Particle-Number-Conditioned Master Equation and Its Application in Quantum Measurement and in Quantum Transport

Xin-Qi Li

Dept. of Physics, Beijing Normal University

Collaborators

J.S. Jin, and J.Y. Luo (HKUST & CAS)
F. Li, S-K Wang, and H.J. Jiao (CAS)
Prof. Y.J. Yan (HKUST)
Prof. S.A. Gurvitz (Weizmann Institute of Science)
Prof. W.M. Zhang (Cheng Kong Univ., Taiwan)

Outline:

- Master equation approach to quantum transport

- Quantum measurement of solid-state qubit
  - Example: SET detector, signal-to-noise ratio, etc

- Quantum transport
  - Current fluctuation, full counting statistics
  - Example: double-dot interferometer

rate 1990’

nGF Landauer-Buttiker

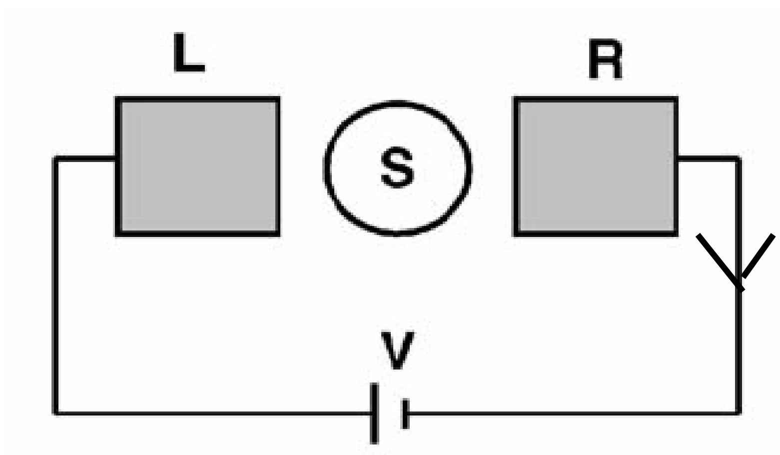
rate Gurvitz (1996), “ + ”
Schrodinger

“ ” “ ”

1 Jauho
quantum shuttle 2

$$\dot{\rho} = -i[H_S, \rho] - R\rho$$

X.Q. Li *et al:*
PRL 94, 066803 (2005)
PRB 71, 205304 (2005)
PRB 76, 085325 (2007)



$$\dot{\rho} = -i[H_S, \rho] - R\rho$$

$$\dot{\rho}^{(n)} = -i[H_S, \rho^{(n)}] - [R_1\rho^{(n)} + R_2\rho^{(n+1)} + R_3\rho^{(n-1)}]$$

Current

$$I(t) = e\frac{dN(t)}{dt} = e\sum_n n\,\mathrm{Tr}[\dot{\rho}^{(n)}(t)]$$

$$= \frac{e}{2}\mathrm{Tr}[\bar{Q}\rho Q + \mathrm{H.c.}],$$

where $\bar{Q} \equiv \tilde{Q}^{(-)} - \tilde{Q}^{(+)}$

Noise spectrum: MacDonald's formula

$$S_I(\omega) = 2\omega\int_0^\infty dt\sin\omega t\frac{d}{dt}\left[e^2\langle n^2(t)\rangle - (\bar{I}t)^2\right]$$

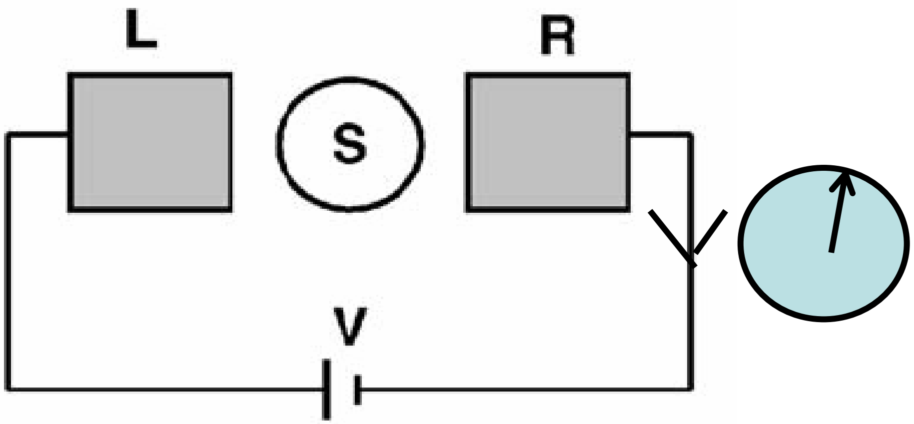
$$\langle n^2(t)\rangle = \sum_n n^2 P(n,t)$$

$$\frac{d}{dt}\langle n^2(t)\rangle = \mathrm{Tr}\left[\bar{Q}\hat{N}(t)Q + \frac{1}{2}\tilde{Q}\rho(t)Q + \mathrm{H.c.}\right]$$

$$\hat{N}(t) \equiv \sum_n n\rho^{(n)}(t) \qquad \frac{d\hat{N}}{dt} = -i\mathcal{L}\hat{N} - \frac{1}{2}\mathcal{R}\hat{N} + \frac{1}{2}(\bar{Q}\rho Q + \mathrm{H.c.})$$

Full counting statistics

Levitov, Lee, & Lesovik:
J. Math. Phys. 37, 4845(1996)



Generating function: $$e^{-F(\chi)} = \sum_n P(n,t) e^{in\chi}$$

$$C_k = -(-i\partial_\chi)^k F(\chi)\big|_{\chi=0}$$

$$C_1 = \bar{n}$$ $$I = eC_1/t$$ **Current**

$$C_2 = \overline{n^2} - \bar{n}^2$$ $$S = 2e^2 C_2/t$$ **Zero-frequency noise**

$$C_3 = \overline{(n-\bar{n})^3}$$ $$F = C_2/C_1$$ **Fano factor**

$$\dot{\rho}^{(n)} = A\rho^{(n)} + C\rho^{(n+1)} + D\rho^{(n-1)}$$

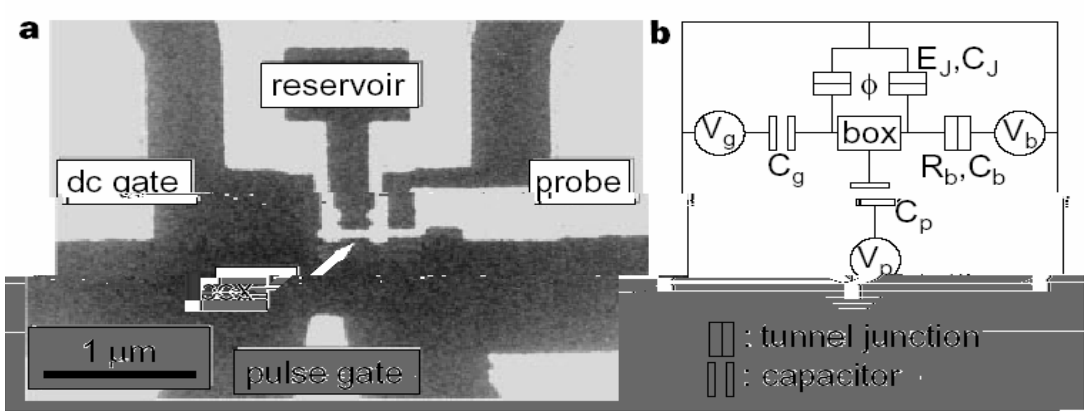
$$S(\chi, t) = \Sigma_n \rho^{(n)}(t) e^{in\chi}$$

$$e^{-F(\chi)} = \mathrm{Tr}[S(\chi, t)]$$

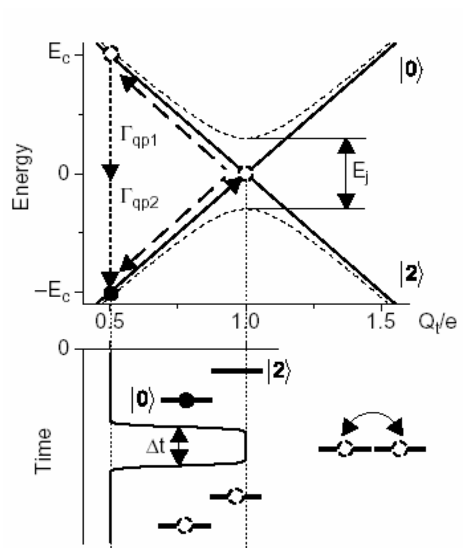
$$\dot{S} = AS + e^{-i\chi}CS + e^{i\chi}DS \equiv \mathcal{L}_\chi S$$

$$F(\chi) = -\lambda_1(\chi)t \qquad \lambda_1(\chi)\big|_{\chi\to 0} \to 0$$

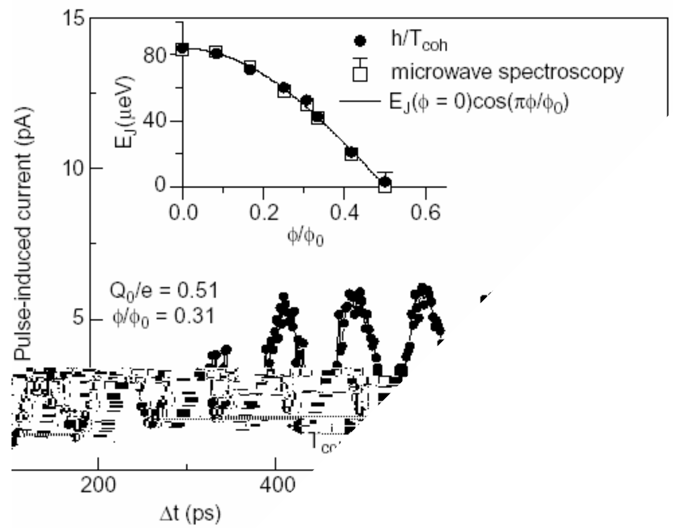
[redacted]



a
reservoir
dc gate
probe
1 μm
pulse gate
b
E_J, C_J
φ
V_g
box
V_b
C_g
R_b, C_b
C_p
V_p
: tunnel junction
: capacitor

