao（邵立晶）
指导老师：B.-Q. Ma（马伯强 教授）

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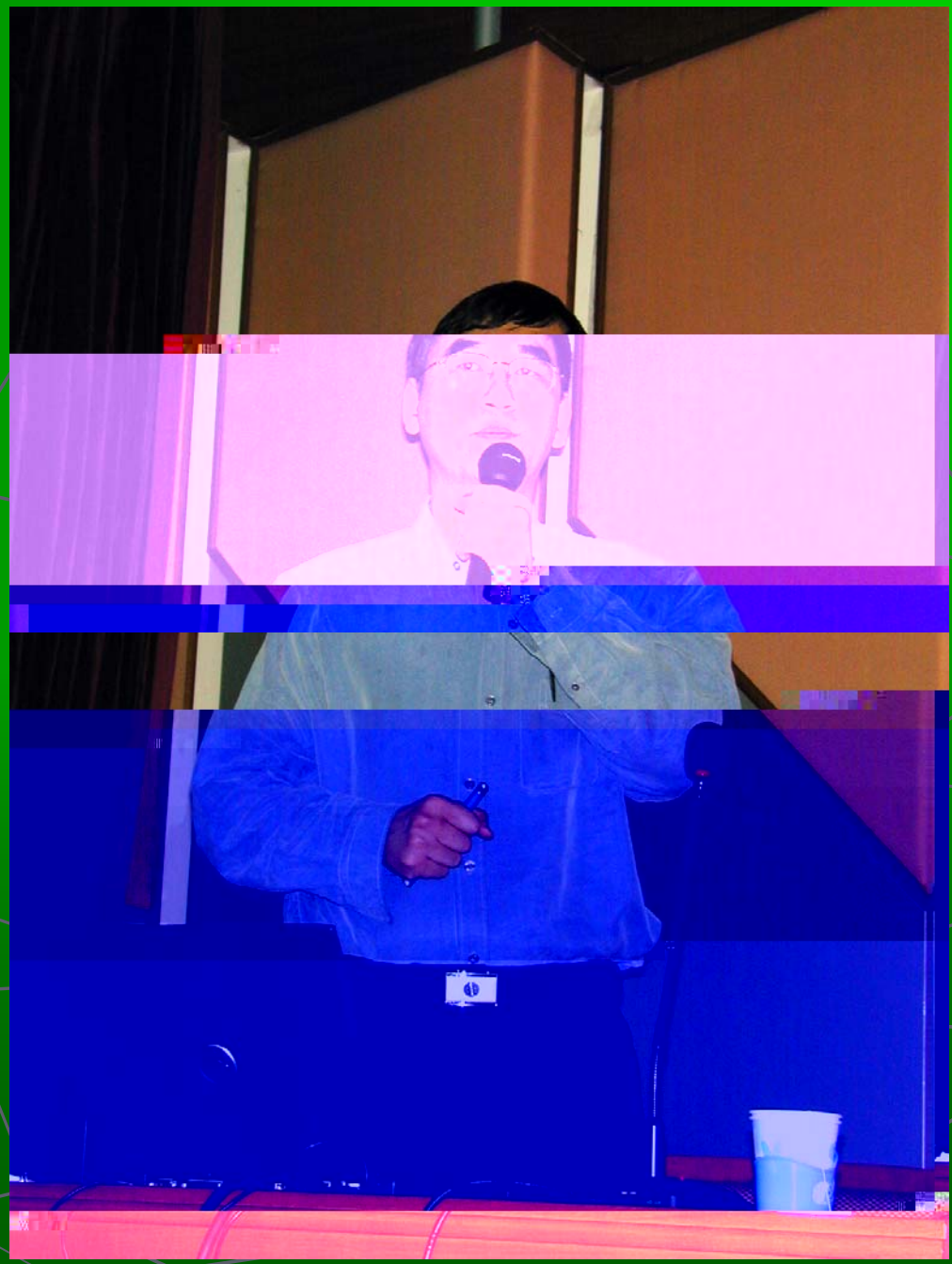

中国物
学会

2010-04-18

My Publication List

- L. Shao, Y.-J. Zhang, B.-Q. Ma, Phys. Lett. B 686 (2010) 136
- L. Shao, Y. Zhang, B.-Q. Ma, Chin. Phys. C (QNP09) in press
- Y. Zhang, L. Shao, B.-Q. Ma, Phys. Lett. B 671 (2009) 30
- Y. Zhang, L. Shao, B.-Q. Ma, Nucl. Phys. A 828 (2009) 390
- Benford
- L. Shao, B.-Q. Ma, Astropart. Phys. 33 (2010) 255
- L. Shao, B.-Q. Ma, Mod. Phys. Lett. A 24 (2009) 3275
- L. Shao, B.-Q. Ma, Physica A (2010) in press
- L. Shao, Z. Xiao, B.-Q. Ma, Astropart. Phys. (2010) in press
- L. Shao, B.-Q. Ma, submitted
- L. Shao, J.-F. Liu, Y.-J. Yang, A. Kong, in preparation

Benford定理

Lijing Shao ( )
B.-Q. Ma ( )

2010-04-23 @

- Benford
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  - Benford
  - Benford

- $e = 1.60 \quad 10^{-19}$ C
- $c = 3 \quad 10^{8}$ m s^{-1}
- $G = 6.67 \quad 10^{-11}$ m^{3} kg^{-1} s^{-2}
- $= 1.055 \quad 10^{-34}$ J s
- $e = 2.718$
- $k_B = 1.38 \quad 10^{-23}$ J K^{-1}
- $m_{Pl} = 2.18 \quad 10^{-8}$ kg
- $_0 = 1.26 \quad 10^{-6}$ N A^{-2}
- $_0 = 8.85 \quad 10^{-12}$ F m^{-2}
- $m_p = 1.67 \quad 10^{-27}$

amu	G	λ_c	ϵ_0
N_A	E_0	m_d	h
μ_B	m_n	m_e	m_p
a_0	μ_n	e	m_p
k	μ_0	R	c

Physics for Scientists and Engineers, 3rd ed. Serway (1990)

Burke & Kincanon, Am. J. Phys. 59 (1991) 952

Table I. Distribution of first digits for 20 physical constants.

	mks system		English system		
k	Frequency	%	Frequency	%	$\log(k+1)/k$
1	8	40	7	35	0.30
2	2	10	2	10	0.18
3	1	5	3	15	0.12
4	0	0	1	5	0.10
5	2	10	1	5	0.08
6	3	15	3	15	0.07
7	0	0	0	0	0.06
8	2	10	1	5	0.05
9	2	10	2	10	0.05

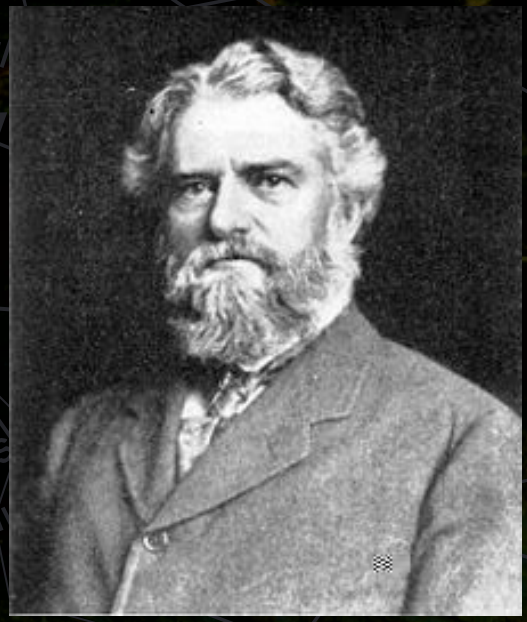
Benford

Newcomb

- 1881 Newcomb

2

- [Newcomb, Am. J. Math. 4 (1881) 39]



- Simon Newcomb (1835 – 1909)
-
.
-

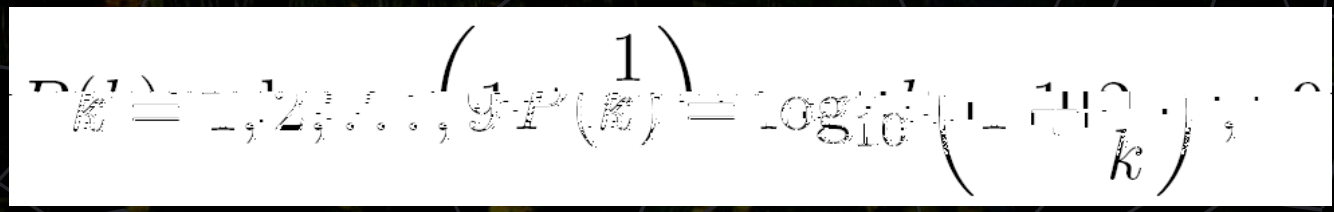
Benford

- 1938 Benford Again!
- [Benford, Proc. Am. Phil. Soc. 78 (1938) 551]
- ...
- **Frank Benford** (1883 – 1948)

109
20

Benford

- Benford's law the first digit law
  the significant digit law



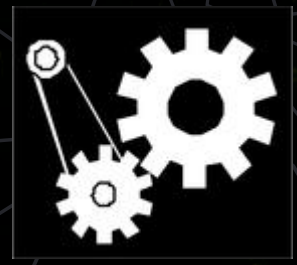
- n-

$$P_{\mathrm{Ben}}(d_1, d_2, ..., d_n) = \log_{10}\left[1 + \left(\sum_{i=1}^{n} d_i \times 10^{n-i}\right)^{-1}\right]$$

15 $\log_{10}(1+1/15)$

Benford

- . . . Benford 1938
- Burke & Kincanon 1991
- α Buck et al. 1993, Ni & Ren 2008
- Leemis et al. 2000
- Pain 2008
- Dorogovtsev et al. 2006
- eBay Giles 2007
- Torres et al. 2007
- Tolle et al. 2000, Snyder et al. 2001, Berger et al. 2005
- . . .