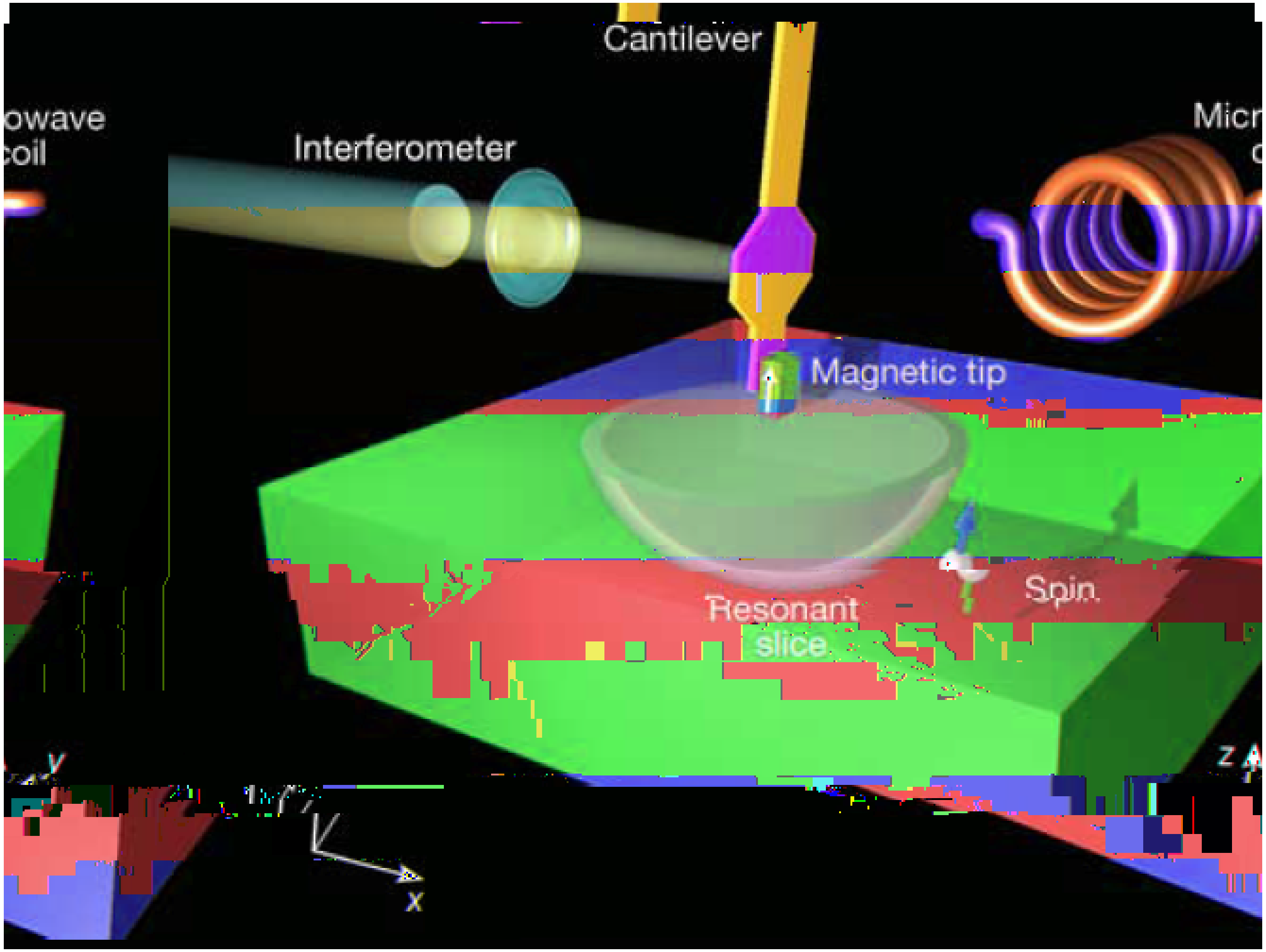


E_c
0
$-E_c$
Energy
Γ_{qp1}
Γ_{qp2}
E_J
|0⟩
|2⟩
0.5
1.0
1.5
Q_t/e
0
Time
Δt



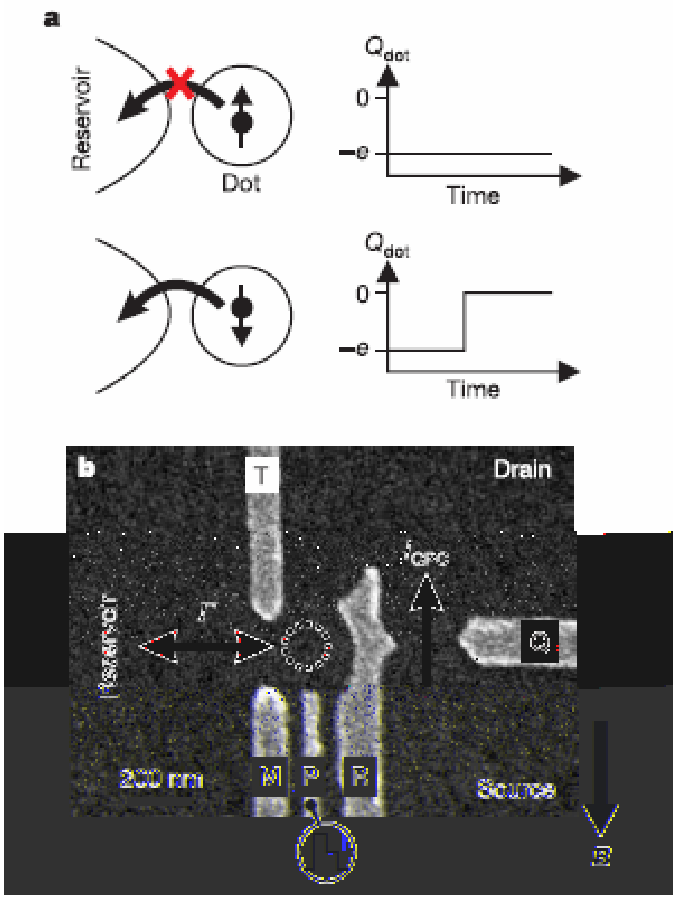
15
10
5
Pulse-induced current (pA)
h/T_{coh}
microwave spectroscopy
$E_J(\phi = 0)\cos(\pi\phi/\phi_0)$
E_J(μeV)
80
40
0
0.0
0.2
0.4
0.6
ϕ/ϕ_0
$Q_0/e = 0.51$
$\phi/\phi_0 = 0.31$
200
400
Δt (ps)

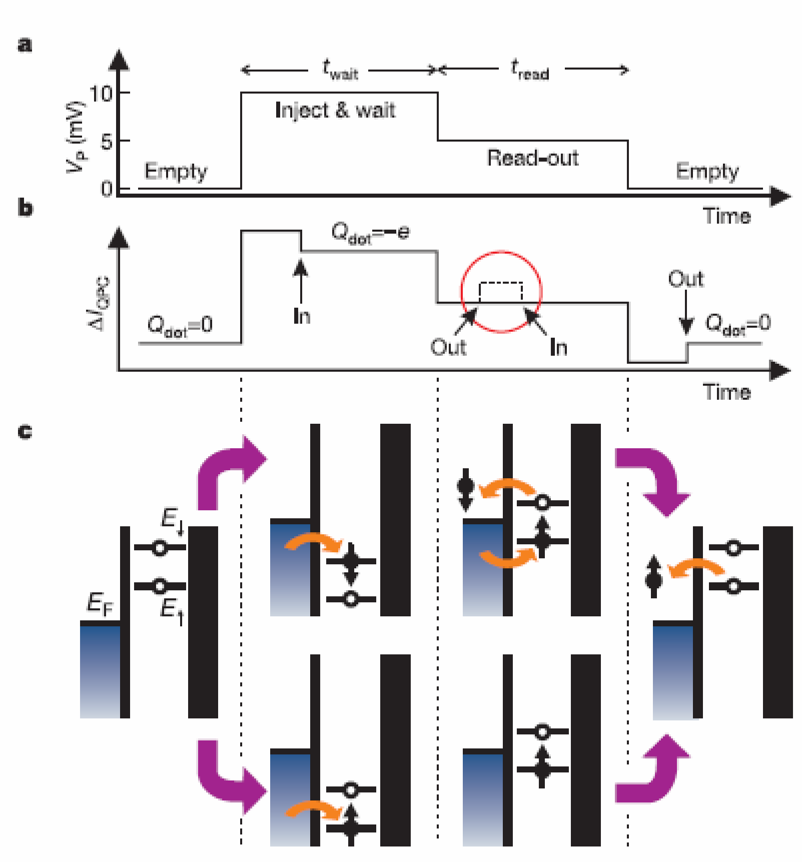
Nature **430**, 329 (2004)



QPC

L.P. Kouwenhoven, et al., Nature 430, 431 (2004)

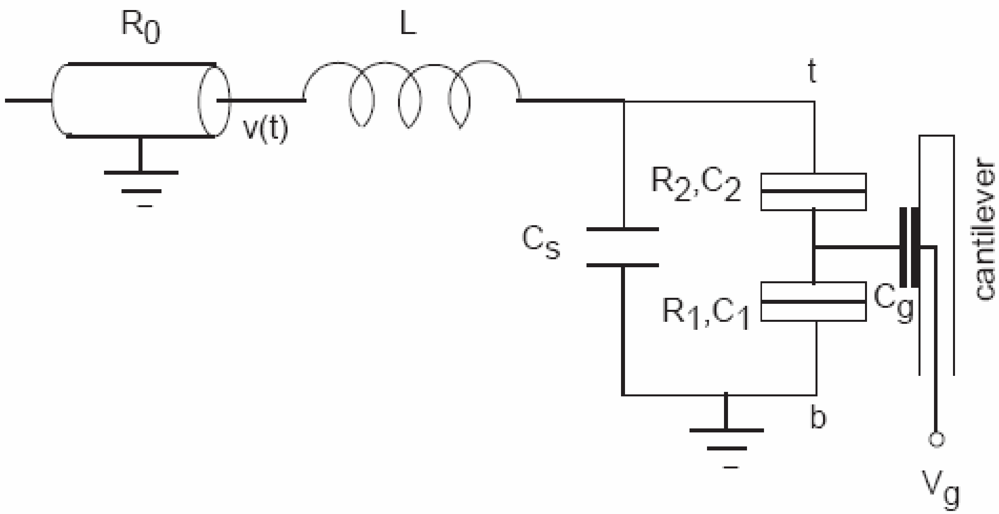




QPC

SET

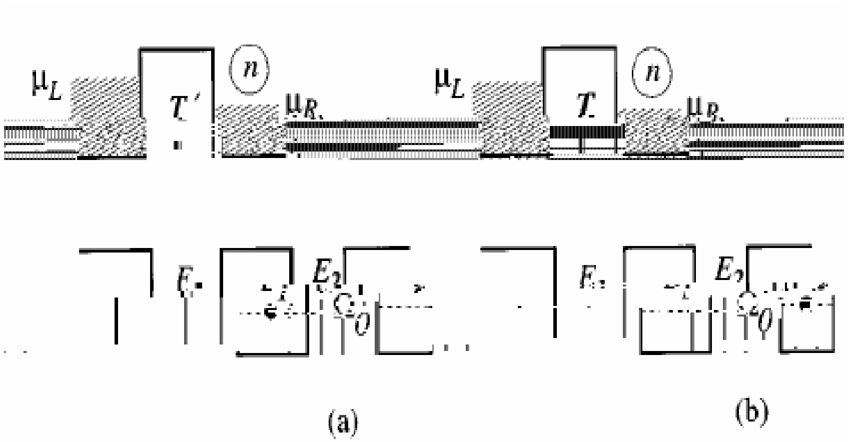
M. Blencowe / Physics Reports 395 (2004) 159–222



Circuit diagram of the rf-SET displacement detector.

QPC / SET

Gurvitz: PRB 56, 15215 (1997)



Non-trivial points:
- signal-to-noise ratio
- quantum efficiency of meas.
- quantum trajectory under meas.

Schoen: PRB (98); RMP(00); PRL(02)

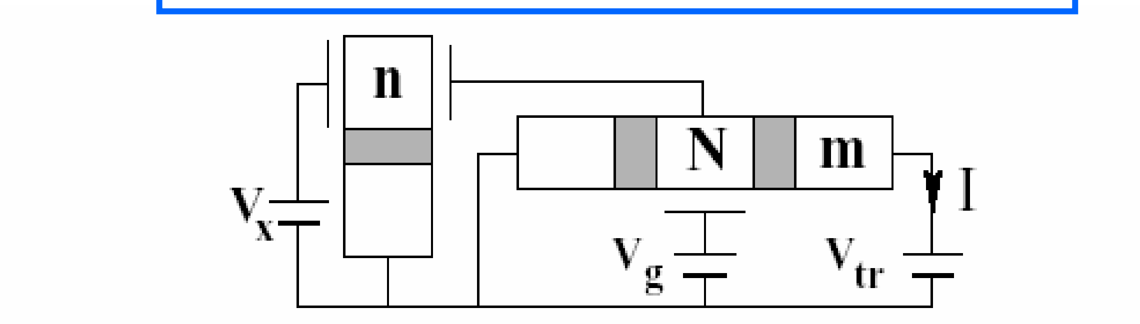


FIG. 1. The circuit of a qubit and a SET used as a meter.

1. Gurvitz *et al*: "n"-resolved Bloch Equation

PRB 56, 15215 (1997); PRL85, 812ˆ 2000)

PRL89,018301(2002); PRL91,066801 (2003)

2. Goan, Wiseman and Milburn: quantum trajectory

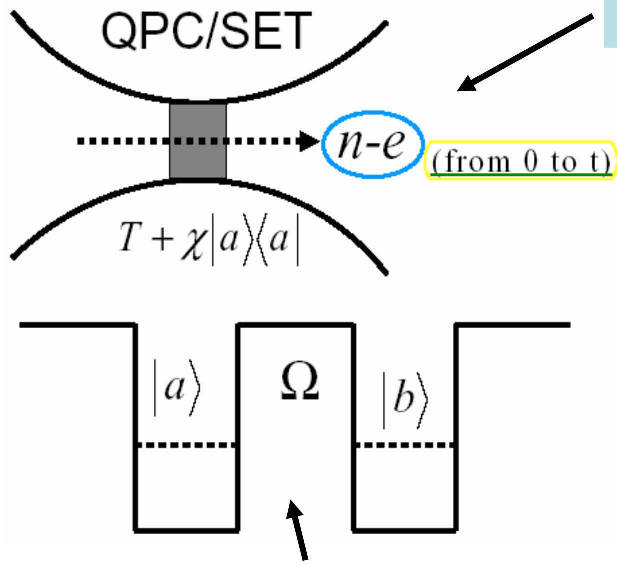
PRB 63,125326(2001); 64,235307(2001)

3. Korotkov, Averin *et al*: Bayesian Approach

PRB 60, 5737 (1999) PRB 63, 085312 (2001)

PRB 64, 165310 (2001) PRB 67, 075303 (2003)

Remarks: !

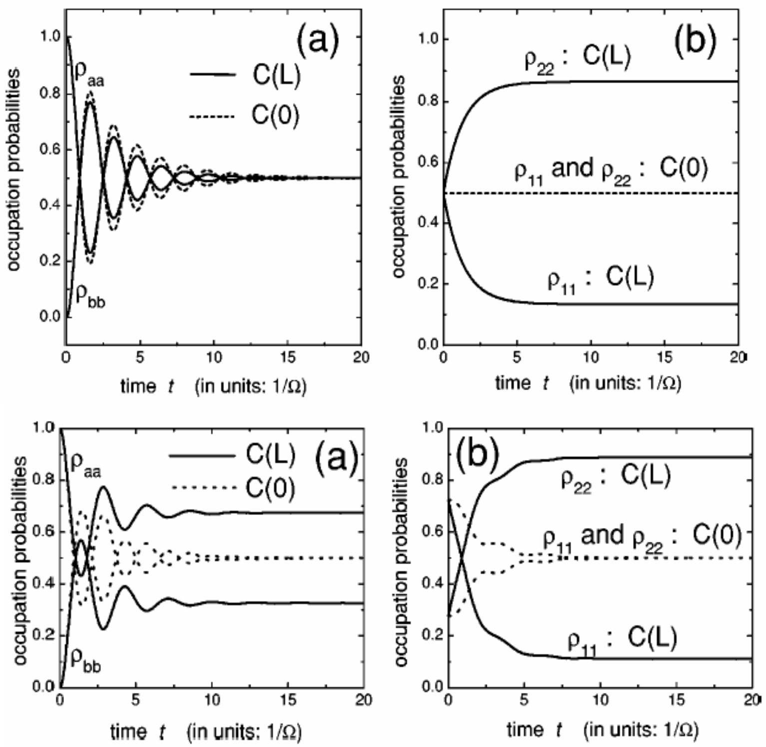


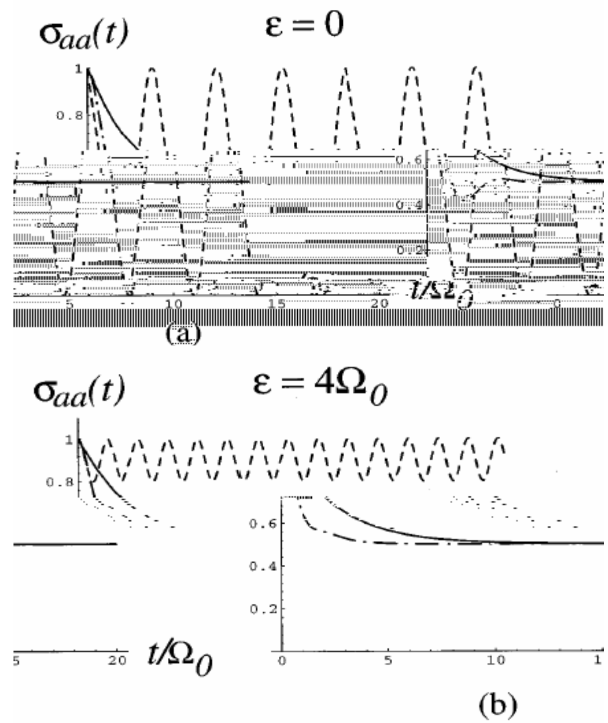
( )

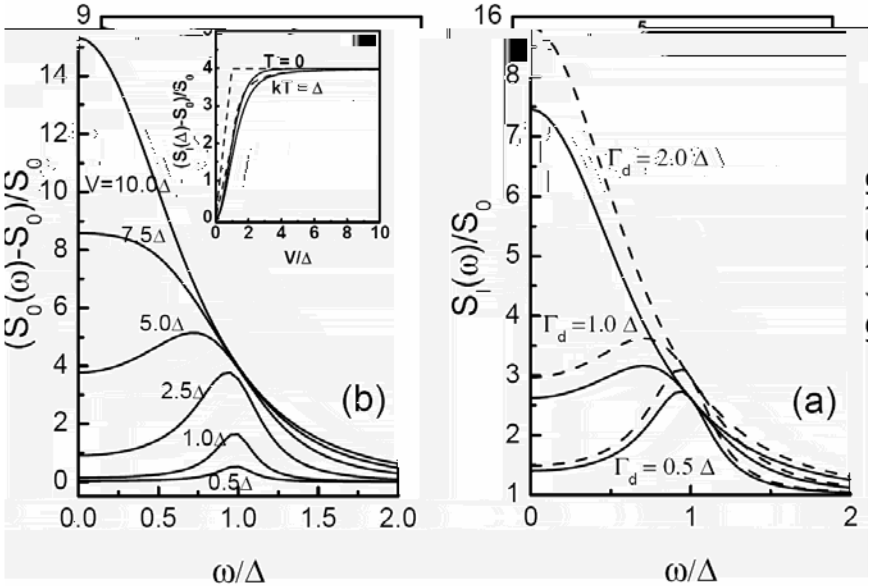
X.Q.Li *et al*, PRB<u>69</u>, 085315 (2004); PRL<u>94</u>, 066803 (2005).

$$\dot{\rho}^{(n)} = -i\mathcal{L}\rho^{(n)} - \frac{1}{2}\Big\{[Q\tilde{Q}\rho^{(n)} + \text{H.c.}] - [\tilde{Q}^{(-)}\rho^{(n-1)}Q + \text{H.c.}] - [\tilde{Q}^{(+)}\rho^{(n+1)}Q + \text{H.c.}]\Big\}$$

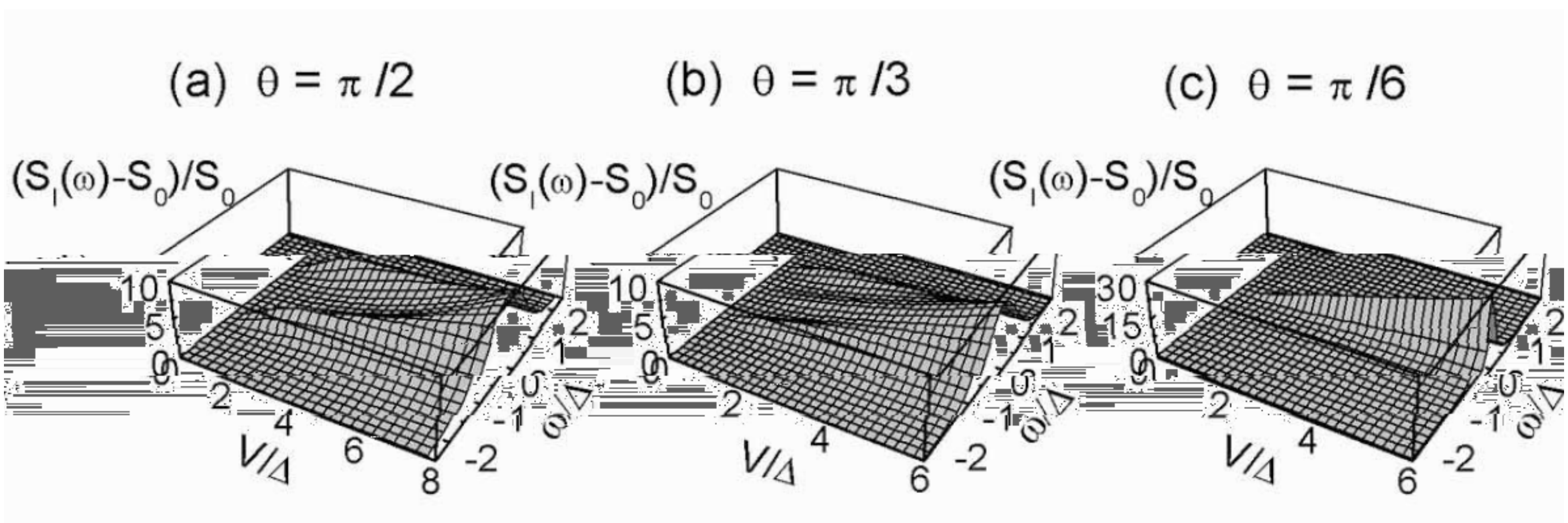
( )


(a)
(b)
ρ_{aa}
ρ_{bb}
C(L)
C(0)
ρ_{22} : C(L)
ρ_{11} and ρ_{22} : C(0)
ρ_{11} : C(L)
occupation probabilities
time t (in units: $1/\Omega$)


$\sigma_{aa}(t)$
$\varepsilon = 0$
t/Ω_0
(a)
$\sigma_{aa}(t)$
$\varepsilon = 4\Omega_0$
t/Ω_0
(b)



(S0(ω)-S0)/S0
V=10.0Δ
7.5Δ
5.0Δ
2.5Δ
1.0Δ
0.5Δ
(b)
ω/Δ
T = 0
kT = Δ
V/Δ
SI(ω)/S0
Γd = 2.0 Δ
Γd = 1.0 Δ
Γd = 0.5 Δ
(a)
ω/Δ



(a) θ = π /2
(b) θ = π /3
(c) θ = π /6
(SI(ω)-S0)/S0
(SI(ω)-S0)/S0
(SI(ω)-S0)/S0
V/Δ
V/Δ
V/Δ
ω/Δ
ω/Δ
ω/Δ

T. M. Stace and S. D. Barrett,
Phys. Rev. Lett. 92, 136802 (2004)

D.V. Averin and A. N. Korotkov,
Phys. Rev. Lett. 94, 069701 (2005);
T. M. Stace and S. D. Barrett,
Phys. Rev.Lett. 94, 069702 (2005).

Application to qubit measurements:

Xin-Qi Li *et al* : Quantum measurement of a solid-state qubit: A unified quantum master equation approach, Phys. Rev. B 69, 085315 (2004)

Xin-Qi Li *et al* : Spontaneous Relaxation of a Charge Qubit under Electrical Measurement, Phys. Rev. Lett. 94, 066803 (2005).

X.N. Hu *et al*: Quantum measurement of an electron in disordered potential, Phys. Rev. B 73, 035320 (2006).