“ ”

- Benford
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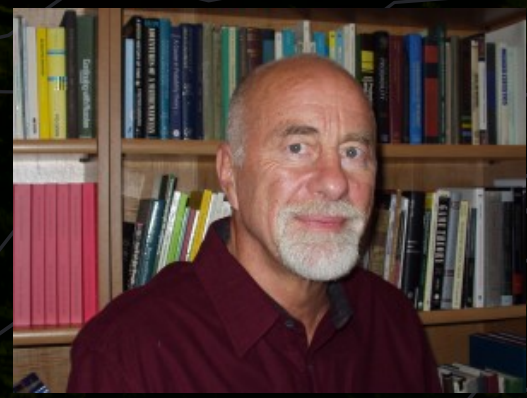
Benford

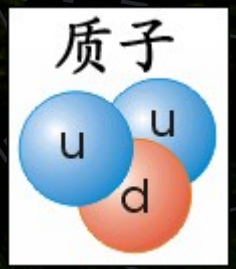
- Nigrini 1996, Nigrini 1999, Rose & Rose 2003
- Al-Marzouki et al. 2005
- Torres et al. 2007
- Diekmann 2007
- Barlow & Bareiss 1985
- Schatte 1988
- Jolion 2001, Fu et al. 2007
- ...
- T.P. Hill



Benford

- Benford

- Benford
- Particle Data Group, Phys. Lett. B 667, 1 (2008)



L. Shao & B.-Q. Ma, Mod. Phys. Lett. A 40 (2009) 3275

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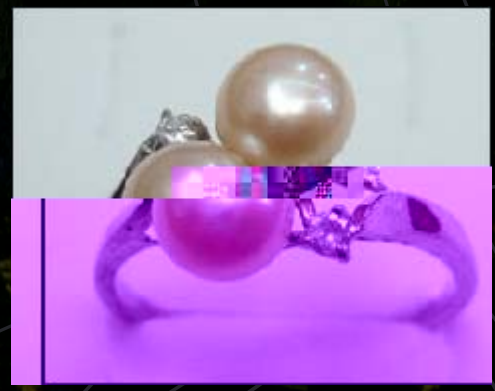

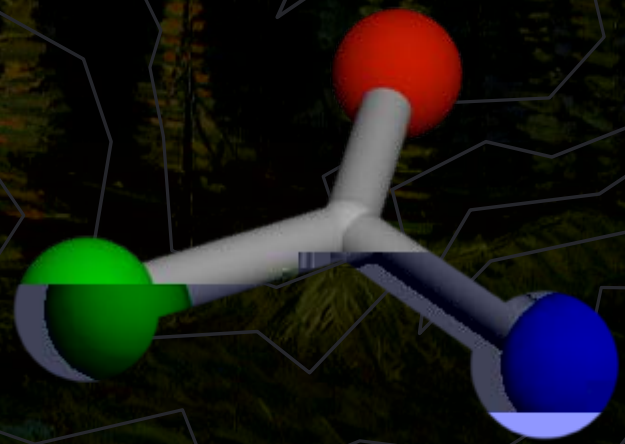

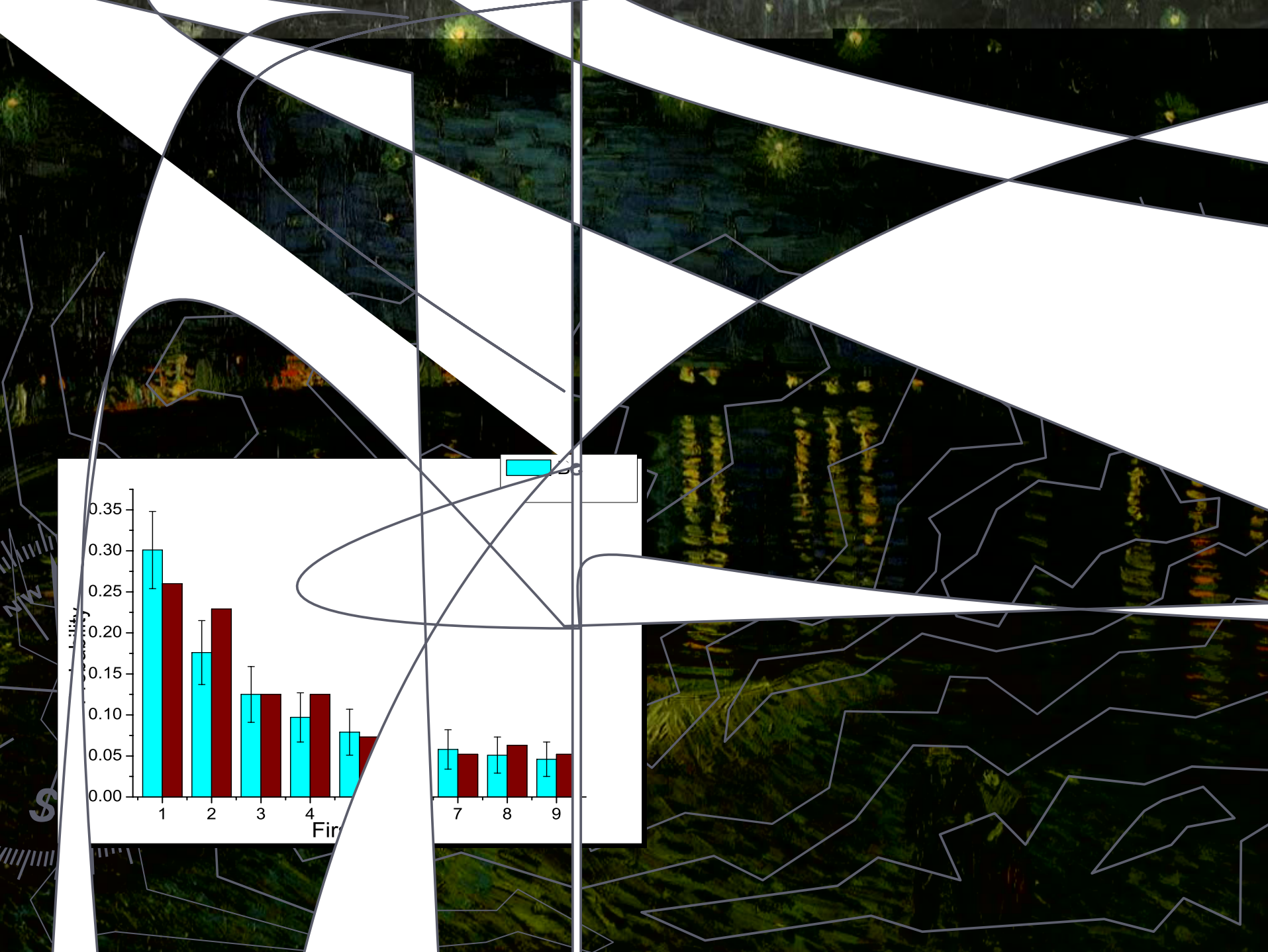
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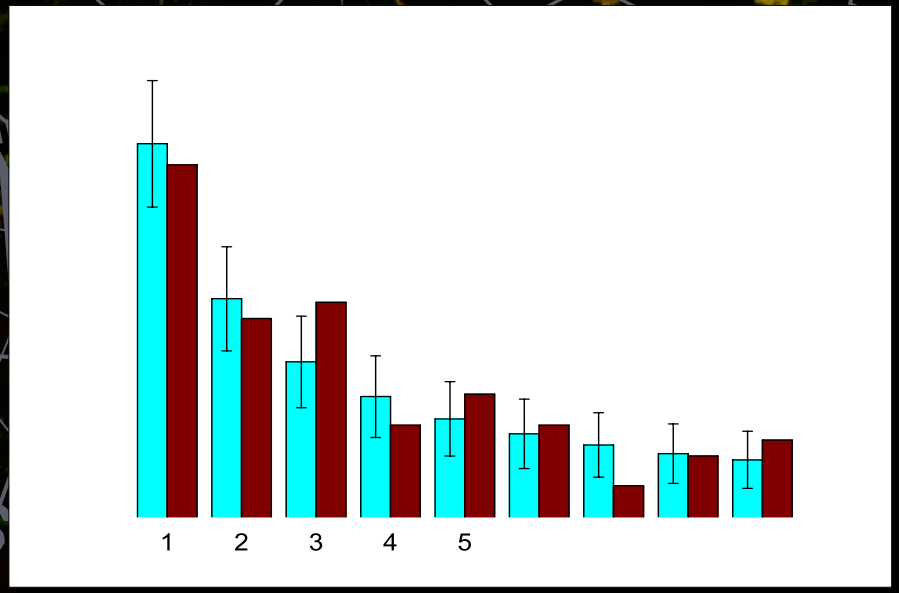
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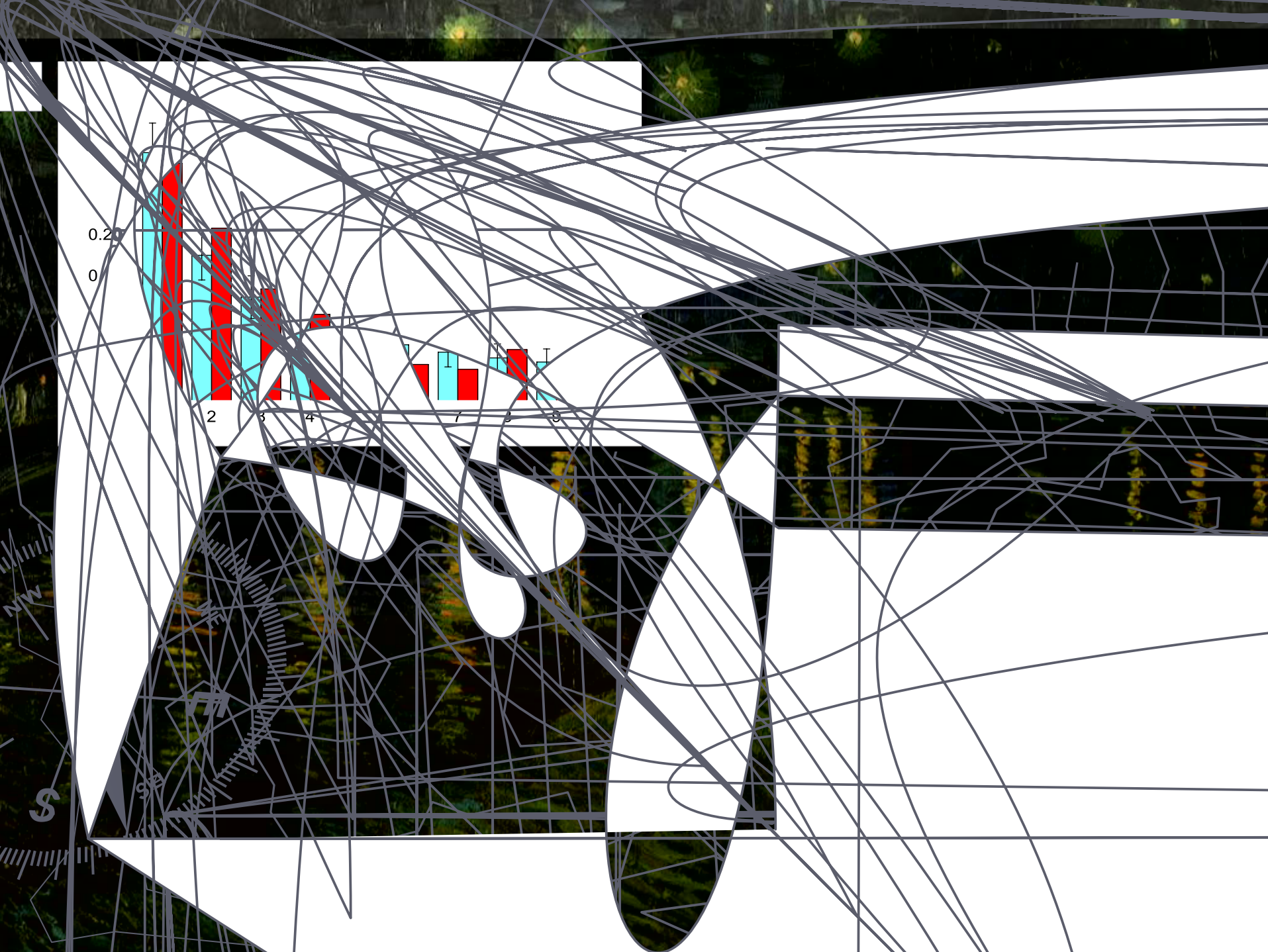
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Benford