J.S. Jin *et al*: Quantum coherence control of solid-state charge qubit by means of a sub-optimal feedback algorithm, PRB 73, 233302 (2006).

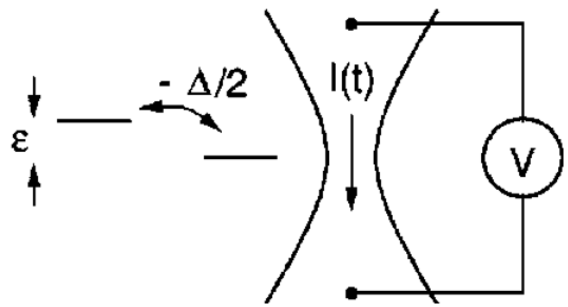
S.K. Wang *et al*: Continuous weak measurement and feedback control of a solid-state charge qubit: physical unravelling of non-Lindblad master equation, Phys. Rev. B 75, 155304 (2007).

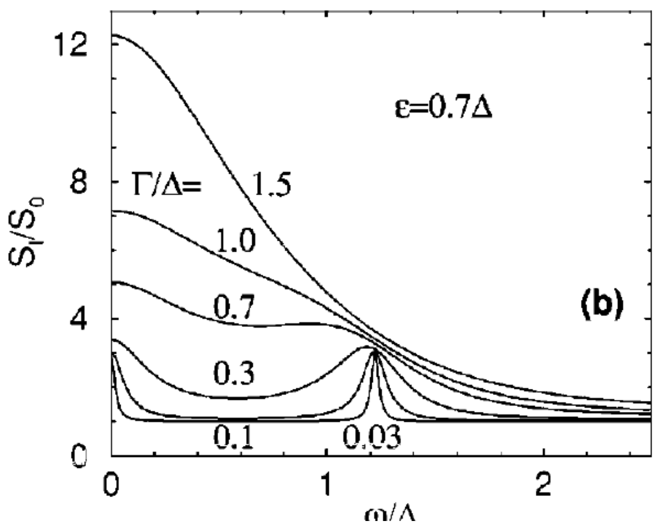
H.J. Jiao *et al*: Quantum measurement characteristics of double-dot single electron transistor, Phys. Rev. B 75, 155333 (2007).

PHYSICAL REVIEW B, VOLUME 64, 165310

Continuous weak measurement of quantum coherent oscillations

A. N. Korotkov and D. V. Averin
Department of Physics and Astronomy, SUNY Stony Brook, Stony Brook, New York 11794-3800
(Received 22 February 2001; revised manuscript received 8 May 2001; published 5 October 2001)





K-A bound:

1) It is shown that the interplay between the information acquisition and the backaction dephasing of the oscillations by the detector imposes a fundamental limit, equal to "4", on the signal-to-noise ratio of the measurement.

2) The limit is universal, e.g., independent of the coupling strength between the detector and system, and results from the tendency of quantum measurement to localize the system in one of the measured eigenstates.

$$\lambda = -2\,\mathrm{Im}\,S_{..}/\hbar$$

$$\ldots(\omega) + (\lambda^2/4)S_{zz}(\omega)$$

$$S_{zz}(\tau) = \langle \sigma_z(\tau)\sigma_z \rangle$$

$$S_I + \frac{\lambda^2\Gamma}{2}\frac{\Omega^2}{(\omega^2-\Omega^2)^2+\omega^2\Gamma^2}$$

$$S_{\max} = \lambda^2/(2\Gamma) = \hbar^2\lambda^2/S_Q$$

$$\lambda = -2\,\mathrm{Im}\,S_{QI}/\hbar$$

$$\hbar^2\lambda^2 = 4(\mathrm{Im}S_{QI})^2 \leq 4|S_{QI}|^2 \leq 4S_Q S_I$$

$$\mathcal{R} = S_{\max}/S_I \leq 4$$

The two conditions needed to reach the "Heisenberg efficiency":

$$(a)\ \ \mathrm{Re}S_{QI} = 0, \qquad (b)\ \ |S_{QI}|^2 = S_Q S_I.$$

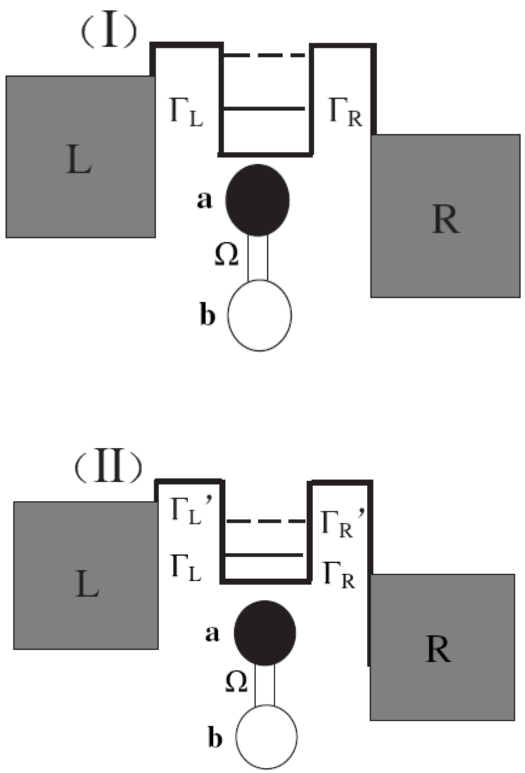
They indicate no lost information either through (a) phase or (b) energy averaging

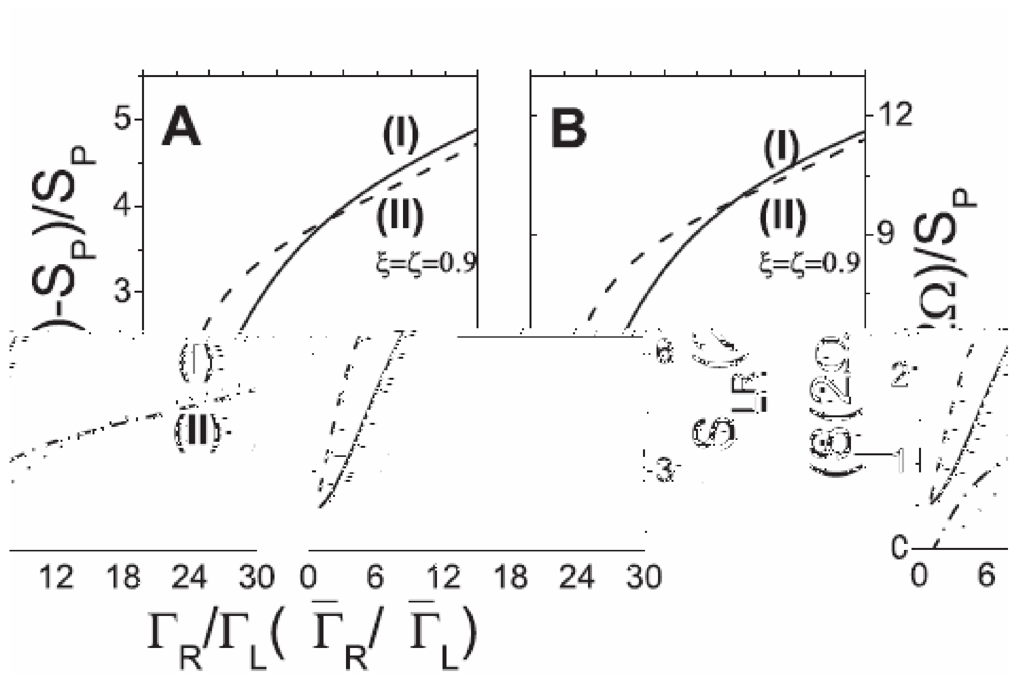
single-dot SET

SET SNR:

Signal-to-Noise Ratio , $SNR \leq 4$

H.J. Jiao *et al*, "Weak Measurement of Qubit Oscillations with Strong Response Detectors: Violation of the Fundamental Bound Imposed on Linear Detectors", Phys. Rev. B 79 075320 (2009)



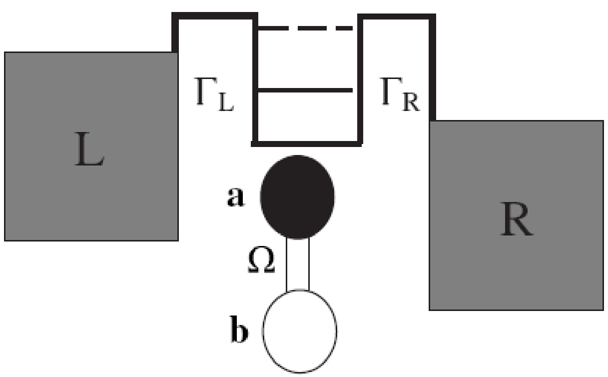


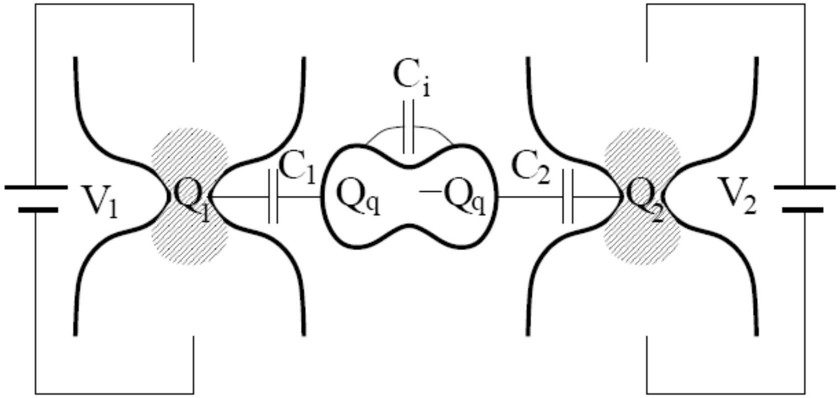
Understanding the violation of Korotkov-Averin bound

$$I(t) = aI_L(t) + bI_R(t)$$

$$\langle I(t)I(0)\rangle : \quad S_{LR}(t) = \langle I_L(t)I_R(0) + I_R(t)I_L(0)\rangle$$