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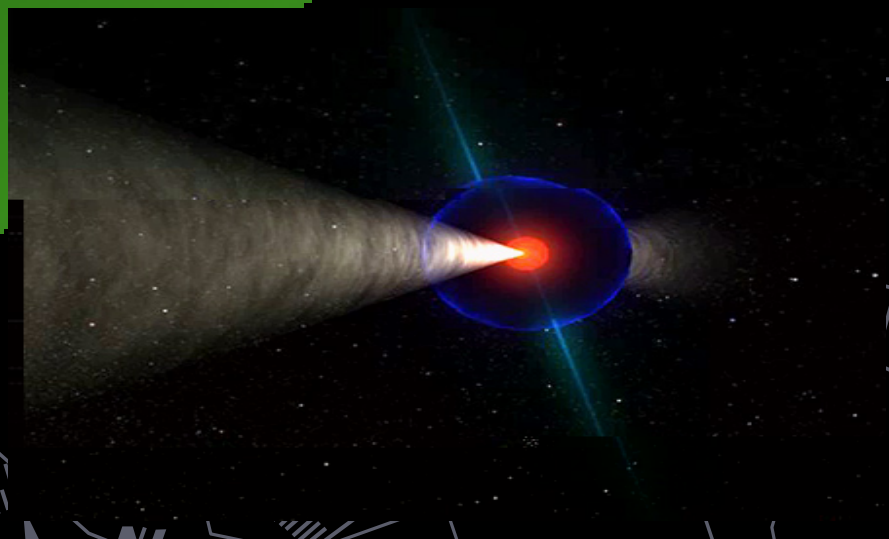
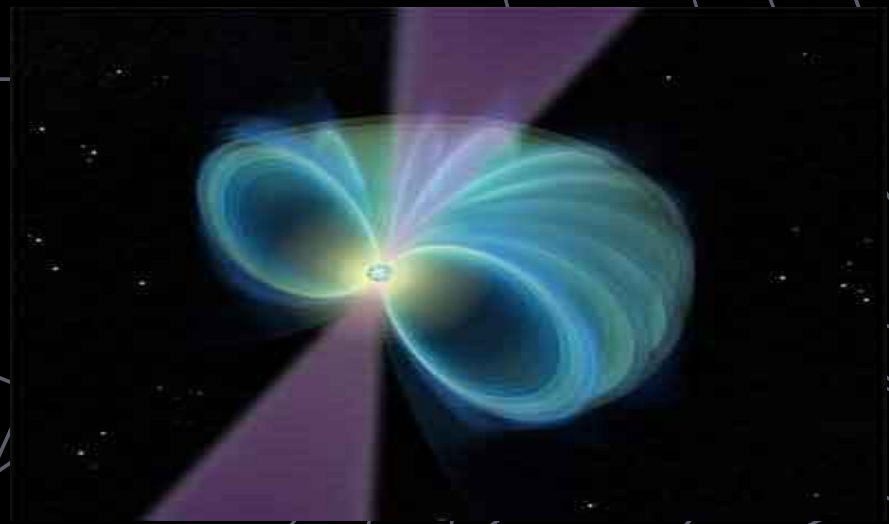
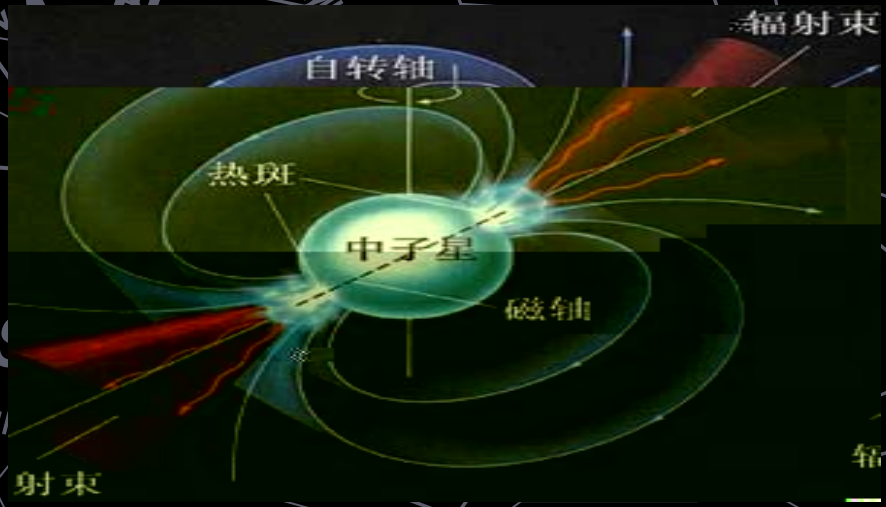
- Australia Telescope National Facility (ATNF) pulsar catalogue (http://www.atnf.csiro.au/research/pulsar/psrcat)

L. Shao & B.-Q. Ma, Astropart. Phys. 33 (2010) 255



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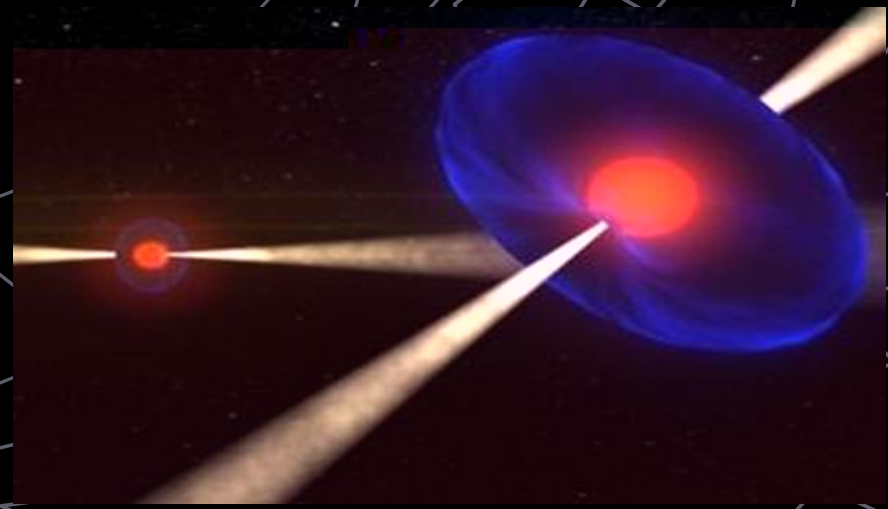

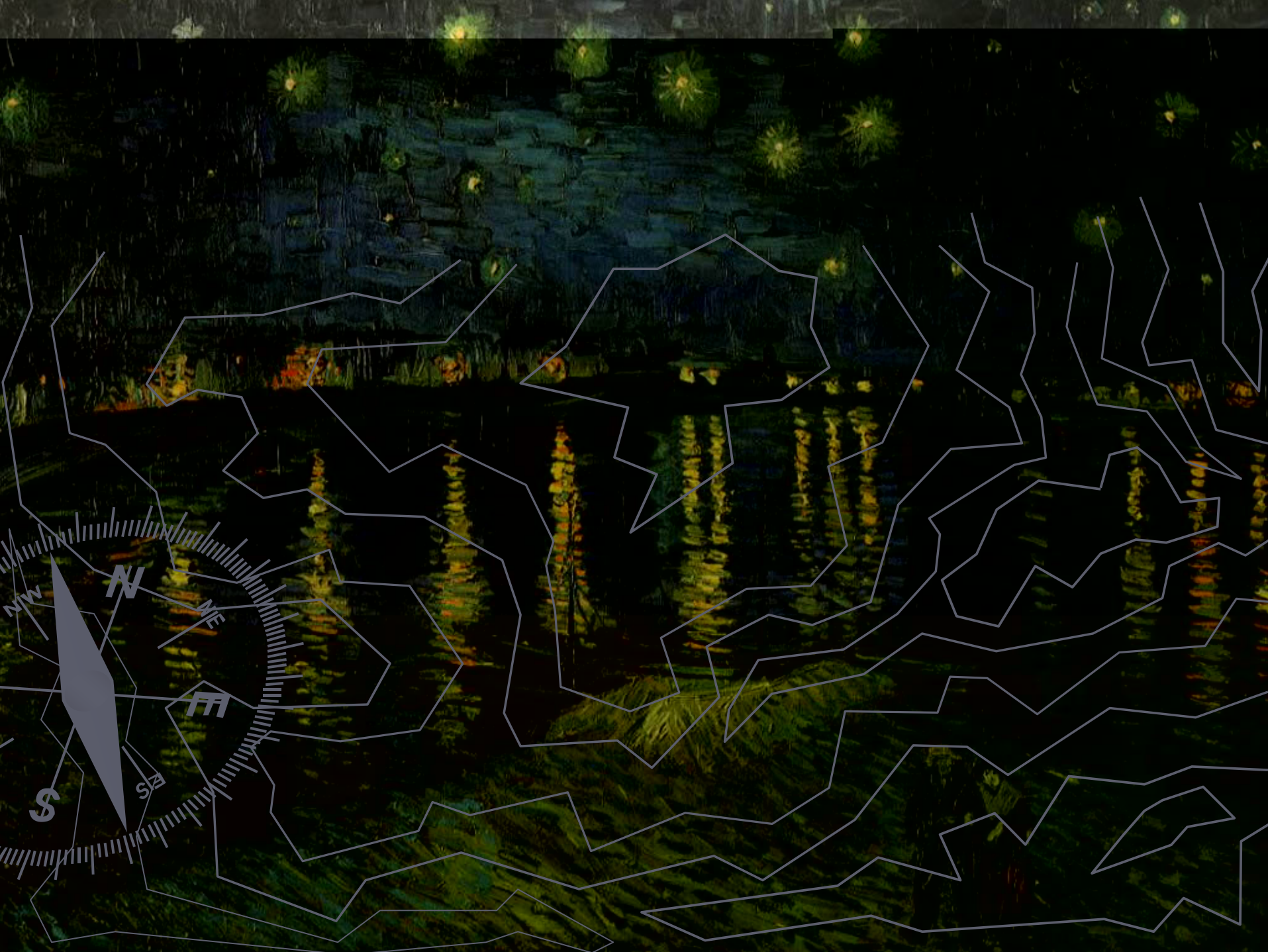
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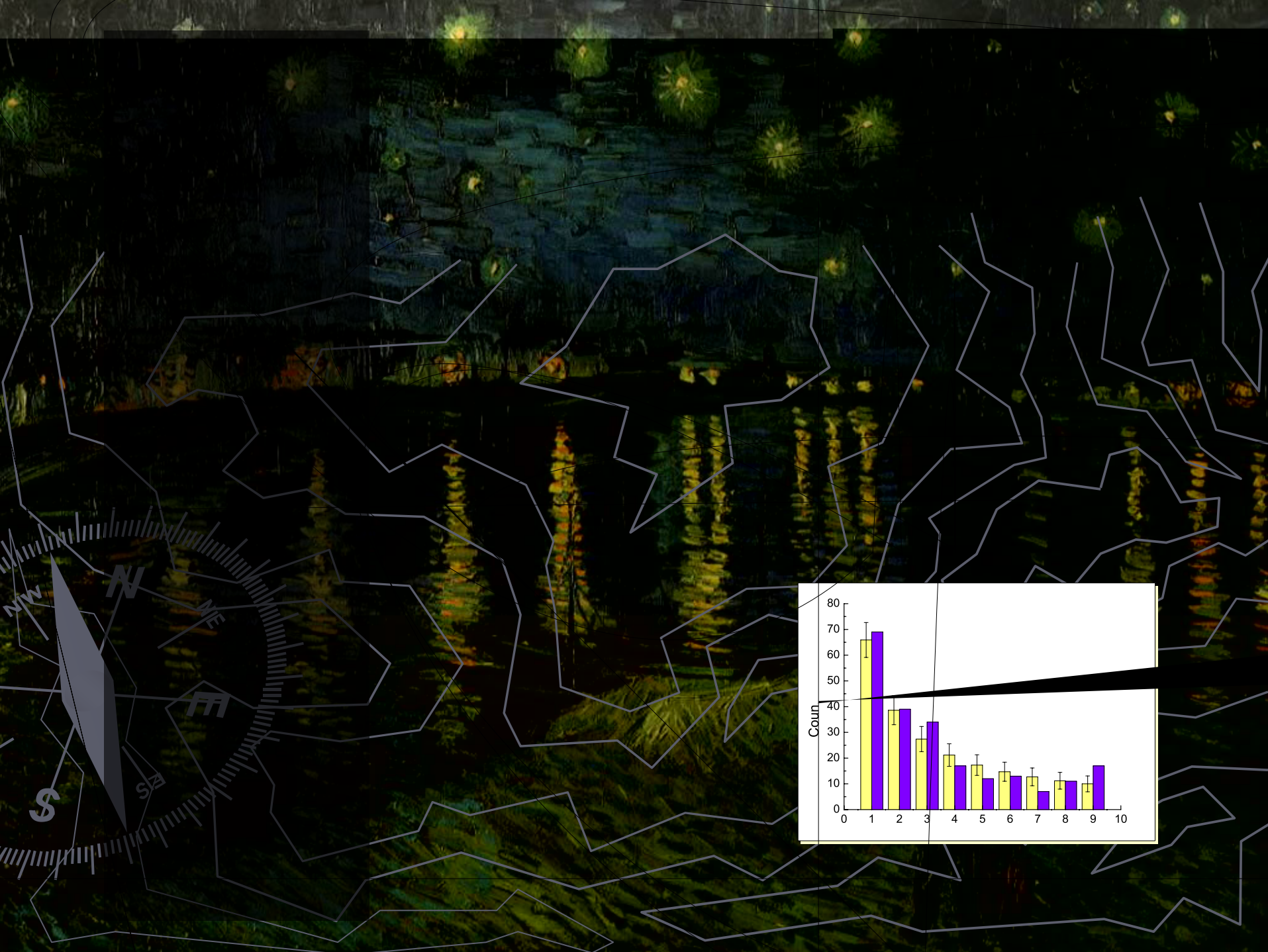
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Benford

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Benford

- Newcomb
  $\{10^{n+f}\}$ f [0,1)
  Benford
- Newcomb (1881) Am. J. Math.

- mantissa a m
  a m $10^n \rightarrow$ m [0.1,1)
- 314.5 $\rightarrow$ 0.3145 0.063 $\rightarrow$ 0.63
  3223 $\rightarrow$ 0.3223

- Benford
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  Benford
- Pinkham (1961) Ann. Math. Statist.

$$\log_{10} N' = \log_{10}(C \times N) = \log_{10} C + \log_{10} N$$

- Benford
- Benford
  (digital)
- Berger, Hill & Morrison (2008) J. Theor. Probab.

- Benford
- $\log_{10}N' = \log_{10}N^n = n \cdot \log_{10}N$
- Shao & Ma (2009) Mod. Phys. Lett. A

- Γ τ
  Benford
- Shao & Ma (2009) Mod. Phys. Lett. A

- Shao & Ma (2009) Astropart. Phys.

- Benford

  ...

- $P(d) = \log_b(1+1/d), \quad d = 1,2,\ldots,b-1$

- Raimi 1969a, Raimi 1969b, Raimi 1976, Raimi 1985
- Hill 1995a, Hill 1995b, Hill 1995c, Hill 1998, Hill et al. 2005

- n- Benford 100 ( ) 1000 ( )
- Hill 1995a,b,c

  Benford

- $m(t+\delta t) = r(t)m(t)$
-
- $\log_{10} m(t+\delta t) = \log_{10} r(t) + \log_{10} m(t)$
-
- $\log_{10} m(t) \rightarrow \infty \rightarrow$

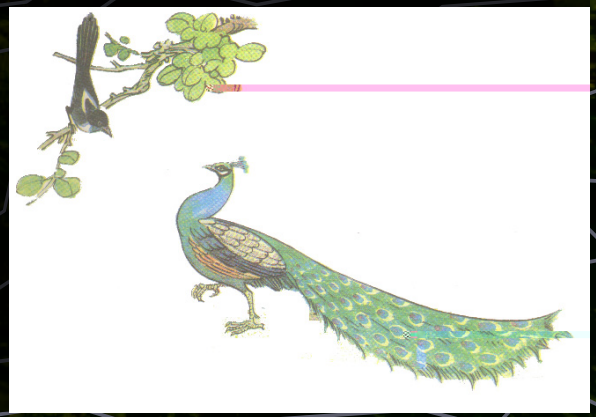
- Pietronero et al. (2001) Physica A

$$P(k) = \frac{\int_k^{k+1} \mathrm{d}\log m(t)}{\int_1^{10} \mathrm{d}\log m(t)} = \frac{\int_k^{k+1} m^{-1}\,\mathrm{d}m}{\int_1^{10} m^{-1}\,\mathrm{d}m} = \log_{10}\left(1+\frac{1}{k}\right)$$