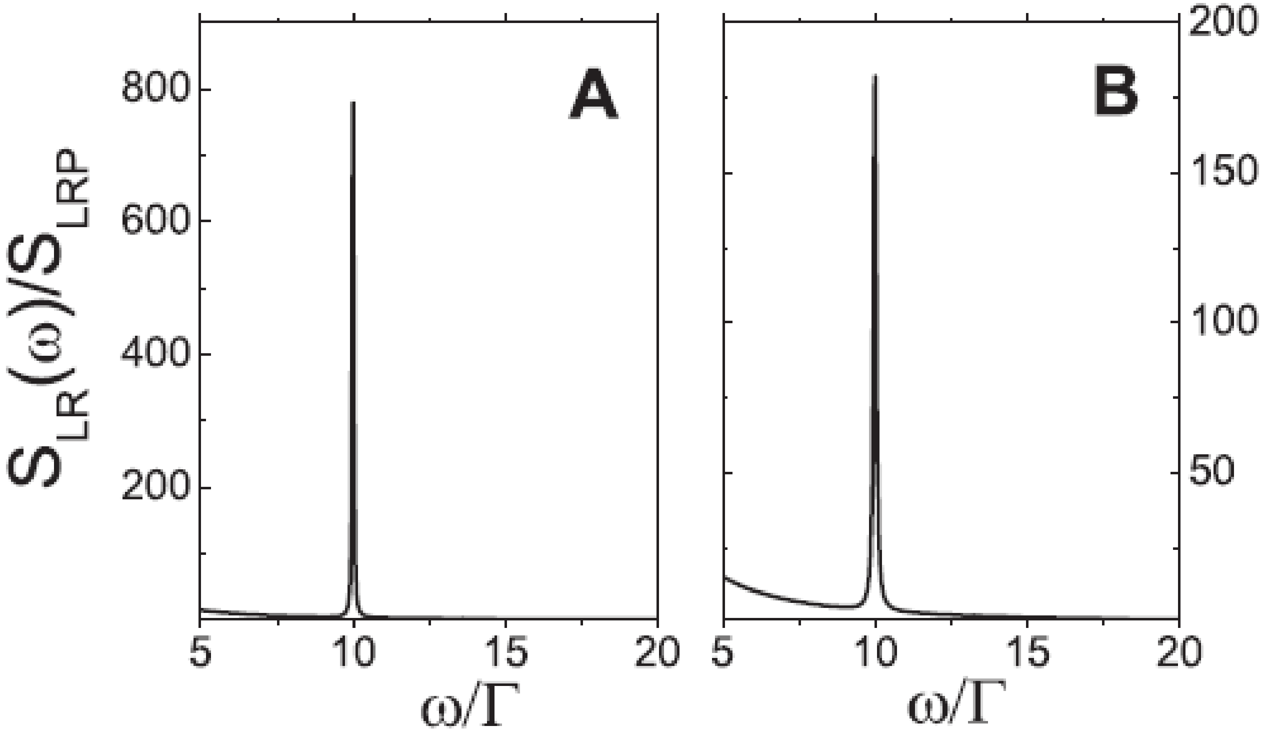
Cross correlation function





A. N. Jordan and M. Buttiker, Phys. Rev. Lett. 95, 220401 (2005).

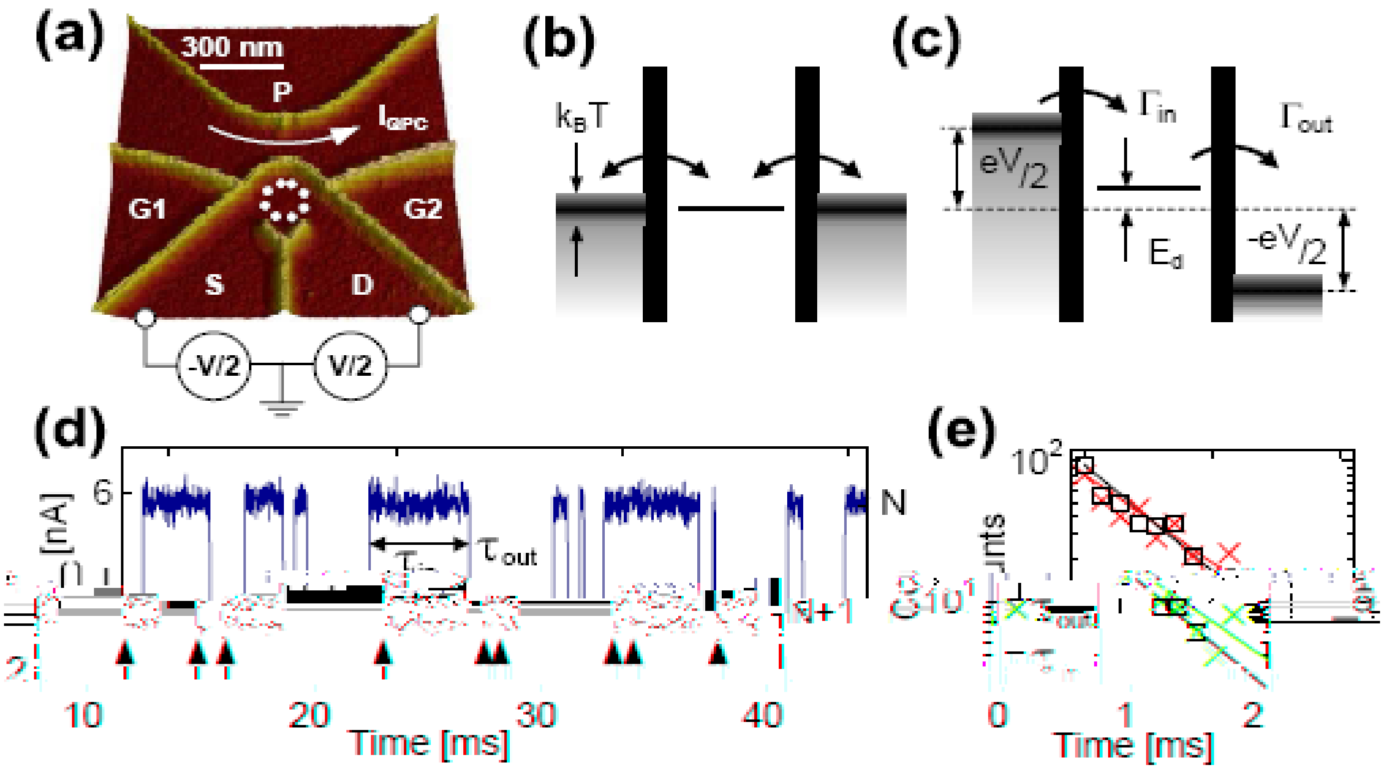
“ ”



A
B
$S_{LR}(\omega)/S_{LRP}$
800
600
400
200
200
150
100
50
5
10
15
20
ω/Γ

K. Ensslin & A.C. Gossard *etal,* PRL 96, 076605 (2006)

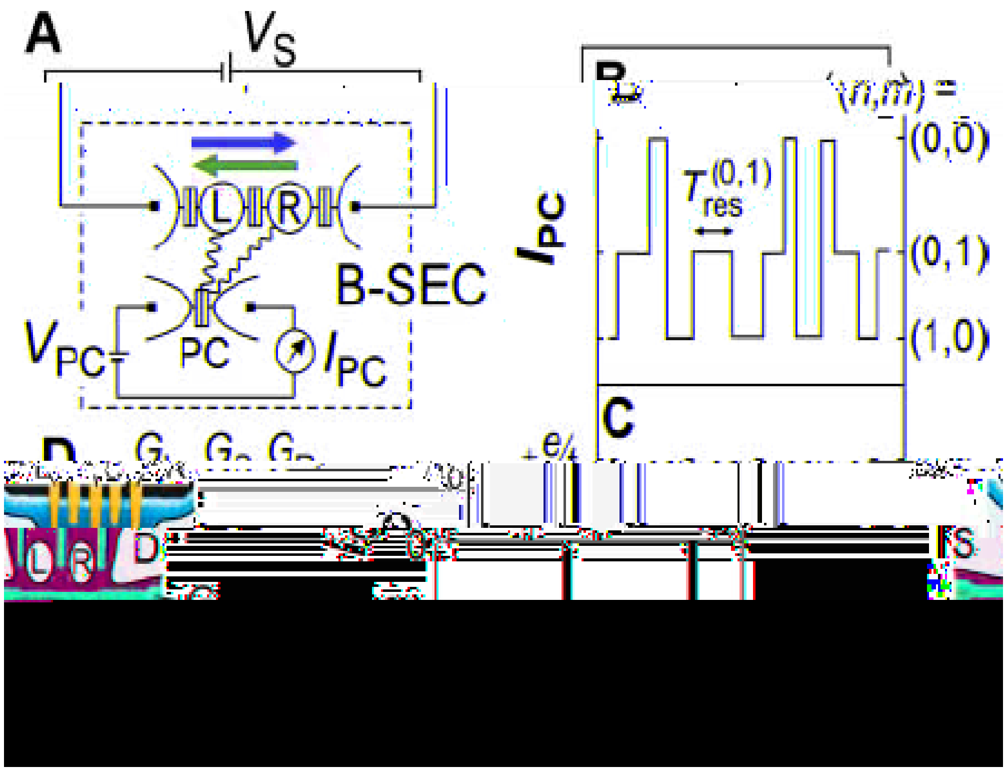
Counting Statistics of Single Electron Transport in a Quantum Dot

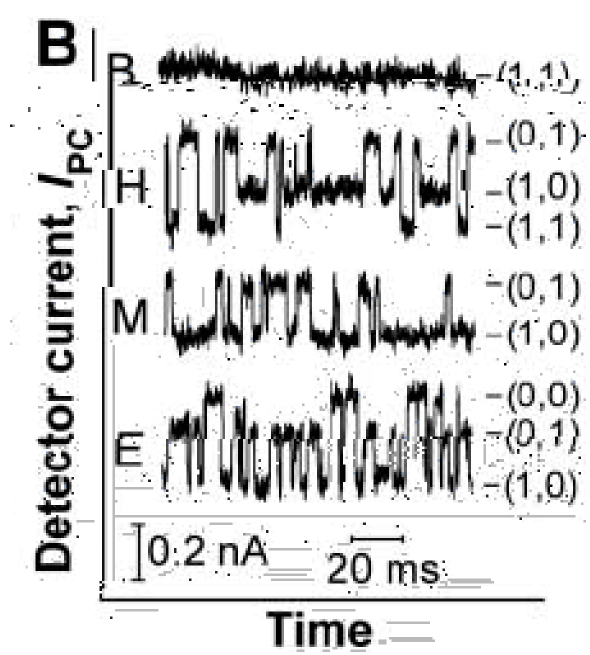


Science 310, 1634 (2006)

Bidirectional Counting of Single Electrons

Toshimasa Fujisawa,[1,2*] Toshiaki Hayashi,[1] Ritsuya Tomita

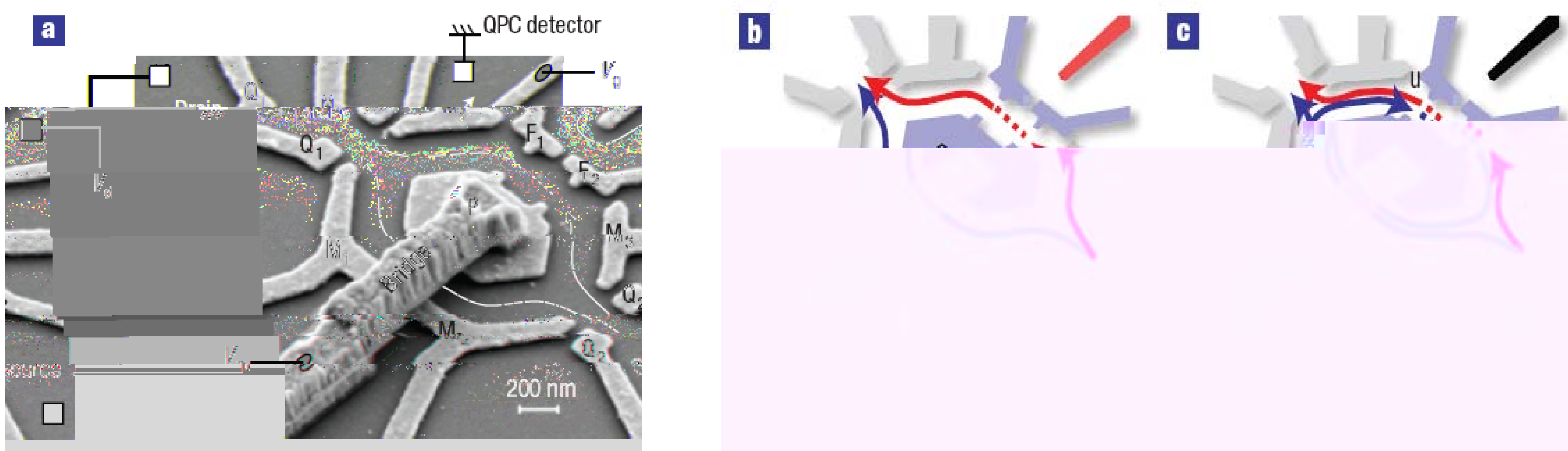




Quantum mechanical complementarity ... Bohm probed in a closed-loop Aharonov– interferometer