- n- Benford
  1/m 1/m
  n- Benford

$$f(m) = \frac{1}{\ln 10} \cdot \frac{1}{m},\ m \in [0.1,\ 1)$$

- Shao & Ma, Physica A (2010) in press



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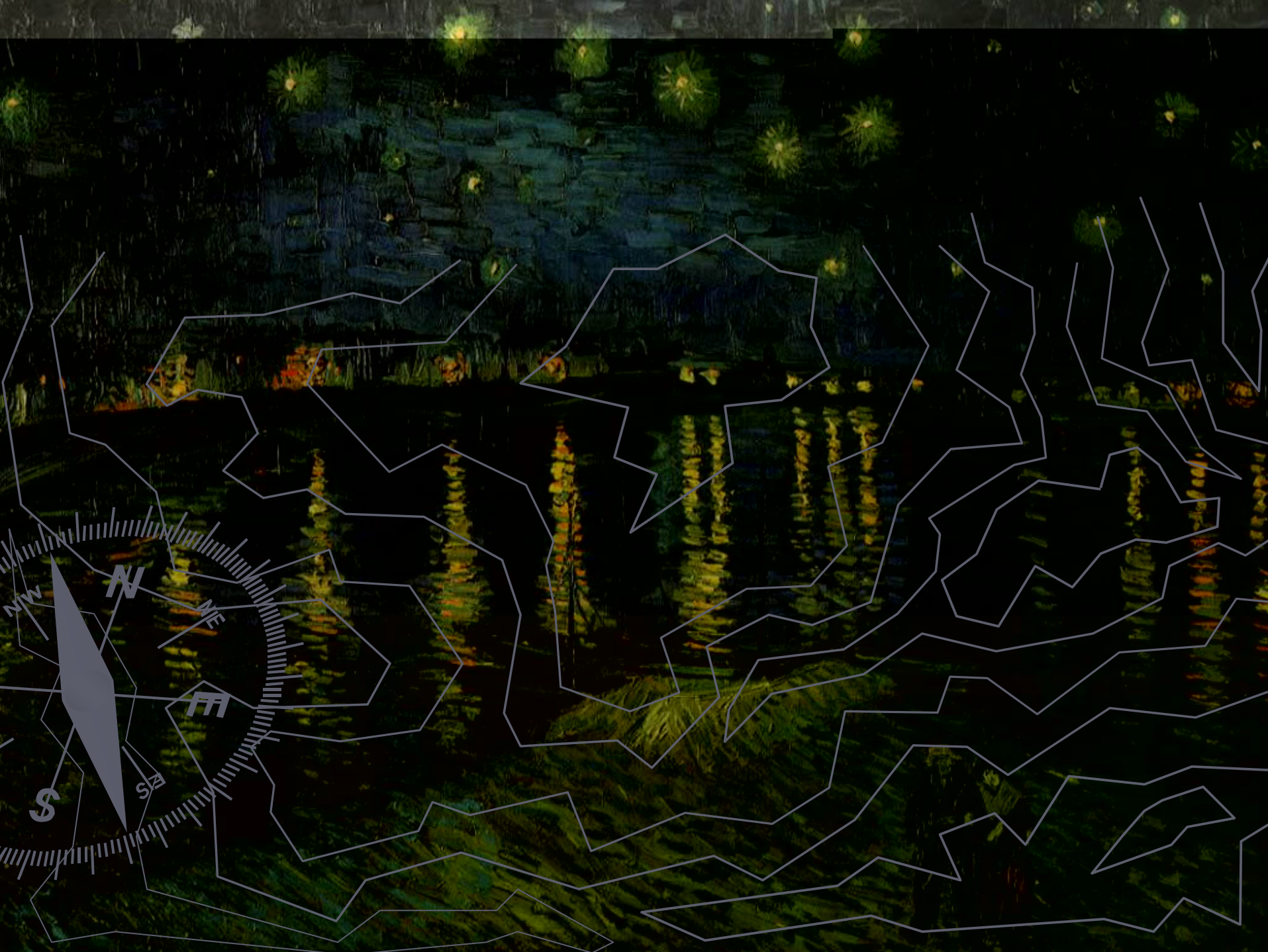
- Benford
- - (BG) -
(FD) - (BE)
- Shao & Ma, Physica A (2010) in press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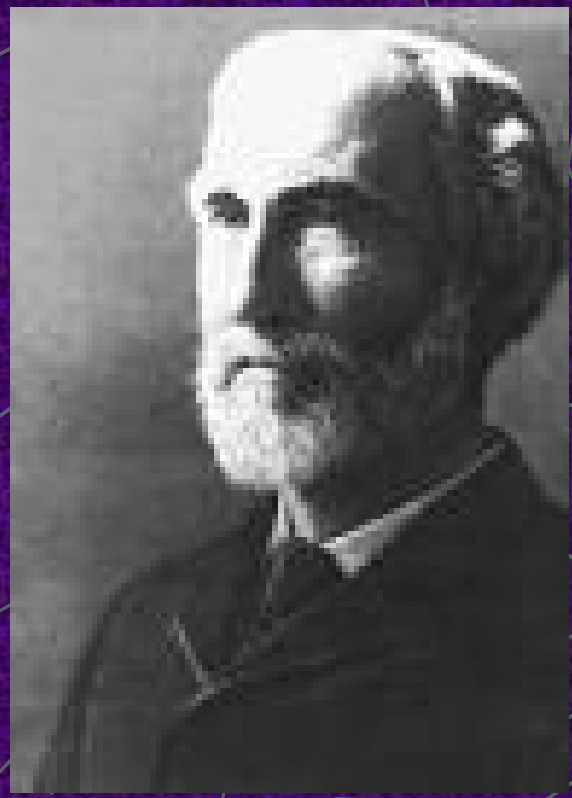
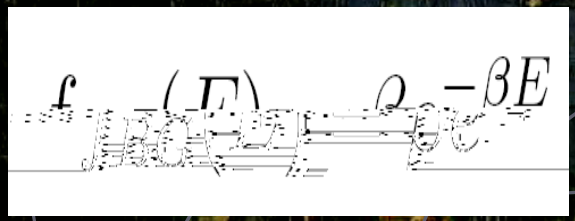


Boltzmann, Wien. Ber. 76 (1877) 373-435.
Gibbs, *Elementary Principles in Statistical Mechanics*, Dover, New York, 1901.

Benford



$$P_{BG}(d;\beta) = \sum_{n=-\infty}^{\infty} \left[e^{-\beta d \cdot 10^n} - e^{-\beta (d+1)\cdot 10^n} \right]$$

Engel & Leuenberger (2003) Stat. Probabil. Lett.
Shao & Ma, Physica A (2010) in press

$$P_{BG}(d;\beta) = P_{BG}(d;10\beta)$$

$$P^{\star}_{BG}(d;\alpha) = P^{\star}_{BG}(d;\alpha+1)$$

$$\int_0^1 P^{\star}_{BG}(d;\alpha)\mathrm{d}\alpha = \log_{10}(1+\frac{1}{d}) \equiv P_{\mathrm{Ben}}(d)$$

-

E. Fermi, Rend. Lincei 3 (1926) 145-149.
P.A.M. Dirac, Proc. R. Soc. A 112 (1926) 661-677.

- Benford

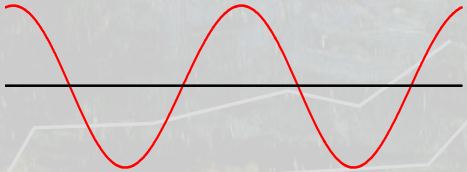
$$f_{FD}(E) = \frac{\beta}{\ln 2}\frac{1}{e^{\beta E}+1}$$

$$P_{FD}(d;\beta) = \sum_{n=-\infty}^{\infty}\frac{1}{\ln 2}\left\{\beta\cdot 10^{n} + \ln\left[\frac{e^{\beta d\cdot 10^{n}}+1}{e^{\beta(d+1)\cdot 10^{n}}+1}\right]\right\}$$

L. Shao & B.-Q. Ma, Physica A (2010) in press

$$P^{\star}_{FD}(d;\alpha) = P_{FD}(d;\beta = 10^{\alpha})$$

$$\int_0^1 P^{\star}_{FD}(d;\alpha)\mathrm{d}\alpha = \log_{10}\left(1+\frac{1}{d}\right) \equiv P_{\mathrm{Ben}}(d)$$



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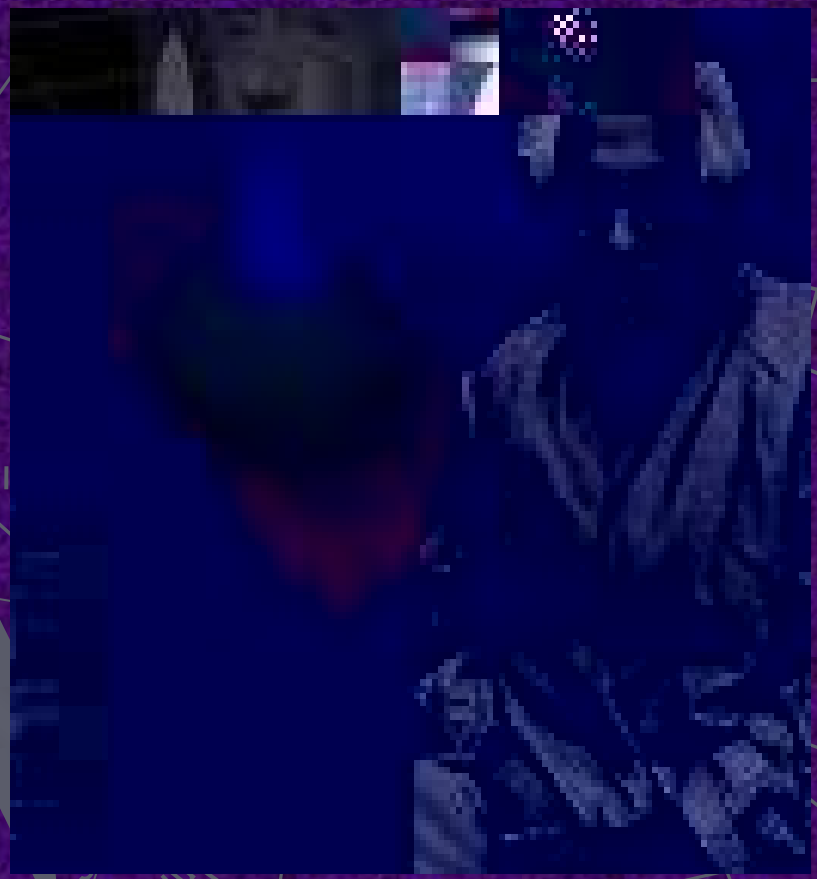

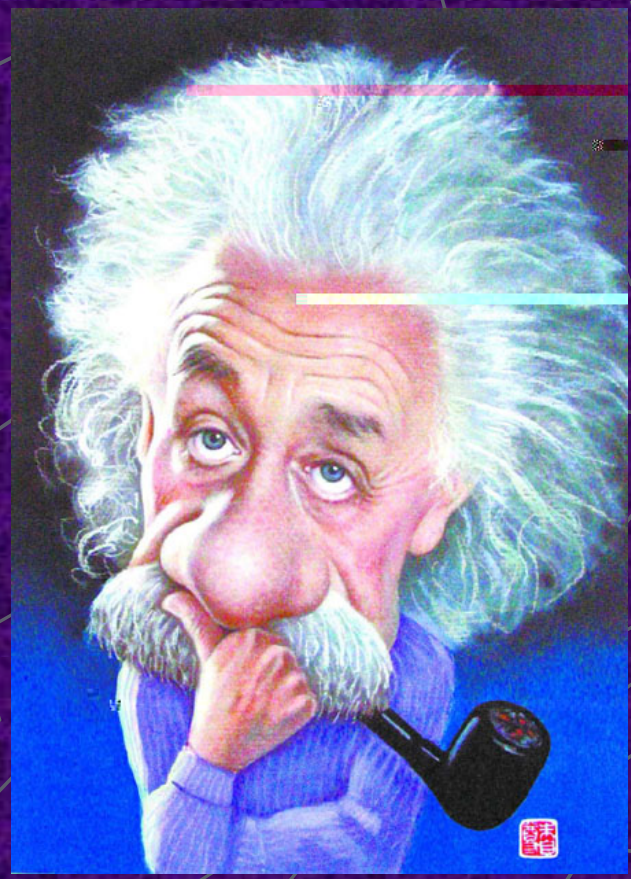

S.N. Bose, Z. Phys. 26 (1924) 178-181.
A. Einstein, Sitz. Ber. Preuss. Akad. Wiss. 22 (1924) 261-267.

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Benford

$$f_{BE}(E) \propto \frac{1}{e^{\beta E} - 1}$$

- $f_{BE} \rightarrow 1/E \quad E \rightarrow 0$
-

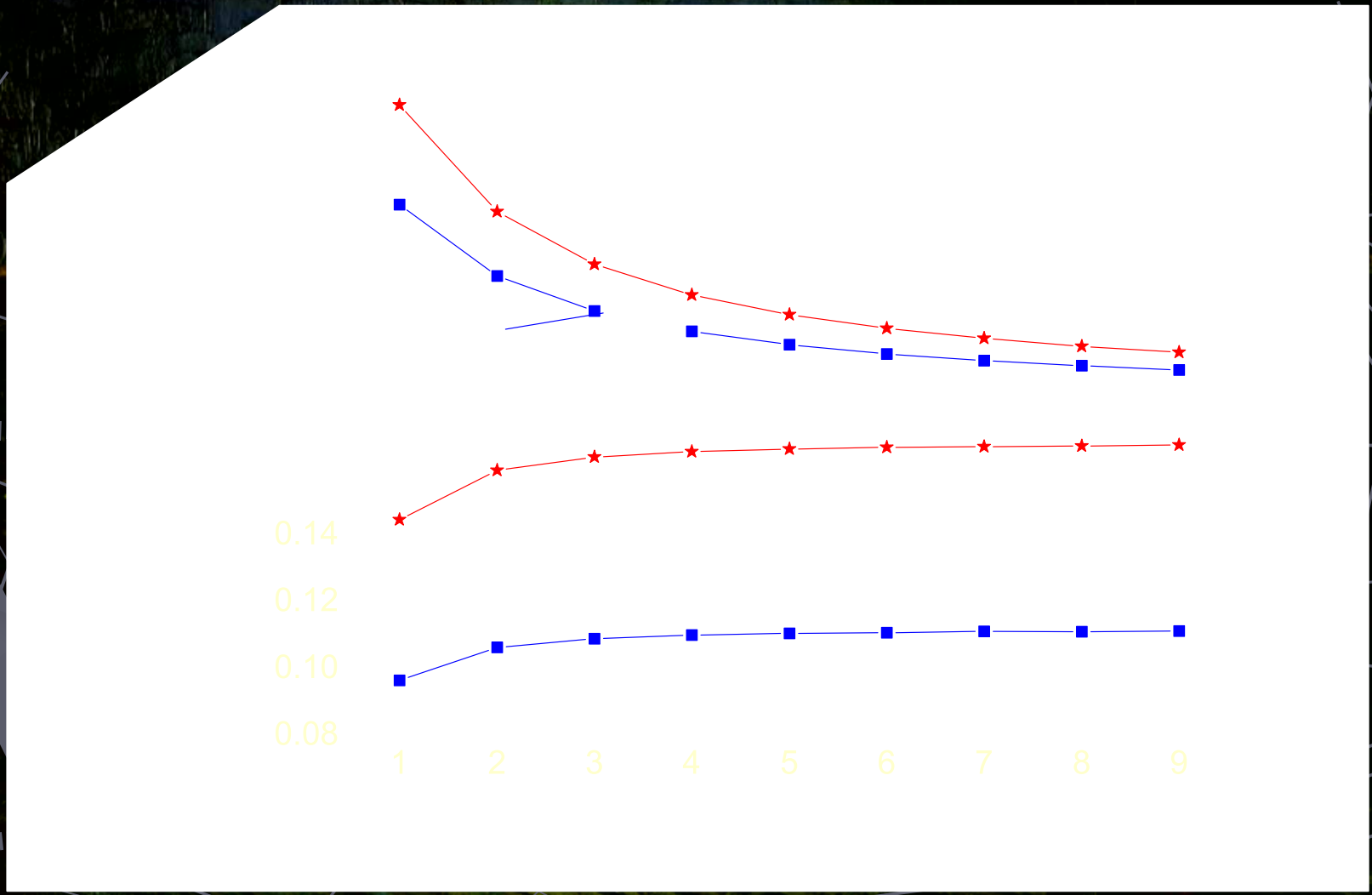
$E \rightarrow 0$

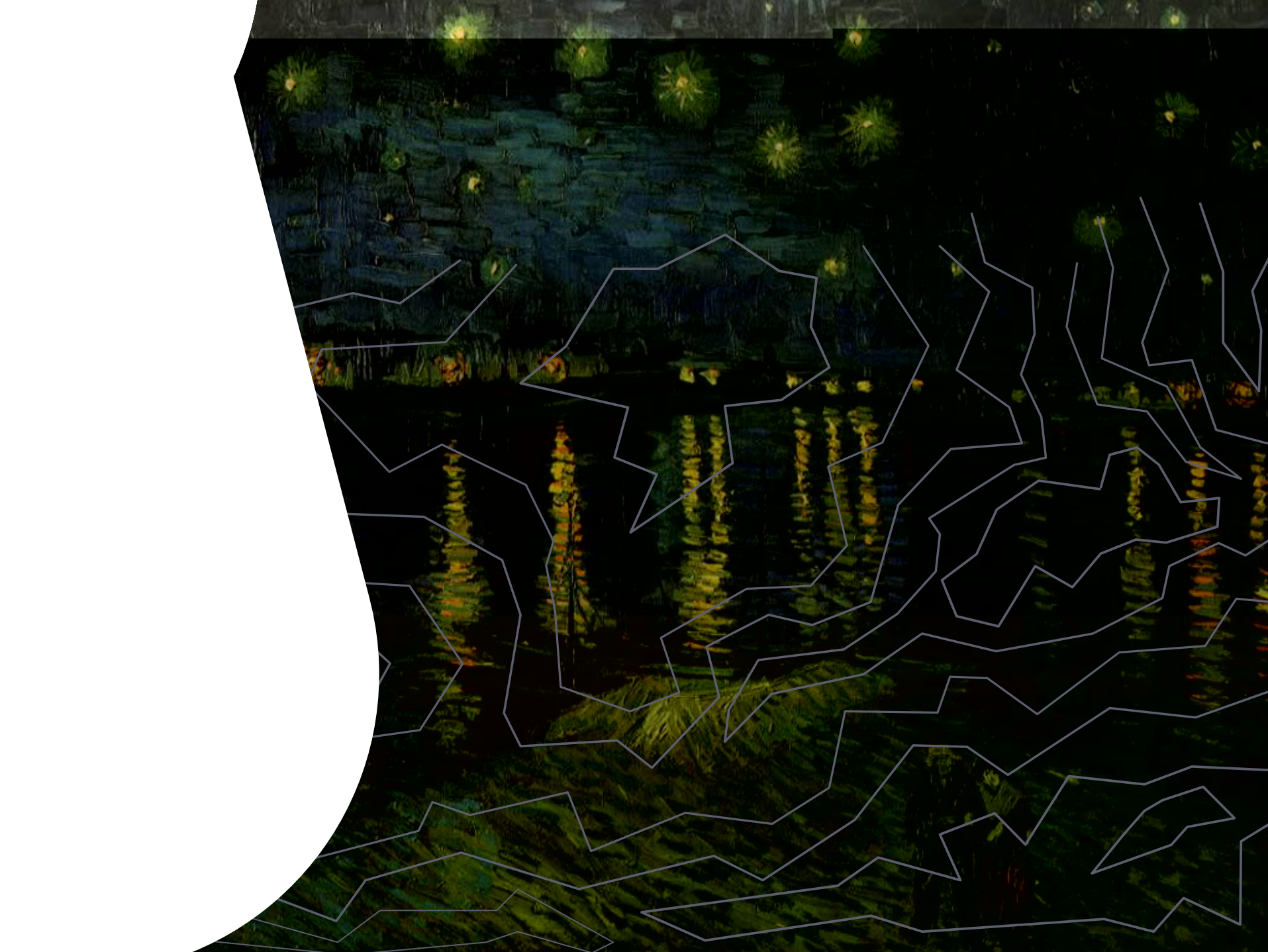
-

Benford

L. Shao & B.-Q. Ma, Physica A (2010) in press

BG FD Benford





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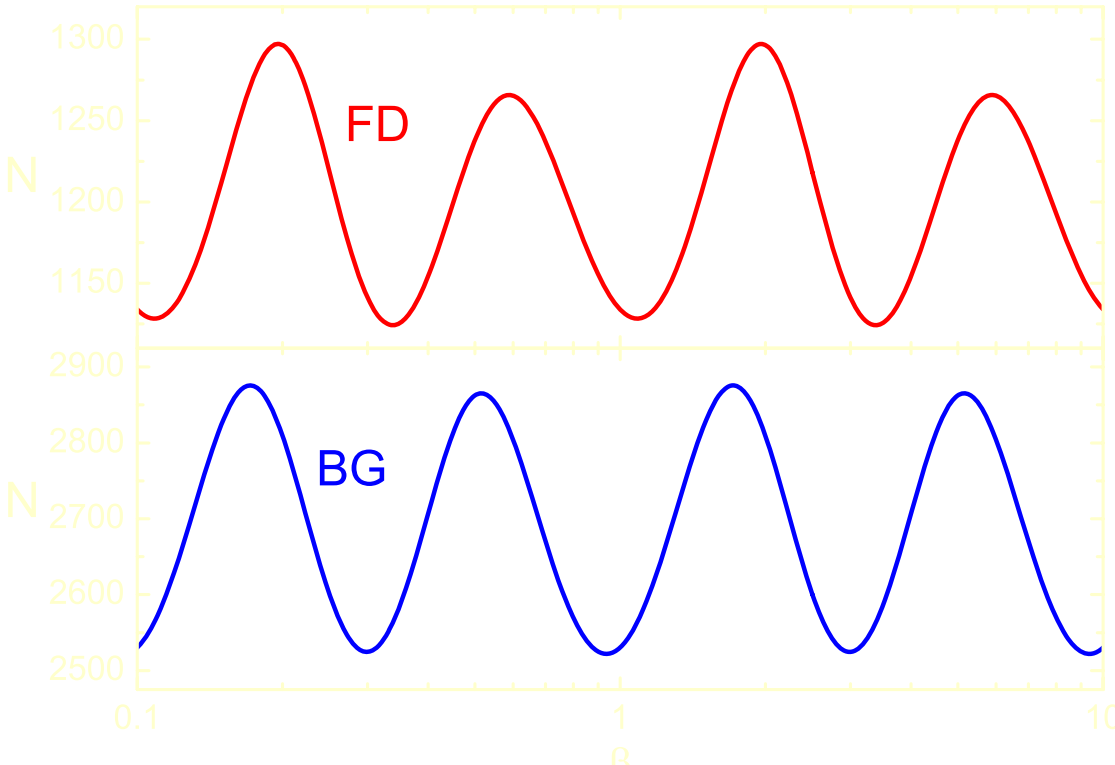