...ONG LEE[1,4*], DONG-IN CHANG[1], GYONG LUCK KHYM[1,2], KICHEON KANG[2], YUNCHUL CHUNG[3*], HU-J...
MINKY SEO[3], MOTY HEIBLUM[5], DIANA MAHALU[5] AND VLADIMIR UMANSKY[5]

[1]Department of Physics, Pohang University of Science and Technology, Pohang 790-784, Republic of Korea
[2]Department of Physics, Chonnam National University, Gwangju 500-757, Republic of Korea
[3]Department of Physics, Pusan National University, Busan 609-735, Republic of Korea
[4]National Center for Nanomaterials Technology, Pohang 790-784, Republic of Korea
[5]Braun Center for Submicron Research, Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel
*e-mail: ycchung@pusan.ac.kr; hjlee@postech.ac.kr



Application to quantum transport:

1˜ **Single and double QDs:** ***Coulomb staircase, noise spectrum***

X.Q. Li *et al*, PRB 71, 205304 (2005)

J. Y. Luo *et al*, PRB 76, 085325 (2007)

2˜ **QD coupled to FM electrodes:** ***spin-dependent current & fluctuations***

J. Y. Luo *et al*, J. Phys.:Condens.Matter 20, 345215 (2008)

3˜ **Transport through parallel quantum dots:** ***counting statistics, magnetic field switching of current, giant fluctuations of current, harmonic decomposition of the interference pattern***

S.K. Wang *et al*, PRB 76, 125416 (2007)

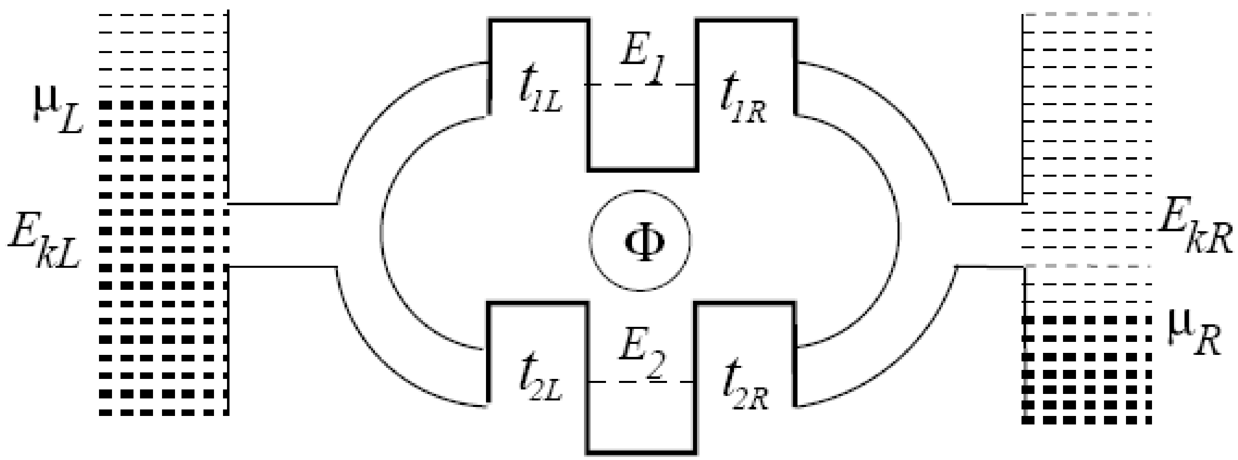
F. Li, X.Q.Li, W.M. Zhang, and S.A. Gurvitz: Europhys. Lett. 88, 37001(2009)

F. Li *et al*, Physica E 41, 521 (2009)

F. Li *et al*, Physica E 41, 1707(2009)

Quantum Transport through Parallel Quantum Dots

F. Li, X.Q.Li, W.M. Zhang, and S.A. Gurvitz: Europhys. Lett. 88, 37001(2009)



$$H = H_0 + H_T + \sum_{\mu=1,2} E_\mu d_\mu^\dagger d_\mu + U d_1^\dagger d_1 d_2^\dagger d_2$$

$$H_0 = \sum_k [E_{kL} a_{kL}^\dagger a_{kL} + E_{kR} a_{kR}^\dagger a_{kR}]$$

$$H_T = \sum_{\mu,k} \left(t_{\mu L} d_\mu^\dagger a_{kL} + t_{\mu R} a_{kR}^\dagger d_\mu \right) + H.c.$$

$$t_{\mu L(R)} = \bar{t}_{\mu L(R)} e^{i\phi_{\mu L(R)}}$$

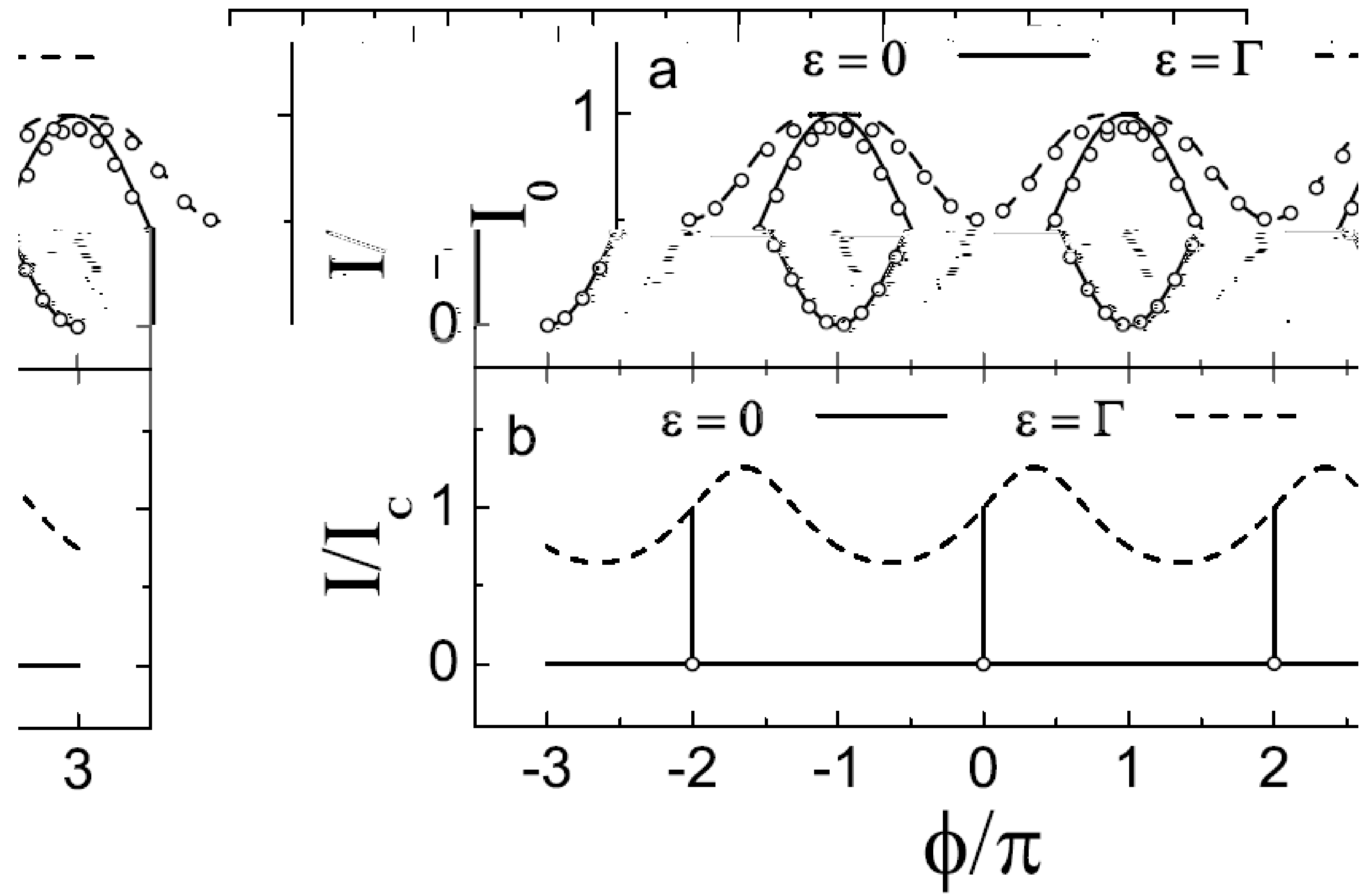
$$\phi_{1L} + \phi_{1R} - \phi_{2L} - \phi_{2R} = \phi$$

$$\phi \equiv 2\pi\Phi/\Phi_0$$

$$\dot{\sigma}_{00} = -2\Gamma_L \sigma_{00} + \Gamma_R(\sigma_{11} + \sigma_{22} + \bar{\sigma}_{12} + \bar{\sigma}_{21})$$
$$\dot{\sigma}_{11} = \Gamma_L \sigma_{00} - \Gamma_R \sigma_{11} - \Gamma_R(\bar{\sigma}_{12} + \bar{\sigma}_{21})/2$$
$$\dot{\sigma}_{22} = \Gamma_L \sigma_{00} - \Gamma_R \sigma_{22} - \Gamma_R(\bar{\sigma}_{12} + \bar{\sigma}_{21})/2$$
$$\dot{\bar{\sigma}}_{12} = e^{i\phi}\Gamma_L \sigma_{00} - \frac{\Gamma_R}{2}(\sigma_{11} + \sigma_{22}) - (i\epsilon + \Gamma_R)\bar{\sigma}_{12}$$

$$I(\phi) = I_C \frac{\epsilon^2}{\epsilon^2 + I_C\left(2\Gamma_R \sin^2\frac{\phi}{2} - \epsilon\sin\phi\right)}$$

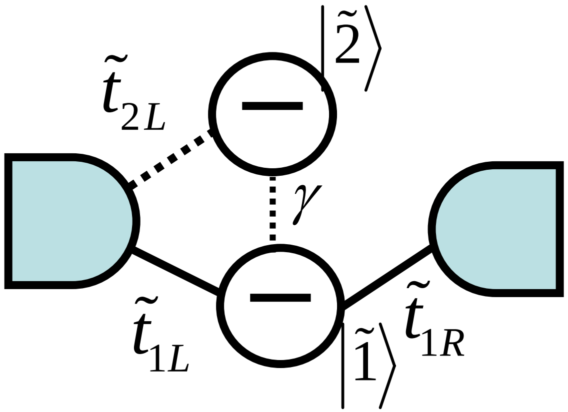
$$I_C = 2\Gamma_L\Gamma_R/(2\Gamma_L + \Gamma_R)$$


a
ε = 0
ε = Γ
I/I_0
1
0
b
ε = 0
ε = Γ
I/I_c
1
0
-3
-2
-1
0
1
2
3
φ/π

State basis transformation:

$$\begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \end{pmatrix} = \frac{1}{\mathcal{N}} \begin{pmatrix} t_{1R} & t_{2R} \\ -t_{2R}^* & t_{1R}^* \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$$

$$\mathcal{N} = (\bar{t}_{1R}^2 + \bar{t}_{2R}^2)^{1/2}$$

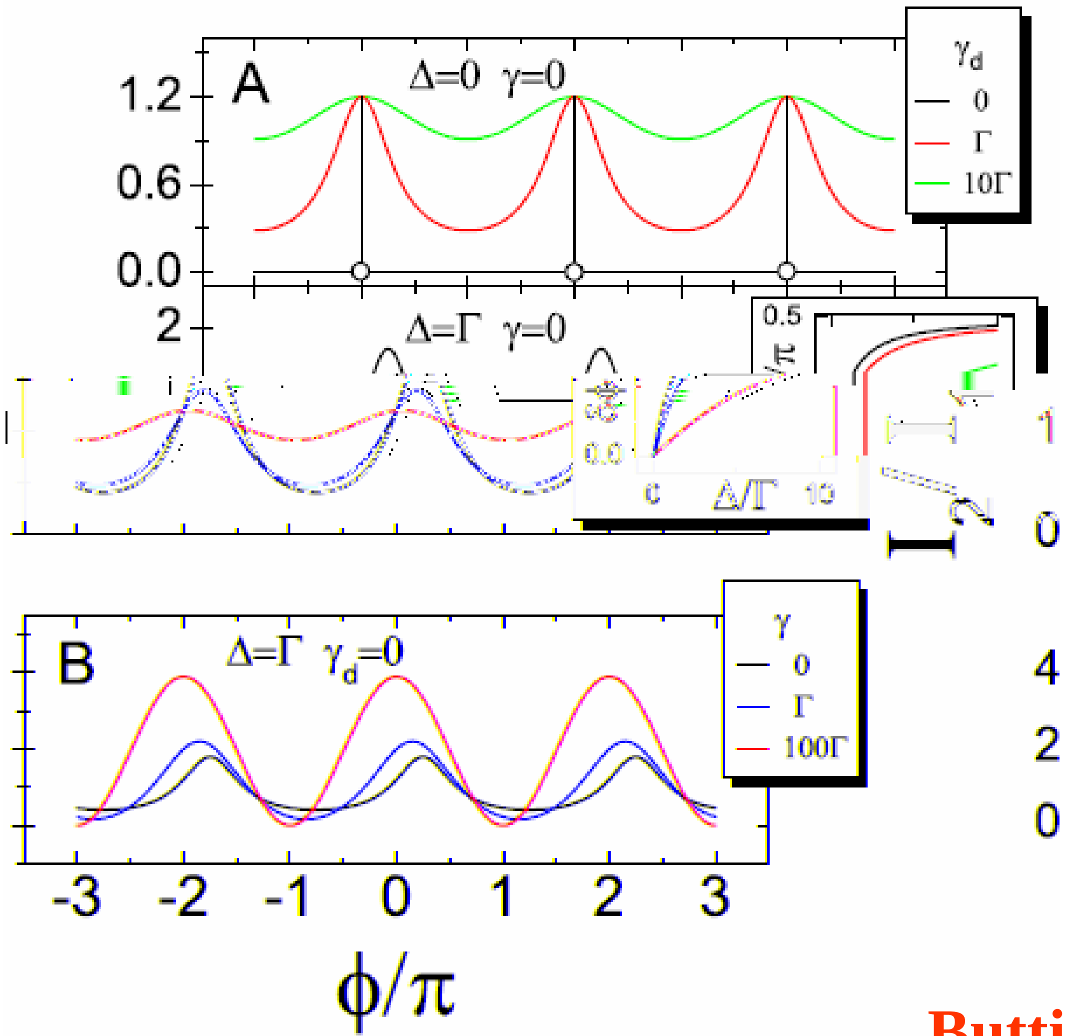


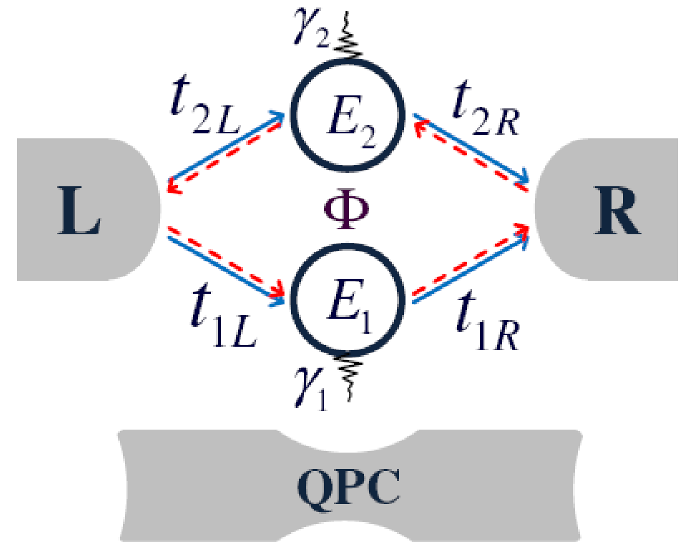
$$\gamma = \varepsilon \bar{t}_{1R} \bar{t}_{2R} / (\bar{t}_{1R}^2 + \bar{t}_{2R}^2)$$

(I) $\widetilde{t}_{2R} = 0$

$$\widetilde{t}_{2L}(\phi) = -e^{i(\phi_{2L} - \phi_{1R})} (\bar{t}_{1L} \bar{t}_{2R}\, e^{i\phi} - \bar{t}_{2L} \bar{t}_{1R}) / \mathcal{N}$$

(II) $\widetilde{t}_{2L} = 0$ for $\phi = 2n\pi$
provided that $\bar{t}_{1L} / \bar{t}_{2L} = \bar{t}_{1R} / \bar{t}_{2R}$





Visibility, dephasing

Close *vs* open …

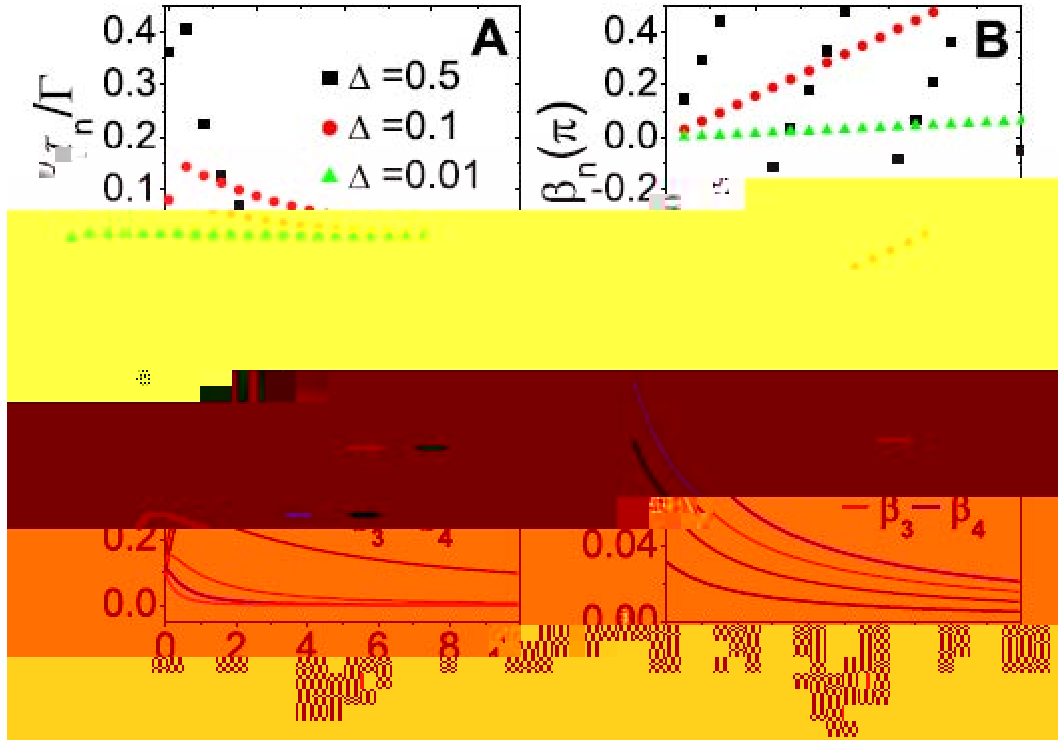
Magnetic asymmetry

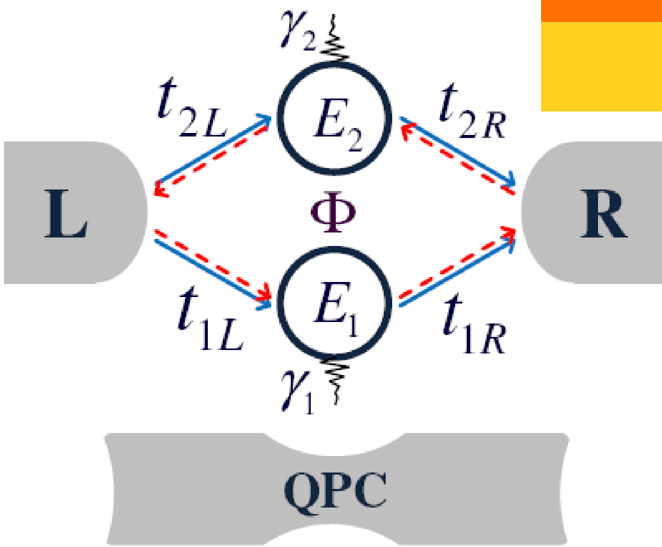
Buttiker, PRL 57, 1761 (1986)

Based on current conservation and time-reversal invariance, the Onsager relation, say, the symmetry relation of transport coefficients under inversion of magnetic feld, will lock the current peaks at …, for *any two-terminal linear transport*.

Harmonic Analysis

$$I(\phi) = I_0 + \sum_{n=1}^{\infty} I_n \cos(n\phi + \beta_n)$$





(1) $\beta_n = n\beta_1$

(2)

1st partial wave $\propto t_2 e^{i\chi_2}$

2nd partial wave $\propto t_1 e^{i\chi_1} t_2^{*} e^{i\tilde{\chi}_2} t_1 e^{i\tilde{\chi}_1}$