L. Shao & B.-Q. Ma, Physica A (2010) in press

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$$f(E) \propto \frac{1}{e^{\beta E} \pm z}$$

$$P(d;\beta,z) = \sum_{n=-\infty}^{\infty} \frac{1}{\ln(1 \pm z)} \left\{ \beta \cdot 10^n + \ln \left[\frac{e^{\beta d \cdot 10^n} \pm z}{e^{\beta (d+1) \cdot 10^n} \pm z} \right] \right\}$$

L. Shao & B.-Q. Ma, Physica A (2010) in press

- Benford
- Raimi “ ”
  Benford
  - Raimi (1969) Sci. Am.
- Hill “ ” “ ”
  Benford
  - Hill (1995) Stat. Sci.

L. Shao & B.-Q. Ma, Physica A (2010) in press

β E

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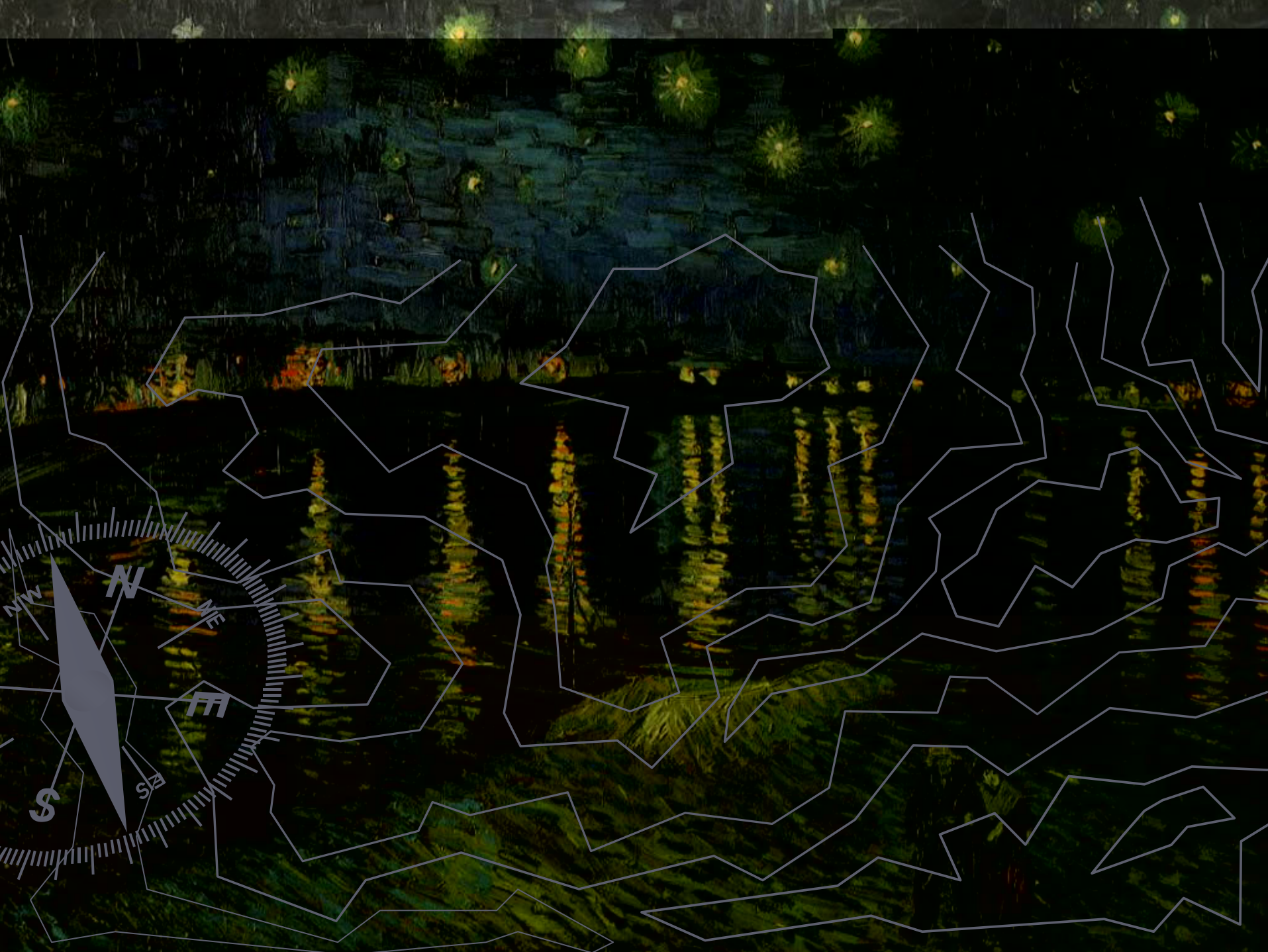
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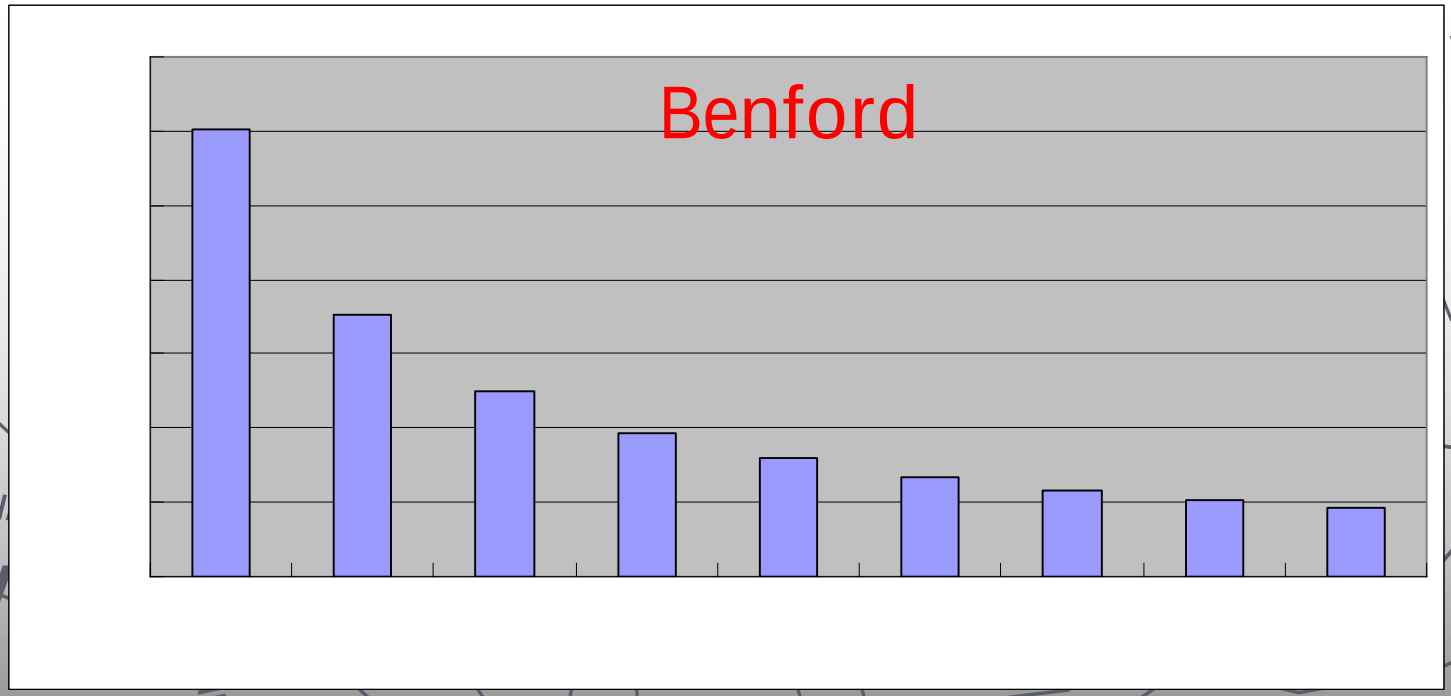

The Other Part

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- (2005-2007)
  → (2007)
- (hobbies)
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PLB09 Zhang, Shao & Ma

Benford

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NPA09 Zhang, Shao & Ma

- Albert Kong, Ji-Feng Liu, Yi-Jung Yang
- Shao, Liu, Yang & Kong, in preparation
- Benford
- ApP10 Shao & Ma
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- PLB10 Shao, Zhang & Ma
- Lorentz
- ApP10 Shao, Xiao & Ma
- Lorentz cont.
- Shao & Ma, submitted

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- Where is the PHYSICS
- Enjoy it!

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lshao@pku.edu.cn

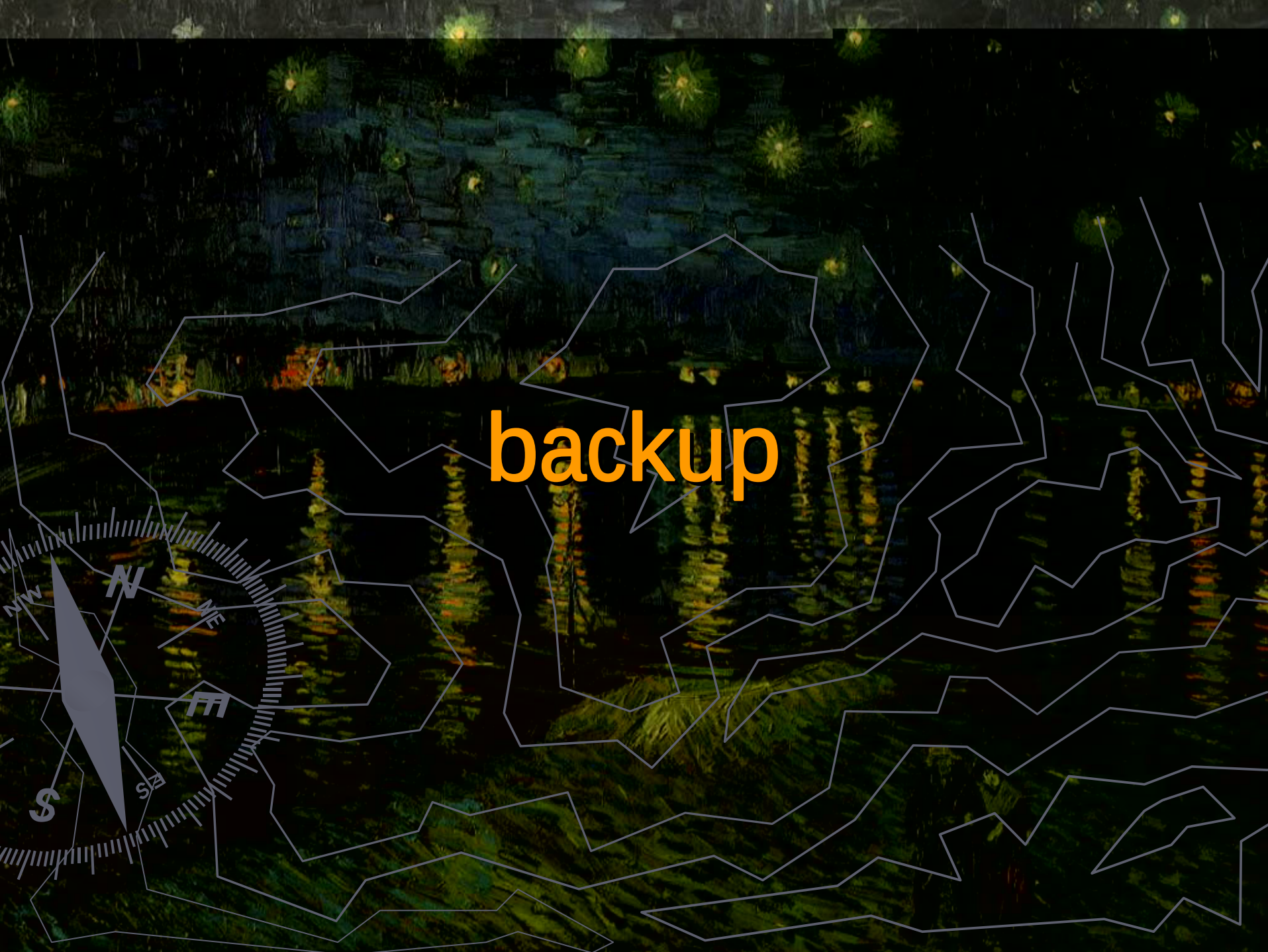
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Table 1. The first digit distribution of the full widths of mesons.

First digit	Case 1 (88)	Case 2 (91)	Case 3 (96)
1	24 (26.5 ± 4.3)	25 (27.4 ± 4.4)	25 (28.9 ± 4.5)
2	22 (15.5 ± 3.6)	22 (16.0 ± 3.6)	22 (16.9 ± 3.7)
3	11 (11.0 ± 3.1)	11 (11.4 ± 3.2)	12 (12.0 ± 3.2)
4	9 (8.5 ± 2.8)	11 (8.8 ± 2.8)	12 (9.3 ± 2.9)
5	5 (7.0 ± 2.5)	5 (7.2 ± 2.6)	7 (7.6 ± 2.6)
6	2 (5.9 ± 2.3)	2 (6.1 ± 2.4)	2 (6.4 ± 2.4)
7	5 (5.1 ± 2.2)	5 (5.3 ± 2.2)	5 $(5.6 \pm 2.3$
8	6 (4.5 ± 2.1)	6 (4.7 ± 2.1)	6 $(4.9 \pm 2.2$
9	4 (4.0 ± 2.0)	4 (4.2 ± 2.0)	5 $(4.4 \pm 2.0$
Pearson χ^2	6.62	6.82	6.32

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Table 2. The first digit distribution of the full widths of baryons.

First digit	Case 1 (65)		Case 2 (72)		Case 3 (81)	
1	21	(19.6 ± 3.7)	22	(21.7 ± 3.9)	23	(24.4 ± 4.1)
2	11	(11.4 ± 3.1)	12	(12.7 ± 3.2)	13	(14.3 ± 3.4)
3	9	(8.1 ± 2.7)	11	(9.0 ± 2.8)	14	(10.1 ± 3.0)
4	6	(6.3 ± 2.4)	6	(7.0 ± 2.5)	6	(7.8 ± 2.7)
5	6	(5.1 ± 2.2)	7	(5.7 ± 2.3)	8	(6.4 ± 2.4)
6	4	(4.4 ± 2.0)	5	(4.8 ± 2.1)	6	(5.4 ± 2.2)
7	1	(3.8 ± 1.9)	1	(4.2 ± 2.0)	2	(4.7 ± 2.1)
8	4	(3.3 ± 1.8)	4	(3.7 ± 1.9)	4	(4.1 ± 2.0)
9	3	(3.0 ± 1.7)	4	(3.3 ± 1.8)	5	(3.7 ± 1.9)
Pearson χ^2	2.57		3.52		4.57	

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Table 3. The first digit distribution of the full widths of hadrons.

First digit	Case 1 (eq. (152))		Case 2 (eq. (168))		Case 3 (eq. (177))	
(53.3 ± 6.1)	1	45	(46.1 ± 5.7)	47	(49.1 ± 5.9)	4
(31.2 ± 5.1)	2	33	(26.9 ± 4.7)	34	(28.7 ± 4.9)	3
(22.1 ± 4.4)	3	20	(19.1 ± 4.1)	22	(20.4 ± 4.2)	2
(17.2 ± 3.9)	4	15	(14.8 ± 3.7)	17	(15.8 ± 3.8)	1
(14.0 ± 3.6)	5	11	(12.1 ± 3.3)	12	(12.9 ± 3.4)	1
(11.8 ± 3.3)	6	6	(10.2 ± 3.1)	7	(10.9 ± 3.2)	
(9.5 ± 3.0)	7	7	(10.3 ± 3.1)	6	(8.9 ± 2.9)	6
10	(7.8 ± 2.7)	10	(8.3 ± 2.8)	10	(9.1 ± 2.9)	8
7	(7.0 ± 2.6)	8	(7.5 ± 2.7)	10	(8.1 ± 2.8)	9
4.82		4.39		4.62		Pearson χ^2

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Table 1
The first digit distributions of the period, frequency, spin down ages, and their time derivatives of pulsars.

Physical Quantity	Notation	Data points	$\chi^2(8)$	p-value
Barycentric period of the pulsar (s)	P0	1825	50.552	0.0001

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Table 2
The first digit distributions of the power quantities of pulsars.

Physical Quantity	Notation	Data points	$\chi^2(8)$	p-value
Spin down energy loss rate (ergs/s)	Edot	1664	6.601	0.5802
Energy flux at the Sun (ergs/kpc^2/s)	Edotd2	1656	9.938	0.2694
Radio luminosity at 400 MHz (mJy kpc^2)	R_Lum	663	10.083	0.2592
Radio luminosity at 1400 MHz (mJy kpc^2)	R_Lum14	1391	2.673	0.9532
Mean flux density at 400 MHz (mJy)	S400	663	10.446	0.2351
[illegible] (mJy)	[illegible]	[illegible]	[illegible]	[illegible]

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Table 3

The first-digit distributions of the kinematic quantities of pulsars.

Notation	Data points	$\chi^2(8)$	p-value	Physical Quantity
PMDec	178	4.840	0.7745	Proper motion in declination (mas/yr)
PMRA	178	8.421	0.3935	Proper motion in right ascension (mas/yr)
PMElat	39	13.057	0.1099	Proper motion in ecliptic latitude (mas/yr)
PMElong	83	11.695	0.1653	Proper motion in ecliptic longitude (mas/yr)
PMTot	220	7.527	0.4810	Total proper motion (mas/yr)
VTrans	219	11.855	0.1578	Transverse velocity - based on DIST (km/s)

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Table 4
The first digit distributions of the expected minimum companion mass and median companion mass of pulsars.

points	$\chi^2(8)$	p-value	Physical Quantity	Notation	Data
0	31.331	0.0001	Minimum companion mass assuming i=90 degrees and neutron star mass is 1.35 $M_\odot$	MinMass	14
0	32.781	0.0001	Median companion mass assuming i=60 degrees	MedMass	14

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Table 1: The maximum deviations of the first digit distributions from Benford's law f the BG distribution and the FD distribution.

d First Digit	$P_{\rm Ben}(d)$ Benford	Max $\lvert\Delta P(d)\rvert$ BG	Max $\lvert\Delta P(d)\rvert$ FD	Max $\lvert\Delta P(d)/P_{\rm Ben}(d)\rvert$ BG	Max $\lvert\Delta P(d)/P_{\rm Ben}(d)\rvert$ FD
1	0.301	0.0291	0.0435	9.66%	14.5%
2	0.176	0.0187	0.0281	10.6%	15.9%
3	0.125	0.0136	0.0204	10.9%	16.3%
4	0.097	0.0107	0.0160	11.0%	16.5%
5	0.079	0.0088	0.0131	11.1%	16.6%
6	0.067	0.0074	0.0111	11.1%	16.6%
7	0.058	0.0064	0.0097	11.1%	16.7%
8	0.051	0.0057	0.0085	11.1%	16.7%
9	0.046	0.0051	0.0076	11.1%	16.7%

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