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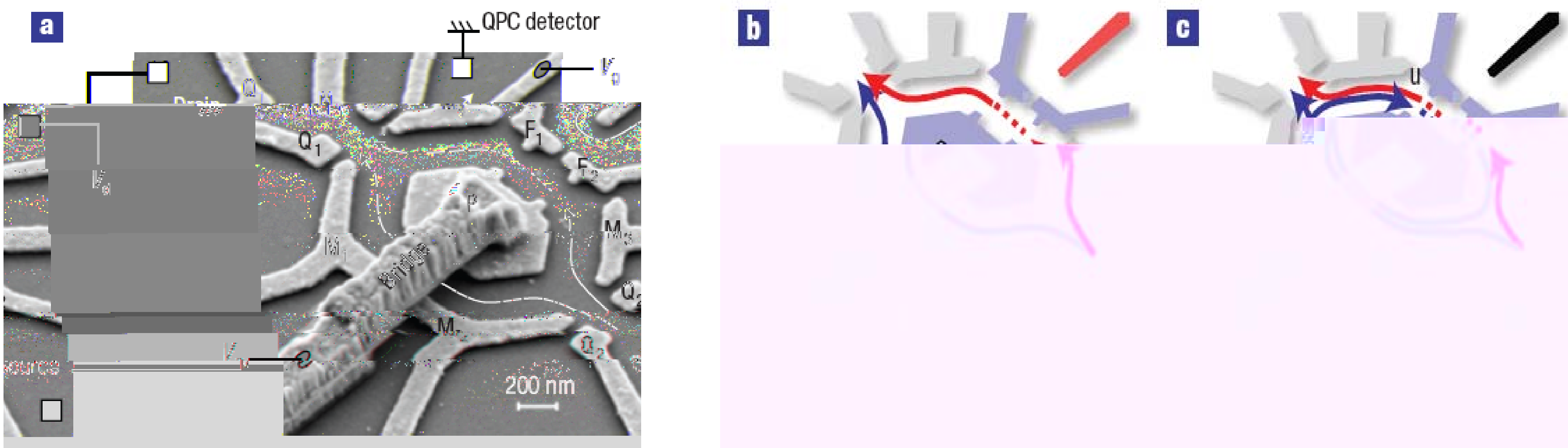
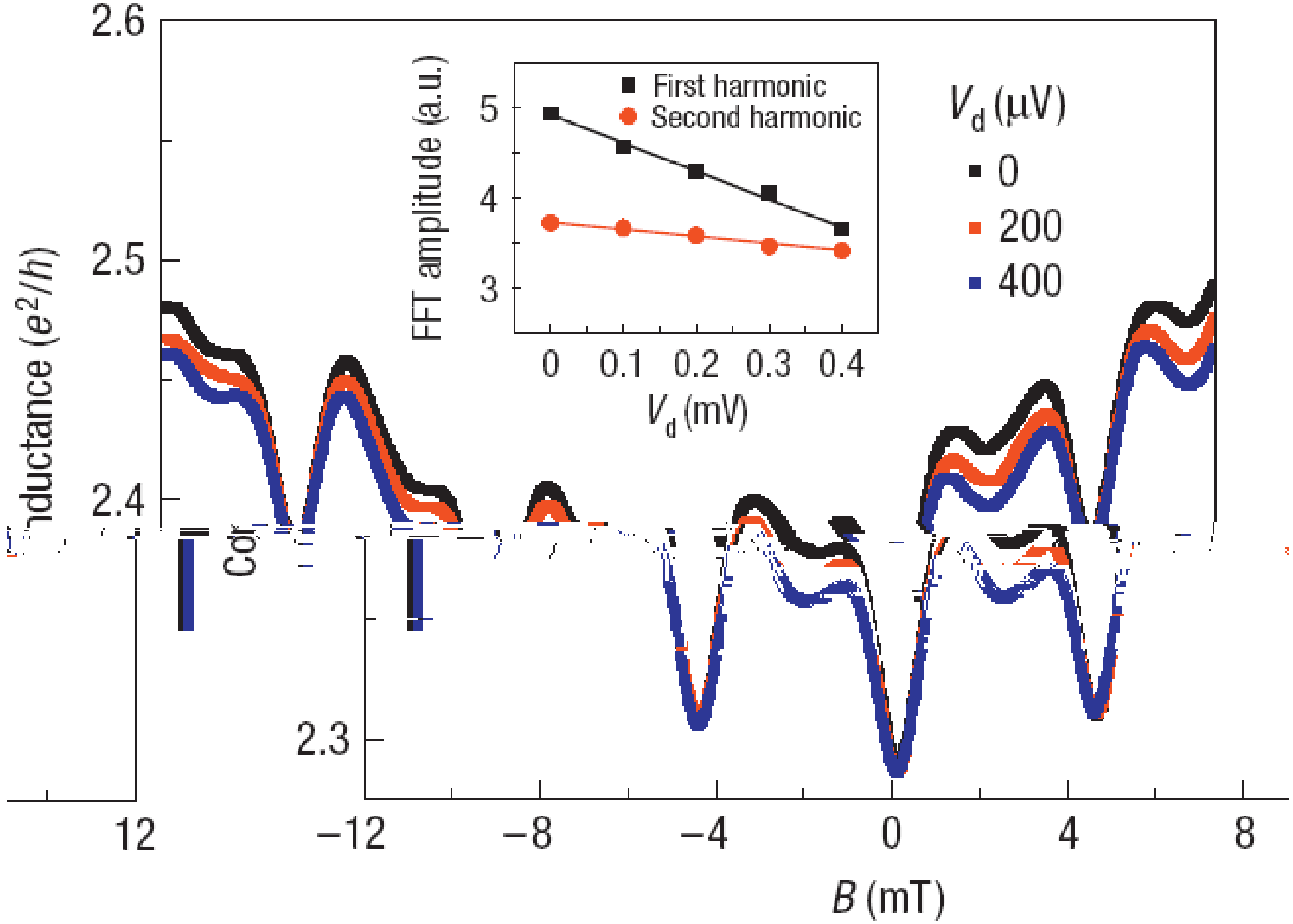
Quantum mechanical complementarity ... Bohm probed in a closed-loop Aharonov–Bohm interferometer

...ONG LEE[1,4*], DONG-IN CHANG[1], GYONG LUCK KHYM[1,2], KICHEON KANG[2], YUNCHUL CHUNG[3*], HU-J... MINKY SEO[3], MOTY HEIBLUM[5], DIANA MAHALU[5] AND VLADIMIR UMANSKY[4,5]

[1]Department of Physics, Pohang University of Science and Technology, Pohang 790-784, Republic of Korea
[2]Department of Physics, Chonnam National University, Gwangju 500-757, Republic of Korea
[3]Department of Physics, Pusan National University, Busan 609-735, Republic of Korea
[4]National Center for Nanomaterials Technology, Pohang 790-784, Republic of Korea
[5]Braun Center for Submicron Research, Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel
*e-mail: ycchung@pusan.ac.kr; hjlee@postech.ac.kr





2.6
2.5
2.4
2.3
onductance (e^2/h)
Co
FFT amplitude (a.u.)
5
4
3
0
0.1
0.2
0.3
0.4
V_d (mV)
First harmonic
Second harmonic
V_d (μV)
0
200
400
12
−12
−8
−4
0
4
8
B (mT)

Strongly super-Poissonian fluctuations, giant Fano factor

$$S_R(\omega) = \frac{8\Gamma_L\Gamma_R[2\Gamma_L\Gamma_R\Delta^2 - \Delta^4 + 3\Delta^2\omega^2 - 2\omega^2(\Gamma_R^2+\omega^2)]\bar{I}}{[(2\Gamma_L+\Gamma_R)\Delta^2 - (2\Gamma_L+3\Gamma_R)\omega^2]^2 + \omega^2(2\Gamma_L\Gamma_R + 2\Gamma_R^2 + \Delta^2 - \omega^2)^2} + 2\bar{I}$$

Limiting order:

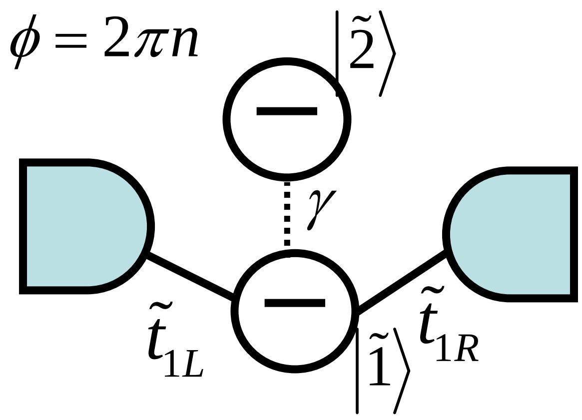
(i) First $\omega \to 0$ then $\Delta \to 0$

$$F \equiv \frac{S_R(0)}{2\bar{I}} = \frac{8\Gamma_L^2\Gamma_R^2 + (4\Gamma_L^2+\Gamma_R^2)\Delta^2}{(2\Gamma_L+\Gamma_R)^2\Delta^2}$$

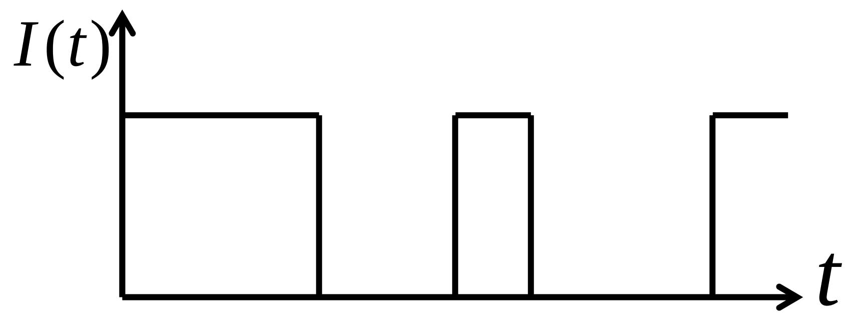
Divergent !!

(ii) First $\Delta \to 0$ then $\omega \to 0$

$$F = \frac{\Gamma_L^2 + \Gamma_R^2}{(\Gamma_L+\Gamma_R)^2}$$



$\gamma \propto \Delta = |E_1 - E_2|$



Enhanced Shot Noise in Tunneling through a Stack of Coupled Quantum Dots

P. Barthold,[1,*] F. Hohls,[1,2] N. Maire,[1] K. Pierz,[3] and R. J. Haug[1]

[1]Institut für Festkörperphysik, Universität Hannover, Appelstrasse 2, D-30167 Hannover, Germany
[2]Cavendish Laboratory, University of Cambridge, Madingley Road, Cambridge CB3 0HE, United Kingdom
[3]Physikalisch-Technische Bundesanstalt, Bundesallee 100, D-38116 Braunschweig, Germany

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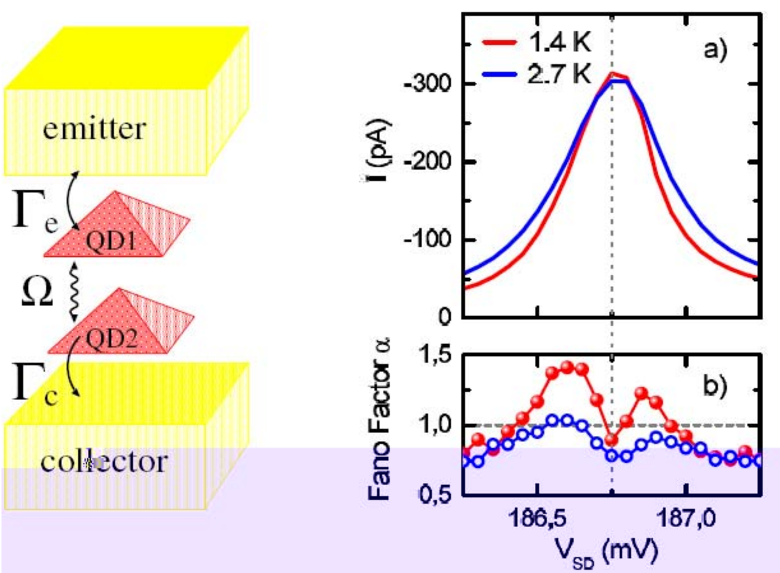
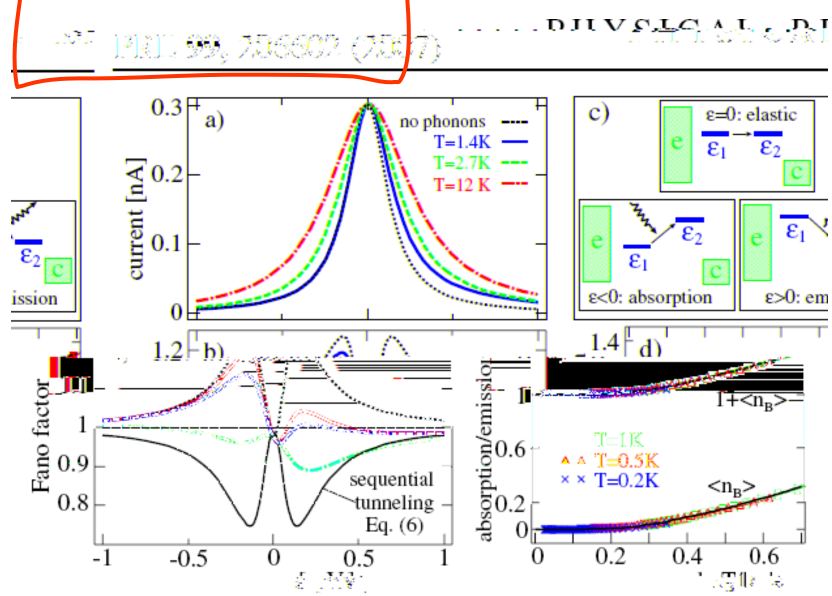
We have investigated the noise properties of the tunneling current through vertically coupled self-assembled InAs quantum dots. We observe super-Poissonian shot noise at low temperatures. For increased temperature this effect is suppressed. The super-Poissonian noise is explained by capacitive coupling between different stacks of quantum dots.





Summary

- **Master equation approach to quantum transport**

- **Quantum measurement of solid-state qubit**
  - **Example: SET detector, signal-to-noise ratio, etc**

- **Quantum transport**
  - **Current fluctuation, counting statistics**
  - **Example: double-dot interferometer**